3D numerical simulation of tranport and turbulence in the edge plasma

P. Tamain[1], Ph. Ghendrih[1], E. Tsitrone[1], V. Grandgirard[1], Y. Sarazin[1], X. Garbet[1]

TORE SUPRA

Association EURATOM-CEA

CEA / DSM / Département de **R**echerches sur la **F**usion **C**ontrôlée

CEA-Cadarache, 13108 ST PAUL-LEZ-DURANCE (France)

Abstract

Plasma edge turbulence is a key issue for fusion devices since it determines plasma profiles through turbulent fluxes as well as transport barriers.

We introduce a global 3D fluid code for Transport and Turbulence (T&T) in the edge plasma, including both the Scrape-Off-Layer (SOL) and the outermost closed field lines. Electrostatic two-fluids flux-driven balance equations are solved in a simple geometry, but this choice does not affect the main features of Edge-SOL physic. A detailed description of the model and of the numerical scheme is given.

First simulation results are presented, focusing on validation cases. Conservation laws as well as linear growth rates are checked. Eventually, we discuss the physical issues that are within the scope of such a 3D global code.

Edge T&T issues

- Open questions linked to transport and/or turbulence in the edge plasma :
  - turbulence driving mechanisms : ITG, interchange, Kelvin-Helmholtz ?
  - turbulence spreading : SOL → Core, Core → SOL ?
  - transport barriers formation (LH transition) : shear flows, interplay with turbulence
  - validity of the flute hypothesis :parallel/perpendicular transport interplay
  - large scale flows : equilibrium flow distribution, SOL flows asymmetry
  - matter deposition by particle injection
  - …

• Existing codes : Uedge, Edge2D, B2, EMC3, GLF23, Tokam, ETAI3D, BOUT…

Global 3D transport and turbulence code across Last Closed Flux Surface (LCFS)

3D edge T&T model

- Main hypotheses :
  - fluid description : $\omega \ll \omega_{coll}$
  - electrostatic turbulence : $\beta \ll 1$
  - quasi-neutrality : $n_e = Z n_i$
  - Braginskii closure : $\omega_{ci} \gg \omega_{coll}$

- Orderings :

$$\partial_t \sim \omega \ll \omega_{ci} \qquad \nabla n_e \sim \nabla \tilde{n}_e \Rightarrow \frac{\tilde{n}_e}{n_e} \sim \frac{L_{turb}}{L_\perp} \ll 1$$

- First version isothermal : $T_e = T_i = T_0$

- Drift speeds :
  - electric drift : $\vec{u}_E = \vec{v}_E = \dfrac{\vec{B} \wedge \vec{\nabla}\Phi}{B^2}$
  - diamagnetic drift : $\vec{u}_* = \dfrac{\vec{B} \wedge \vec{\nabla} p_i}{Z e n_i B^2}$ $\qquad \vec{v}_* = -\dfrac{\vec{B} \wedge \vec{\nabla} p_e}{e n_e B^2}$

    does not contribute to advection (diamagnetic cancellation)

  - polarization drift (only for ions) :

$$\vec{u}_p = \frac{m_i}{ZeB^2} \vec{B} \wedge \left[\partial_t + \left(u_{\|} \vec{b} + \vec{u}_E \right) . \vec{\nabla} \right] \left(\vec{u}_E + \vec{u}_* \right)$$

$|\vec{u}_p| \ll |\vec{u}_E| , |\vec{u}_*|$ but $\vec{\nabla} . \vec{j}_* \sim \dfrac{\vec{j}_*}{L_B} \sim \dfrac{\vec{j}_p}{L_{turb}} \sim \vec{\nabla} . \vec{j}_p$

- Balance equations for :
  - matter

$$\partial_t N_e + B\nabla_\parallel \frac{N_e M}{B} - B\nabla_\parallel \frac{J_\parallel}{B} + \frac{1}{B}\left[\Phi, N_e\right] =$$
$$-BN_e\left[\Phi, \frac{1}{B^2}\right] + B\left[N_e, \frac{1}{B^2}\right] + S_{N_e} + \vec{\nabla}_\perp . \left(D_{\perp N_e}\vec{\nabla}_\perp N_e\right)$$

  - charge

$$\partial_t W + M\nabla_\parallel W + \frac{1}{B}\left[\Phi, W\right] =$$
$$(1+Z)\frac{B^3}{N_e}\left[N_e, \frac{1}{B^2}\right] + Z\frac{B^3}{N_e}\nabla_\parallel \frac{J_\parallel}{B} + \vec{\nabla}_\perp . \left(D_{\perp W}\vec{\nabla}_\perp W\right)$$

with the vorticity $W = \nabla_\perp^2 \Phi + \frac{1}{Z}\nabla_\perp^2 \ln N_e$

– parallel momentum

$$\partial_t M + M\nabla_{\|} M + \frac{1}{B}[\Phi, M] =$$
$$-(1+Z)\nabla_{\|} \ln N_e + \frac{Z}{N_e}\vec{\nabla}.\left(D_M \vec{\nabla} M\right) + \frac{Z}{N_e} S_M$$

– parallel current (Ohm's law)

$$\nabla_{\|} \ln N_e - \nabla_{\|}\Phi - \frac{J_{\|}}{\sigma_{\|}} = 0$$

• Adimensionned quantities :

$$t \rightarrow \frac{t}{\omega_{ci}} \qquad \omega_{ci} = \frac{eB_0}{m_i} \qquad l \rightarrow \rho_L l \qquad \rho_L = \frac{\sqrt{m_i T_0}}{eB_0}$$

$$\Phi \rightarrow \frac{T_0}{e}\Phi \qquad u_{\|} \rightarrow c_s M = \sqrt{\frac{T_0}{m_i}} M \qquad j_{\|} \rightarrow n_0 e c_s J_{\|}$$

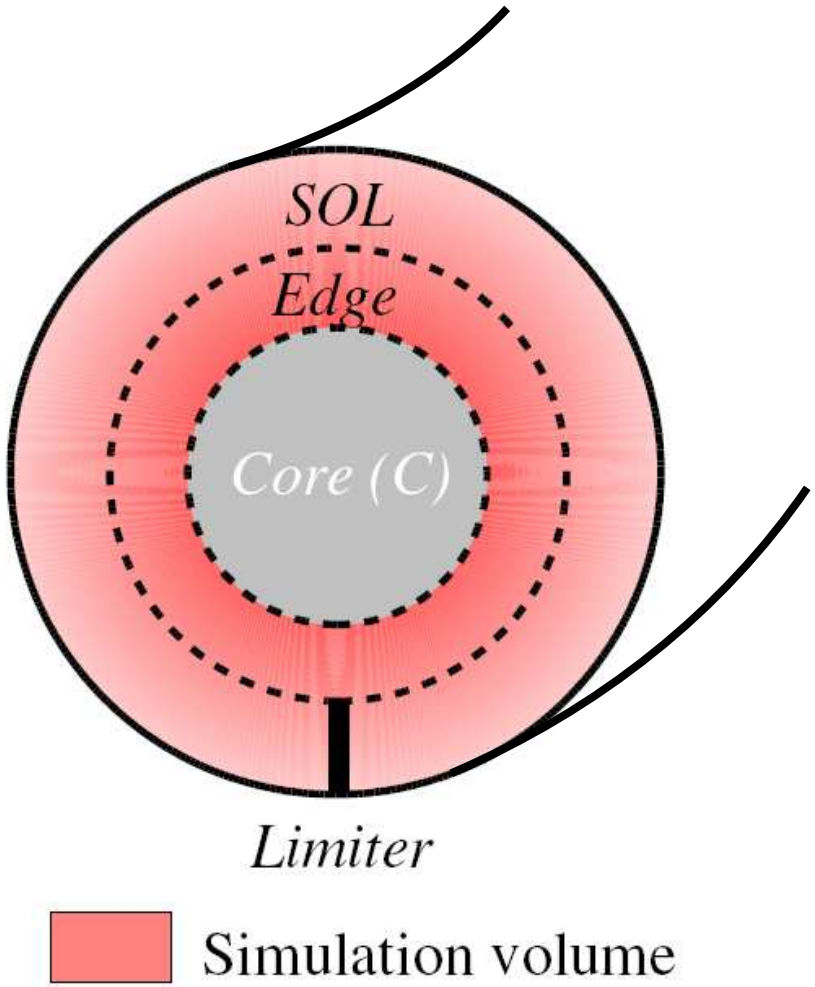
- Operators : $\nabla_{\parallel} = \frac{1}{R_0}\left(\partial_\phi + \frac{1}{q}\partial_\theta\right)$

$$[f,g] = \left(\vec{\nabla}_\perp f \wedge \vec{\nabla}_\perp g\right).\vec{b} = \frac{1}{r}\left(\partial_r f.\partial_\theta g - \partial_\theta f.\partial_r g\right)$$

Geometry

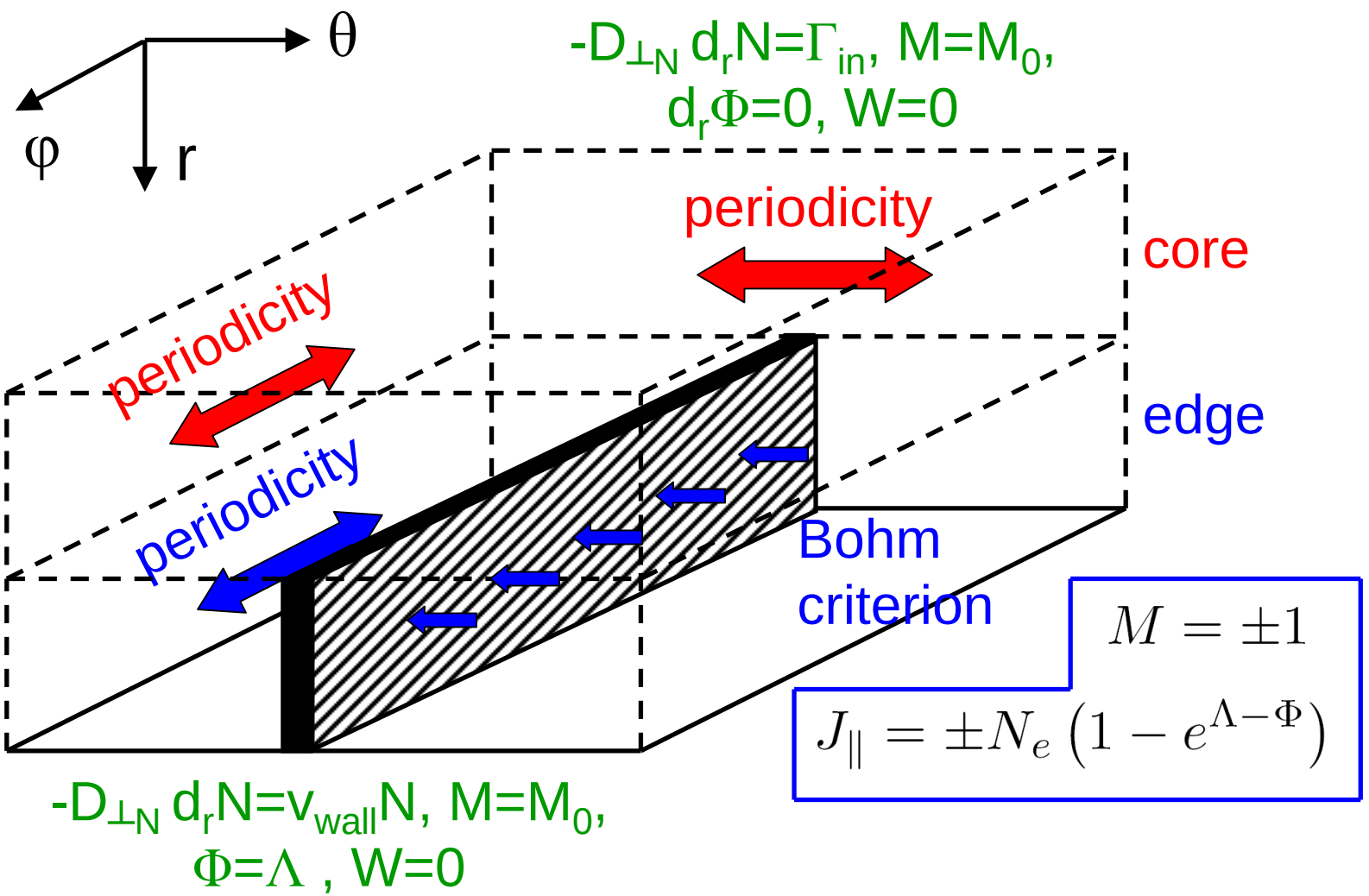
- A simplified geometry…
  - full toroidal plasma
  - concentric circular flux surfaces
  - large aspect ratio
  - limiter configuration

- … including main physical features
  - curvature effects
  - Core/SOL transition with plasma-wall interaction boundarie conditions



Boundary conditions

- 2 execution modes : core or core+SOL
- In core+SOL mode :
- In core mode : Dirichlet BC at outer flux surface


θ
φ
r
-D⊥N drN=Γin, M=M0,
drΦ=0, W=0
periodicity
core
periodicity
edge
periodicity
Bohm
criterion
$M = \pm 1$
$J_{\parallel} = \pm N_e \left(1 - e^{\Lambda - \Phi}\right)$
-D⊥N drN=vwallN, M=M0,
Φ=Λ , W=0

Numerical scheme

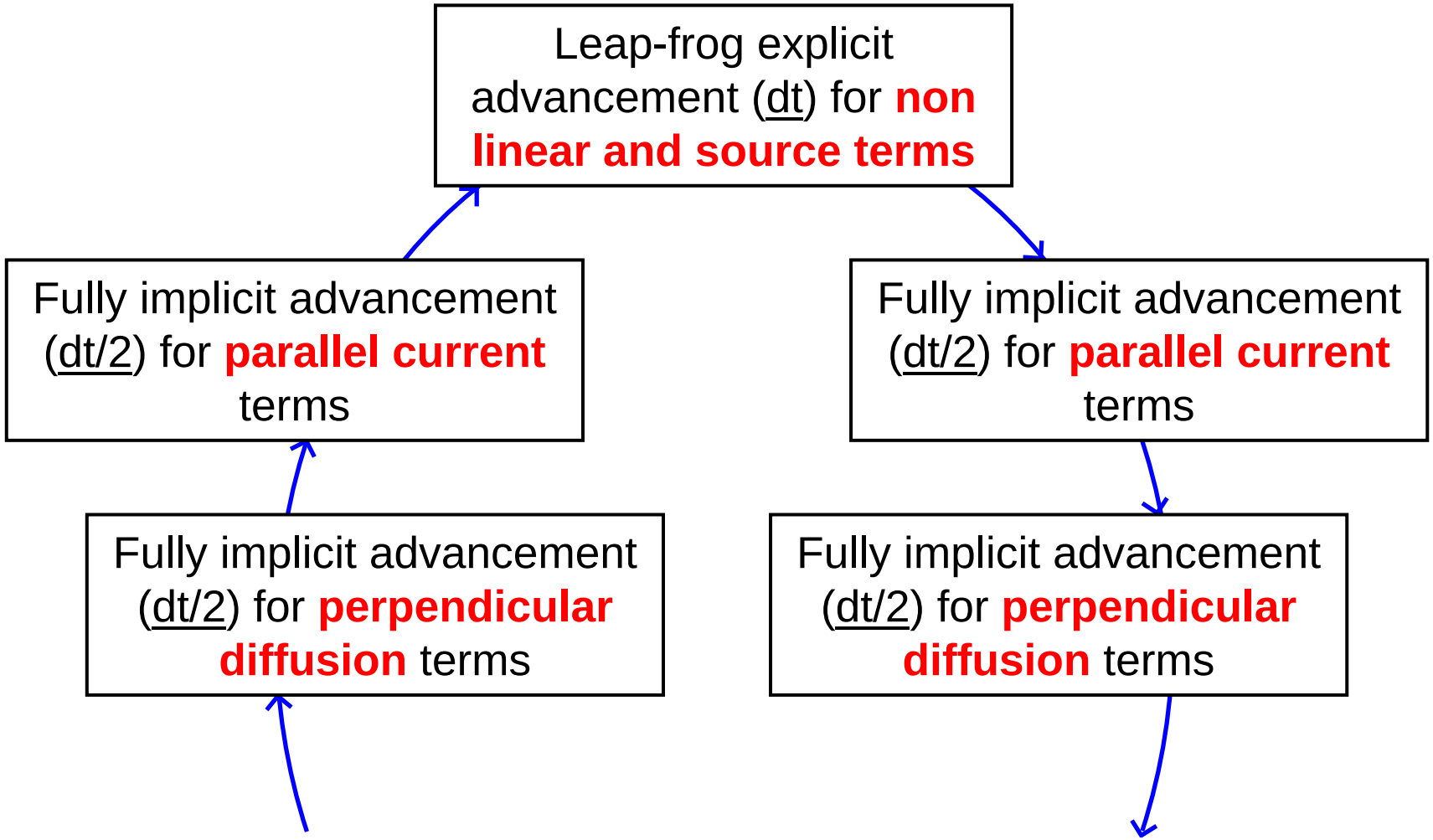
- (r, θ, φ) coordinates with structured mesh

- Finite differences scheme in radial and poloidal direction, semi-spectral in toroidal direction

- Second order time-splitting scheme :

- Parallel current terms treated appart :
  - very fast dynamic => implicit
  - $J_{||}$, Φ, W interplay => 3D coupling

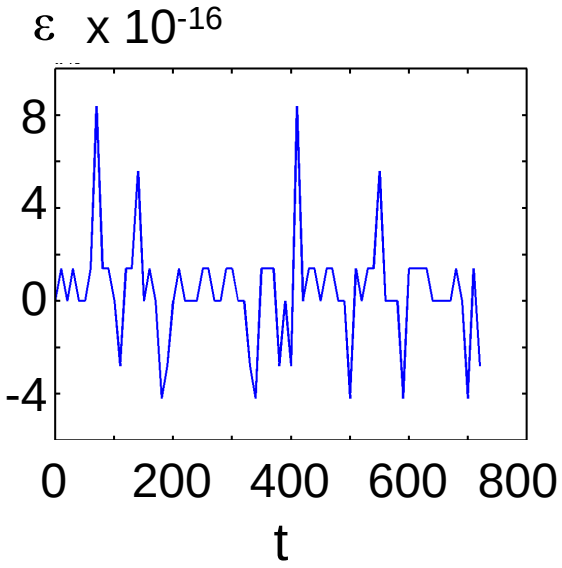
➡ direct implicit method with sparse solver (MUMPS)

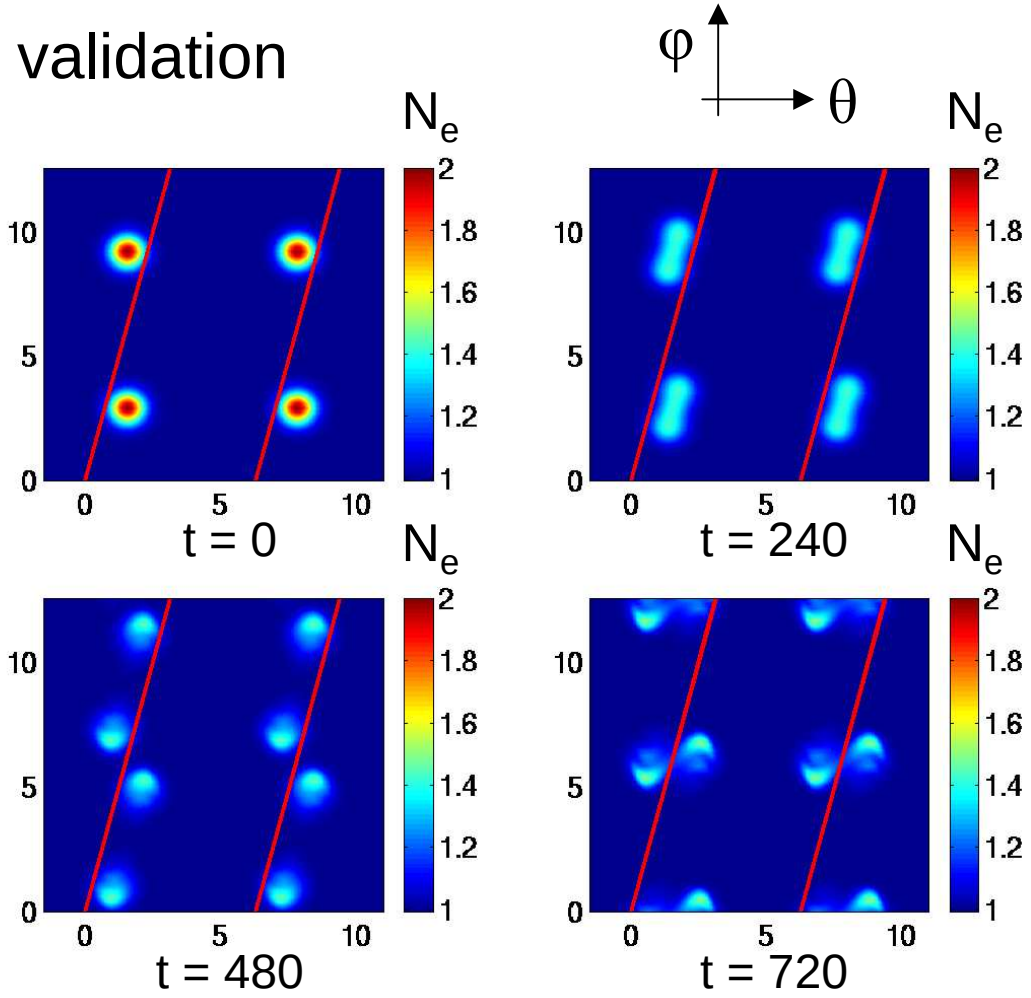

Leap-frog explicit advancement (dt) for non linear and source terms
Fully implicit advancement (dt/2) for parallel current terms
Fully implicit advancement (dt/2) for parallel current terms
Fully implicit advancement (dt/2) for perpendicular diffusion terms
Fully implicit advancement (dt/2) for perpendicular diffusion terms

Validation

- Parallel transport terms validation

$$\begin{cases} \partial_t N_e + \nabla_{\parallel} N_e M = 0 \\ \partial_t M + M \nabla_{\parallel} M = \\ \qquad -(1+Z)\,\nabla_{\parallel} \ln N_e \end{cases}$$





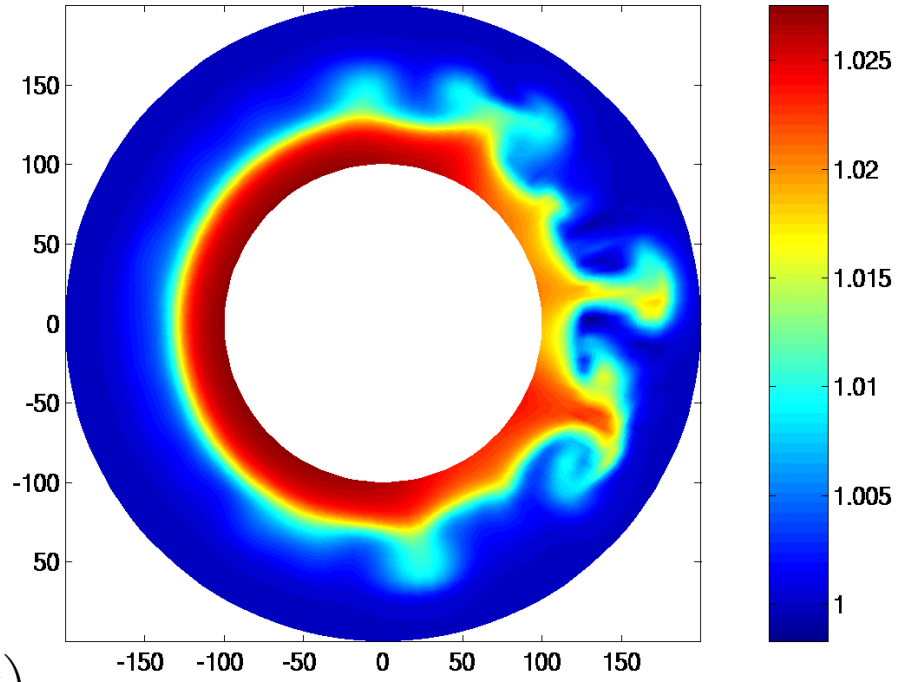
- Simplified 3D interchange turbulence validation

$$\partial_t N_e + \nabla_{\parallel} N_e M + \frac{1}{B}\left[\Phi, N_e\right] = \vec{\nabla}_{\perp}.\left(D_{\perp N_e}\vec{\nabla}_{\perp} N_e\right)$$

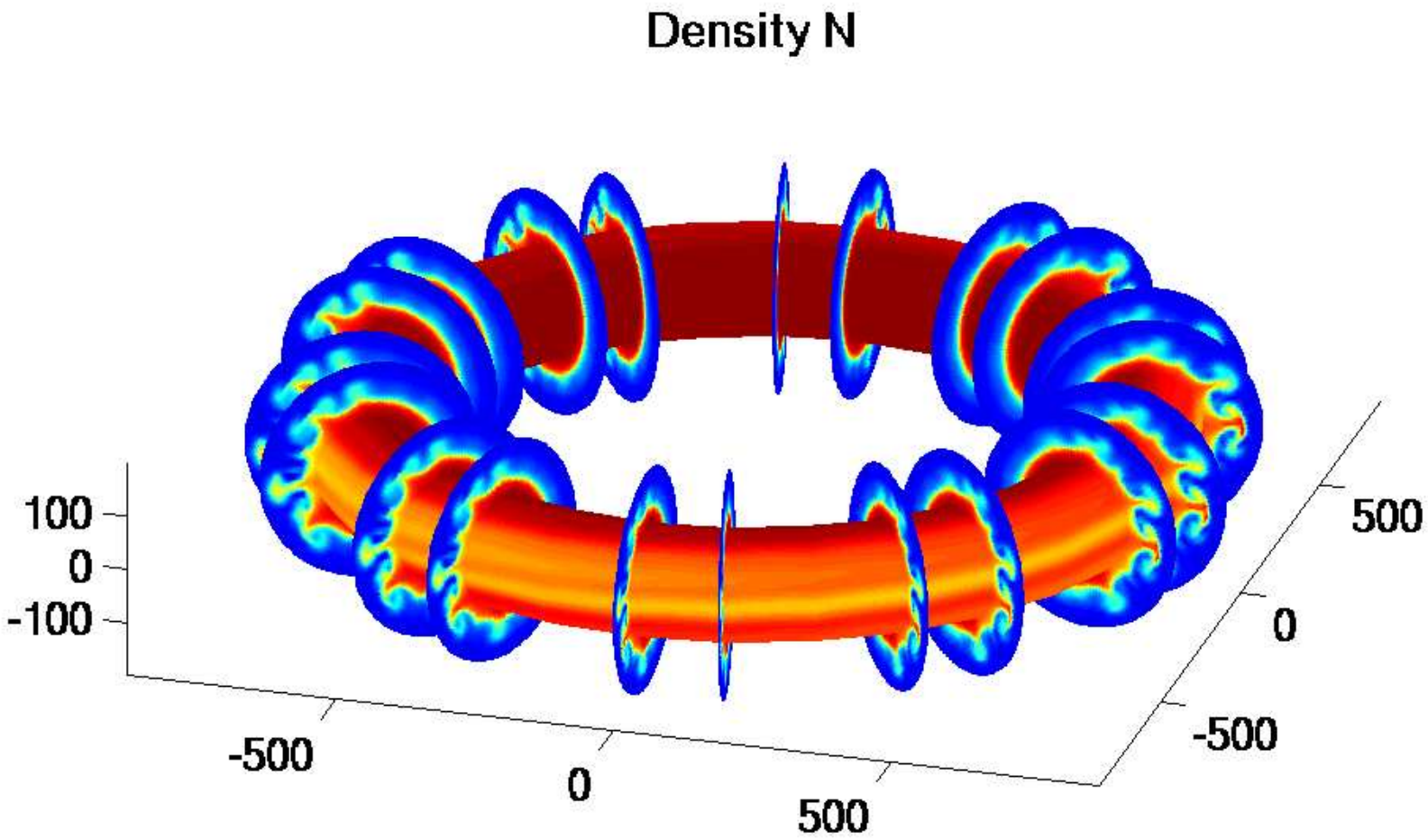
$$\partial_t W + M\nabla_{\parallel} W + \frac{1}{B}\left[\Phi, W\right] = -\left(1+Z\right)\frac{B^3}{N_e}\frac{1}{r}\partial_r\frac{1}{B^2}\partial_\theta N_e + \vec{\nabla}_{\perp}.\left(D_{\perp W}\vec{\nabla}_{\perp} W\right)$$

$$\partial_t M + M\nabla_{\parallel} M + \frac{1}{B}\left[\Phi, M\right] = -\left(1+Z\right)\nabla_{\parallel}\ln N_e + \frac{Z}{N_e}\vec{\nabla}.\left(D_M\vec{\nabla} M\right)$$

$$W = \nabla_{\perp}^2\Phi$$



Poloidal plot
of density

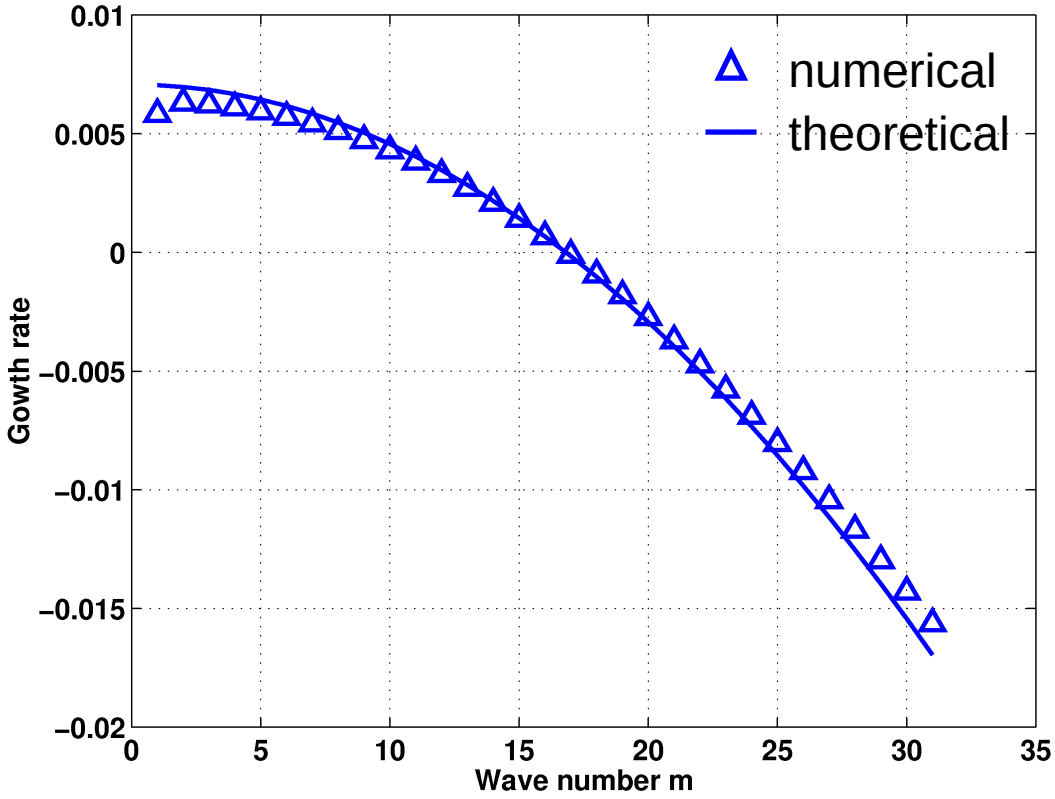

Density N
100
0
-100
-500
0
500
500
0
-500

Growth rates comparison (on a simplified linearized system)

$$\begin{cases} \partial_t \tilde{N}_e - \frac{1}{r}\partial_\theta \tilde{\Phi} \frac{\partial_r \bar{N}_e}{\bar{N}_e} + \left[\tilde{\Phi}, \tilde{N}_e\right] = D_{\perp N_e} \nabla_\perp^2 N_e \\ \partial_t \nabla_\perp^2 \tilde{\Phi} + \left[\tilde{\Phi}, \nabla_\perp^2 \tilde{\Phi}\right] + g \frac{1}{r} \frac{\partial_\theta \tilde{N}_e}{\bar{N}_e} = D_{\perp W} \nabla_\perp^4 \tilde{\Phi} \end{cases}$$

with $g = (1+Z) B^3 \partial_r \frac{1}{B^2}$

$$\gamma = \frac{1}{\sqrt{k_r^2 + \frac{m^2}{r^2}}} \sqrt{-g \frac{\partial_r \bar{N}_e}{\bar{N}_e} \frac{m}{r}} - \frac{D_{\perp N_e} + D_{\perp W}}{2} \left(k_r^2 + \frac{m^2}{r^2}\right)$$



Conclusion

- 3D fluid modelling of electrostatic turbulence in the edge plasma (core or core +SOL)

- Advancement state : first step of development over, validation process in progress with encouraging preliminary results

- Very large potential field of application : large scale flows, particle injection, edge turbulence, transport barriers…

- Future work : further validation cases (balance laws, SOL BCs…), energy conservation integration