Particle and impurity transport in tokamaks – a review of theoretical issues

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Background: density peaking

- ITER performance depends on density profile.
  \[ P_{\text{fusion}} = n_D n_T \left< \sigma_{\text{fusion}} \nu \right> E_{\text{fusion}} \]

- Density peaking observed in most present tokamak and stellarator devices
  - Different mechanisms in different devices?
  - Which ones extrapolate to ITER?
  - Will impurities accumulate?
  - What about alpha particles?

- Theoretically expected peaking mechanisms:
  - core fuelling
  - classical mechanisms
  - turbulence-driven transport

From ITER Technical Basis
Classical mechanisms
Neoclassical particle transport

- Fluid treatment: take toroidal projection of the momentum equation

\[
\frac{m_n dV}{dt} = Z e n (E + V \times B) - \nabla p - \nabla \cdot \vec{\pi} + \vec{F}
\]

multiply by \( R \) and flux-surface average: \( \langle \Gamma \cdot \nabla \psi \rangle = -\langle R (Z n e E_\phi + F_\phi) \rangle \)

- Note:
  - Transport is “automatically ambipolar”: \( \sum_a \langle n_a V_a \cdot \nabla \psi \rangle = 0 \)
  - Electron transport is smaller than ion transport. Impurities go in and bulk ions go out, or vice versa.
  - Ware pinch: Friction and electric force do not balance in trapped region. Hence

\[
\langle \Gamma \cdot \nabla \psi \rangle \sim -\varepsilon^{1/2} n e R E_\phi
\]

- Explains ITB formation in C-Mod (Ernst, 2004)
- Why counter-NBI heating produces density peaking (MAST, ASDEX, JFT-2M)
Neoclassical impurity transport

- Diamagnetic effect of bulk ion banana orbits gives rise to parallel friction force on impurities:
  - $dn_i/dr < 0 \Rightarrow$ co-current friction, inward impurity flux
  - $dT_i/dr < 0 \Rightarrow$ counter-current thermal force, outward impurity flux

- Hence, in the banana regime, the radial impurity flux becomes

$$\Gamma_z = L_{i11} \frac{d \ln n_i}{d \psi} - L_{i12} \frac{d \ln T_i}{d \psi} + O(Z^{-1})$$

- Also an impurity Ware pinch
  - Relatively small, of order

$$\frac{j_\Omega}{j_{BS}} \sqrt{\frac{m_e}{m_i}} \times \text{gradient-driven transport}$$
Beam-driven electron transport

- Transport due to friction between electrons and beam ions

\[ \langle n_e \mathbf{V}_e \cdot \nabla \psi \rangle = -\frac{p_e R^2 B^2 \Phi}{m_e \tau_e} \langle \Omega_e^{-2} \rangle \left[ l_{11}^{e} \left( \frac{d \ln p_e}{d \psi} - \frac{j_b B}{p_e RB \phi Z_{\text{eff}}} \right) + l_{12}^{e} \frac{d \ln T_e}{d \psi} + l_{11}^{i} \frac{d \ln p_i}{d \psi} + l_{12}^{i} \frac{d \ln T_i}{d \psi} + \ldots \right] \]

- Has the same coefficient as ordinary neoclassical diffusion, and is somewhat larger than the Ware pinch per unit of current driven

\[ \langle n_e \mathbf{V}_e \cdot \nabla \psi \rangle = -\frac{f_i m_e R}{e^2 \tau_{ee}} 0.63 e^{-1/2} j_{\text{NBICD}} - 0.85 j_{\Omega} + \ldots \]

- Effectively shifts the NBI fuelling profile about one banana width (of the beam ions)
  - outward for co-injection
  - inward for counter-injection.
Particle pinch from current drive

- Wave-particle interaction (ECCD, LHCD) affects ion-electron friction force
  - LHCD: electrons are pushed to higher $v_\parallel$
  - ECCD: electrons are heated to reduce friction

- This causes a Ware-like pinch, proportional to the collisionality of resonant electrons

- Current drive pinch therefore smaller than the corresponding Ware pinch.

- For ECCD in the linear approximation

\[
\frac{\langle n_e V_e \cdot \nabla \psi \rangle / j_\parallel \big|_{\text{ECCD}}}{\langle n_e V_e \cdot \nabla \psi \rangle / j_\parallel \big|_{\text{Ohmic}}} = \alpha Z_{\text{eff}} \left( \frac{v_T}{v_{\text{res}}} \right)^3, \quad 1 < \alpha < 4.2
\]
Turbulence

- Kinetic theory
  - Curvature pinch
  - Thermodiffusion
- Fluid theory
- Numerical simulations
- Impurity and alpha-particle transport
Curvature pinch (1)

- Existence of “magnetic drive” seen from drift kinetic equation (Smolyakov, 1993) written in variables ($\mu$, $W=mv^2/2$)

$$\frac{\partial f}{\partial t} + (v_\parallel + v_d + v_{EB}) \cdot \nabla f + \left[ m(v_{\perp}^2 / 2 + v_\parallel^2) v_{EB} \cdot \nabla \ln B - ev_{\parallel} E_\parallel \right] \frac{\partial f}{\partial W} = C(f)$$

suggesting an effective replacement

$$v_{EB} \cdot \nabla f_0 \rightarrow v_{EB} \cdot \left[ \nabla \ln f_0 - \frac{m}{T} \left( \frac{v_{\perp}^2}{2} + v_\parallel^2 \right) \nabla \ln B \right] f_0$$

- Tokamak particle motion is characterised by 3 adiabatic invariants (action integrals):

$$\mu = \frac{mv_{\perp}^2}{2B} \quad \text{gyro - motion}$$

$$J = \int mv_{\parallel} ds \quad \text{bounce motion}$$

$$\langle \psi \rangle_{\text{time}} \quad \text{toroidal precession}$$
Curvature pinch (2)

- Turbulence with $\omega <<$ bounce frequency conserves $\mu$ and $J$. Hence, if transport is diffusive, then it is of the form (Isichenko et al, 1995)

$$\frac{\partial f}{\partial t} = C(f) + \frac{\partial}{\partial \psi} \left( D(\mu, J, \psi) \frac{\partial f}{\partial \psi} \right)_{\mu, J}$$

without a pinch term, because Liouville’s theorem guarantees incompressible phase-space motion.

- Expand in $\tau_{ei} / \tau_E << 1 \Rightarrow f = f_0 + f_1 + ..., \text{with } f_0 \text{ Maxwellian}.$

- Density moment and energy moments give

$$\langle \Gamma_e \cdot \nabla \psi \rangle = -\frac{4\pi^2 m_e}{V'(\psi)} \int D \left( \frac{\partial f_0}{\partial \psi} \right)_{\mu, J} d\mu dJ = -\hat{D} \frac{dn}{d\psi} - \hat{V}_n$$

$$\langle Q_e \cdot \nabla \psi \rangle = -\frac{4\pi^2 m_e}{V'(\psi)} \int \frac{m_e v^2}{2} D \left( \frac{\partial f_0}{\partial \psi} \right)_{\mu, J} d\mu dJ$$
Curvature pinch (3)

- Thus:
  - Thermodiffusion, caused by the dependence of D on energy
  - Curvature pinch, generic in toroidal geometry, usually inward
  - Geometric term adds to the usual thermodynamic forces (Garbet et al, 2005)
  - Energy exchange between turbulence and electrons.
  - Details depend on how D depends on pitch angle. For instance, if D is independent of bounce point location, then

\[
\frac{\dot{V}}{D} = -\frac{4}{3R} \left( s + \frac{3}{8} \right), \quad s = \frac{d \ln q}{d \ln r} \quad \Rightarrow \quad n_e(\psi) \rightarrow n_0 \left[ 1 - \frac{4}{3R} \int_0^\psi \left( s + \frac{3}{8} \right) d\psi' \right]
\]
Thermodiffusion

- Quasilinear (QL) gyrokinetic (GK) theory: Consider GK equation for nonadiabatic part of the distribution function

\[ i v_\parallel \nabla_\parallel g_a + (\omega - \omega_{da}) g_a = \frac{e\phi}{T_a} (Z_a \omega - \omega_{sa}^T) f_{a0}, \quad \omega_{da} = k_\perp \cdot v_{da}, \quad \omega_{sa}^T = \frac{T_a}{eB_0^2} k_\perp \times B \cdot \nabla \ln f_{a0} \]

and work out the quasilinear flux from the non-adiabatic density response

\[ \langle \Gamma_a \cdot \nabla \psi \rangle = -Z_a e \langle R E_{\phi}^* \int g_a d^3v \rangle \]

- Using Krook \( C(f) \) gives pinch under certain conditions (Terry, 1989; Waltz and Dominguez, 1989)
  - Depends on type of mode (ITG, TEM) and parameter regime, particularly collisionality.

- Beyond QL theory (Terry, 2006): nonlinear inward pinch in TEM turbulence
Coppi and Spright (1978) considered QL drift wave turbulence in a resistive plasma (Braginskii eqs, with thermal force and collisional parallel heat conduction), and found

\[ \frac{\tilde{n}_e}{n_e} = \frac{e\phi}{T_e} \left[ 1 + \frac{1.71 i}{\chi|k|^2} \left( \omega + \frac{3\eta_e}{2} - 1 \right) \omega_n \right] \]

For low collisionality, the fluid truncation of Weiland, Jarmen and Nordman (1989)

- no viscosity or friction in momentum equation
- only diamagnetic heat flux

results in

- Particle pinch
- Heat pinch
- Curvature pinch

\[ \eta = \frac{L_n}{L_T} \]
Garbet et al (2003, 2005) derive refined fluid equations for $\omega << \omega_{\text{bounce}}$, recognising that J is conserved for trapped particles, taking moments of the kinetic equation

$$\frac{\partial f}{\partial t} + \psi \left( \frac{\partial f}{\partial \psi} \right)_{\mu,J} + \dot{\alpha} \left( \frac{\partial f}{\partial \alpha} \right)_{\mu,J} = 0,$$

$$\alpha = \varphi - q\theta$$

and assuming that f is Maxwellian (but different on each field line), giving

$$\frac{dn}{dt} = -n \nabla \cdot \mathbf{v}_\perp = -2 \left( \frac{1}{4} + \frac{2s}{3} \right) \frac{1}{B^3} \left( n \nabla \phi + \frac{\nabla p}{Ze} \right)$$

where $= 1$ in Weiland's model.

If temperature fluctuations are small this result can be written

$$\frac{d}{dt} \left( \frac{n(r)}{n_{\text{can}}(r)} \right) = 0,$$

$$n_{\text{can}}(r) = 1 - \frac{4}{3R} \int_0^r \left( s + \frac{3}{8} \right) dr'$$

In the QL theory, the transport is proportional to $d(n/n_{\text{can}})/dr$ rather than $dn/dr$. 
Trapped vs circulating electrons (1)

- Jenko, Dannert and Angioni (EPS 2005) report different transport properties for trapped and circulating electrons in collisionless GK simulations:
  - ITG: passing electrons nearly adiabatic, trapped ones go inward.
  - TEM: passing electrons go inward, trapped ones outward, but different k can drive transport in different directions (also found by Kinsey, 2006). Transport suppressed at high collisionality (Romanelli, 2006)
Hallatschek and Dorland (2005) find in nonlinear GK simulations

- Trapped electrons go out.
- Passing ones go in, giving a net inward pinch.
- This pinch is small: $\Gamma T_e/Q_e \sim 10^{-2}$.
- Attributed to long parallel “tail” in the fluctuation spectrum, violating $\omega \ll k \parallel v_{Te}$.
- Confirmed by Ernst (2004), but disputed by GYRO group.
Role of trapping and collisionality

  - Trapped particles go out, except at low collisionality
  - Passing ones in
  - Total flux inward only at low collisionality
  - Agreement with observed peaked density profile in DIII-D if gradients are adjusted slightly
  - Qualitatively similar for electrons and ions
  - Pinch attributed to temperature gradient rather than magnetic curvature, but depends on magnetic shear

  - QL GK simulations
  - Pinch if dT/dr large
Nonlinear TEM particle transport

- Ernst (2004) finds in nonlinear GK simulations
  - Ware pinch sufficient to explain ITB formation in Alcator C-mod.
  - Ware flux balanced by outward particle flux driven by TEM turbulence, exhibiting “Dimits shift”, attributed to zonal flow generation.
Impurity transport

- Fülöp and Weiland (2006)
  - Neoclassical transport + Weiland model
  - ITER: inward anomalous flux and outward nc transport due to temperature screening

- Angioni and Peeters (2006)
  - Similar model, sign of flux depends on rotation direction of the turbulent fluctuations
  - Explains the dependence of impurity transport on ion and electron temperature gradients

- Estrada-Mila, Candy and Waltz (2005)
  - In a 50-50 DT plasma, T flows inward relative to D
  - Inward He flow at moderate He density gradient
  - Both these NL GK results are reproduced by QL theory in limit of small $k_\theta \rho_a$ and $\omega_d/\omega$
α-particle transport (1)

- Transport reduced because of FLR averaging
  - Analytically predicted
  - Numerically confirmed

- Recent controversy about scaling of fast-ion transport in prescribed random ExB flow
  - Decorrelation Trajectory Method (M. Vlad, Spineanu et al, 2005) suggests transport increasing with $\rho_\alpha$ at high Kubo numbers $K = V_{\text{ExB}} \tau_{\text{corr}}/\lambda_{\text{corr}}$
  - Disputed by Hauff and Jenko (2006), who find transport almost independent of $\rho_\alpha$ at high $K$
    - Modified Decorrelation Trajectory Method
    - Supported by direct numerical simulations

Manfredi and Dendy (1997)
However, in practice $k_\theta \rho_\alpha \sim 0.5$ (Estrada-Mila, Candy and Waltz, 2006)

- Nonlinear GK simulation with Maxwellian trace alphas ($T_\alpha >> T_{e,i}$) suggests larger flux per alpha particle than for thermals
- Alpha particle confinement time still much longer than the slowing-down time, but
- Some broadening (~15%) of the alpha particle density profile

![Graph](image)
Summary

- Classical mechanisms for density peaking:
  - Ware pinch – important at low temperature
  - NBI pinch – fuelling occurs one beam-ion banana width inside or outside deposition radius
  - RF pinch – smaller than Ware pinch if resonant electrons are suprathermal
  - Fuelling – plays important role in present-day experiments

- Turbulence-driven particle pinches:
  - Dominant at low collisionality or large device size
  - Thermodiffusion inward only at low enough collisionality
  - Trapped and circulating ions/electrons have different transport properties

- Impurity transport:
  - Neoclassical transport important – temperature screening prevents central impurity accumulation in banana regime
  - Anomalous flux inward or outward

- Alpha particle transport:
  - Reduction because of gyro-averaging
  - Surprisingly large, but confinement still satisfactory
Open questions

- When and why is the thermodiffusion flux inward/outward?
- What is the role of collisionality, magnetic shear, flux-surface geometry etc?
- Is thermodiffusion more or less important than the curvature pinch?
- Why is the core density profile more peaked in L-mode than in H-mode?
- What is the cause of density transport barriers?
- How much impurity accumulation can we expect in ITER?
  - Standard operation
  - Advanced regimes
- Will D-T separation be a problem?
- Will the ash be removed easily?
- Can we be sure that alpha particles will be confined well enough?
Core fuelling

- Neutral atoms diffuse into the plasma through charge exchange. Diffusion velocity of short-mean-free path neutrals:

\[ V_n = -D(\nabla \ln n_n + 0.76 \nabla \ln T_i), \]

- Continuity equation

\[
\frac{1}{r} \frac{d}{dr} \left( \frac{d}{dr} (n_n V_{nr}) \right) = v_x n_n
\]

has WKB solution

\[
\frac{n_n(r)}{n_n(a)} \sim \frac{1}{\sqrt{rT_i}} \left( \frac{n_i}{v_z} \right)^{1/4} \exp \left( -\int_0^a \frac{dr'}{\lambda(r')} \right)
\]

\[
\lambda^2 = \lambda_x \lambda_z = \frac{\sqrt{T_i/m_i}}{2.93n_i v_z \sigma_x}
\]

\[ D = \frac{T_i}{m_i v_x}, \quad v_x = 2.93n_i (T_i/m_i)^{1/2} \sigma_x \]