# Particle and impurity transport in tokamaks – a review of theoretical issues

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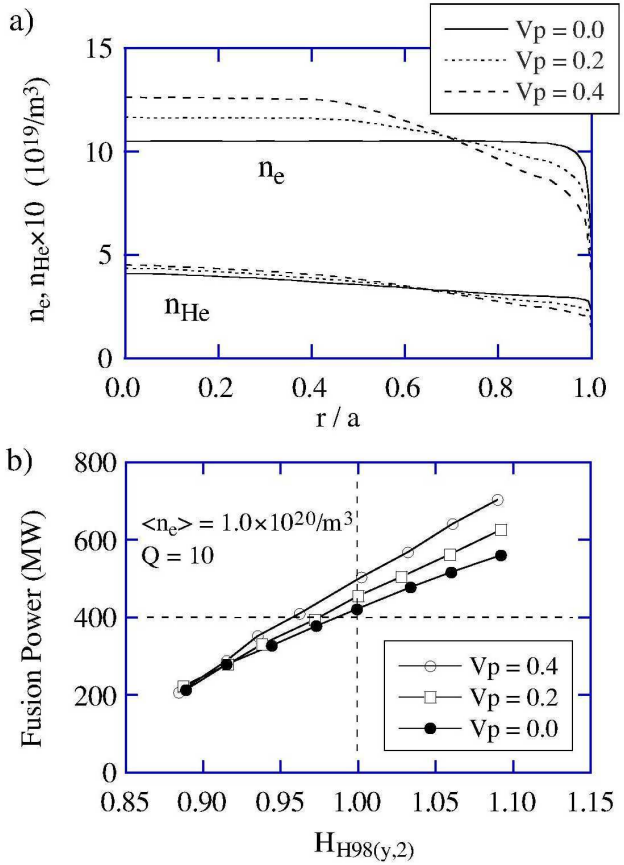
# Background: density peaking

- ITER performance depends on density profile.

$$P_{\text{fusion}} = n_D n_T \langle \sigma_{\text{fusion}} \text{v} \rangle E_{\text{fusion}}$$

- Density peaking observed in most present tokamak and stellarator devices
  - Different mechanisms in different devices?
  - Which ones extrapolate to ITER?
  - Will impurities accumulate?
  - What about alpha particles?

- Theoretically expected peaking mechanisms:
  - core fuelling
  - classical mechanisms
  - turbulence-driven transport




From ITER Technical Basis

# Classical mechanisms

# Neoclassical particle transport

- Fluid treatment: take toroidal projection of the momentum equation

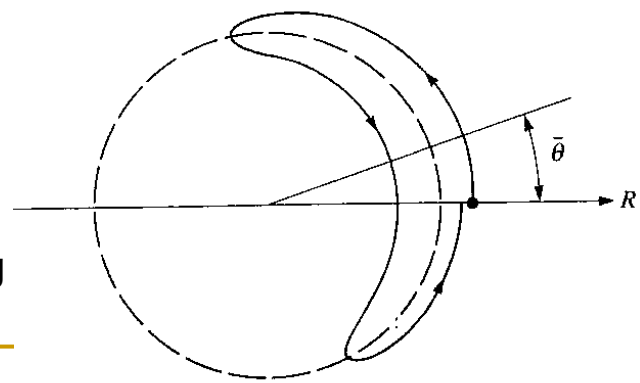
$$mn\frac{d\mathbf{V}}{dt} = Zen(\mathbf{E}+\mathbf{V}\times\mathbf{B}) - \nabla p - \underbrace{\nabla\cdot\overleftrightarrow{\pi}}_{\text{small}} + \mathbf{F}$$

multiply by $R$ and flux-surface average: $\langle \mathbf{\Gamma}\cdot\nabla\psi\rangle = -\langle R(ZneE_\varphi + F_\varphi)\rangle$

- Note:
  - Transport is "automatically ambipolar": $\sum_a \langle n_a \mathbf{V}_a \cdot \nabla\psi\rangle = 0$
  - Electron transport is smaller than ion transport. Impurities go in and bulk ions go out, or vice versa.
  - Ware pinch: Friction and electric force do not balance in trapped region. Hence

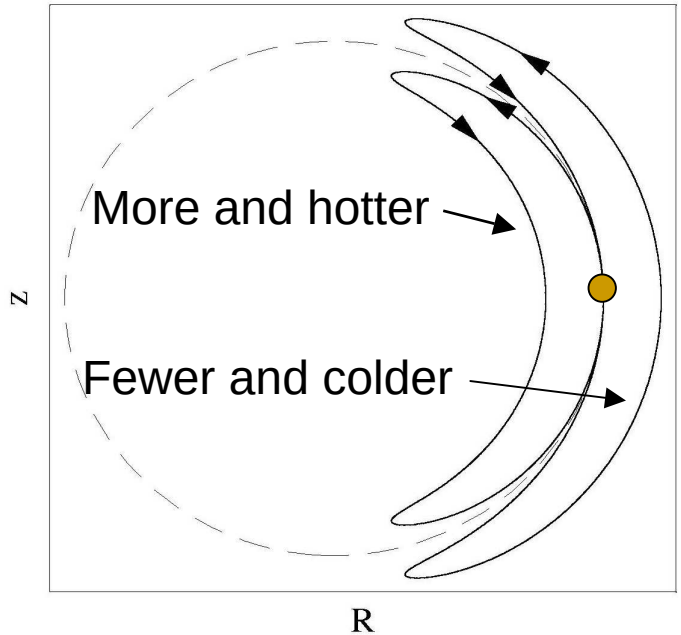
$$\langle \mathbf{\Gamma}\cdot\nabla\psi\rangle \sim -\varepsilon^{1/2} neRE_\varphi$$

    - Explains ITB formation in C-Mod (Ernst, 2004)
    - Why counter-NBI heating produces density peaking (MAST, ASDEX, JFT-2M)



# Neoclassical impurity transport

- Diamagnetic effect of bulk ion banana orbits gives rise to parallel friction force on impurities:
  - $dn_i/dr<0 \Rightarrow$ co-current friction, inward impurity flux
  - $dT_i/dr<0 \Rightarrow$ counter-current thermal force, outward impurity flux




- Hence, in the banana regime, the radial impurity flux becomes

$$\Gamma_z = L_{11}\frac{d\ln n_i}{d\psi} - L_{12}\frac{d\ln T_i}{d\psi} + O\left(Z^{-1}\right)$$

- Also an impurity Ware pinch
  - Relatively small, of order

$$\frac{j_\Omega}{j_{BS}}\sqrt{\frac{m_e}{m_i}} \times \text{gradient-driven transport}$$

# Beam-driven electron transport

- Transport due to friction between electrons and beam ions

$$\langle n_e \mathbf{V}_e \cdot \nabla\psi \rangle = -\frac{p_e R^2 B_\varphi^2}{m_e \tau_e} \langle \Omega_e^{-2} \rangle \left[ l_{11}^e \left( \frac{d \ln p_e}{d\psi} - \frac{\langle j_b B \rangle}{p_e R B_\varphi Z_{\mathrm{eff}}} \right) + l_{12}^e \frac{d \ln T_e}{d\psi} + l_{11}^i \frac{d \ln p_i}{d\psi} + l_{12}^i \frac{d \ln T_i}{d\psi} + \ldots \right]$$

- Has the same coefficient as ordinary neoclassical diffusion, and is somewhat larger than the Ware pinch per unit of current driven

$$\langle n_e \mathbf{V}_e \cdot \nabla\psi \rangle = -\frac{f_t m_e R}{e^2 \tau_{ee}} \left( 0.63 \varepsilon^{-1/2} j_{\mathrm{NBICD}} - 0.85 j_\Omega \right) + \ldots$$

- Effectively shifts the NBI fuelling profile about one banana width (of the beam ions)
  - outward for co-injection
  - inward for counter-injection.

# Particle pinch from current drive

- Wave-particle interaction (ECCD, LHCD) affects ion-electron friction force
  - LHCD: electrons are pushed to higher $v_{||}$
  - ECCD: electrons are heated to reduce friction
- This causes a Ware-like pinch, proportional to the collisionality of resonant electrons
- Current drive pinch therefore smaller than the corresponding Ware pinch.
- For ECCD in the linear approximation

$$\frac{\langle n_e \mathbf{V}_e \cdot \nabla\psi \rangle / j_{||}\big|_{\mathrm{ECCD}}}{\langle n_e \mathbf{V}_e \cdot \nabla\psi \rangle / j_{||}\big|_{\mathrm{Ohmic}}} = \alpha Z_{\mathrm{eff}} \left( \frac{\mathrm{v}_T}{\mathrm{v}_{\mathrm{res}}} \right)^3, \qquad 1 < \alpha < 4.2$$

# Turbulence

- Kinetic theory
  - Curvature pinch
  - Thermodiffusion
- Fluid theory
- Numerical simulations
- Impurity and alpha-particle transport

# Curvature pinch (1)

- Existence of “magnetic drive” seen from drift kinetic equation (Smolyakov, 1993) written in variables ($\mu$, $W=mv^2/2$)

$$\frac{\partial f}{\partial t}+\left(\mathbf{v}_{\parallel}+\mathbf{v}_{d}+\mathbf{v}_{ExB}\right)\cdot\nabla f+\left[m\left(v_{\perp}^{2}/2+v_{\parallel}^{2}\right)\mathbf{v}_{ExB}\cdot\nabla\ln B-ev_{\parallel}E_{\parallel}\right]\frac{\partial f}{\partial W}=C(f)$$

suggesting an effective replacement

$$\mathbf{v}_{ExB}\cdot\nabla f_0\rightarrow\mathbf{v}_{ExB}\cdot\left[\nabla\ln f_0-\frac{m}{T}\left(\frac{v_{\perp}^{2}}{2}+v_{\parallel}^{2}\right)\nabla\ln B\right]f_0$$

- Tokamak particle motion is characterised by 3 adiabatic invariants (action integrals):

$$\mu=\frac{mv_{\perp}^{2}}{2B} \qquad \text{gyro-motion}$$

$$J=\oint mv_{\parallel}ds \qquad \text{bounce motion}$$

$$\langle\psi\rangle_{time} \qquad \text{toroidal precession}$$

# Curvature pinch (2)

- Turbulence with $\omega$<< bounce frequency conserves $\mu$ and J. Hence, if transport is diffusive, then it is of the form (Isichenko et al, 1995)

$$\frac{\partial f}{\partial t} = C(f) + \frac{\partial}{\partial \psi}\left( D(\mu, J, \psi) \frac{\partial f}{\partial \psi} \right)_{\mu, J}$$

  without a pinch term, because Liouville's theorem guarantees incompressible phase-space motion.

- Expand in $\tau_{ei} / \tau_E << 1 \quad \Rightarrow \quad f = f_0 + f_1 + \ldots$, with $f_0$ Maxwellian.

- Density moment and energy moments give

$$\langle \mathbf{\Gamma}_e \cdot \nabla \psi \rangle = -\frac{4\pi^2 m_e}{V'(\psi)} \int D \left( \frac{\partial f_0}{\partial \psi} \right)_{\mu, J} d\mu dJ = -\hat{D} \frac{dn}{d\psi} - \hat{V} n$$

$$\langle \mathbf{Q}_e \cdot \nabla \psi \rangle = -\frac{4\pi^2 m_e}{V'(\psi)} \int \frac{m_e v^2}{2} D \left( \frac{\partial f_0}{\partial \psi} \right)_{\mu, J} d\mu dJ$$

# Curvature pinch (3)

- Thus:
  - Thermodiffusion, caused by the dependence of D on energy
  - Curvature pinch, generic in toroidal geometry, usually inward
  - Geometric term adds to the usual thermodynamic forces (Garbet et al, 2005)
  - Energy exchange between turbulence and electrons.
  - Details depend on how D depends on pitch angle. For instance, if D is independent of bounce point location, then

$$\frac{\hat{V}}{\hat{D}} = -\frac{4}{3R}\left(s + \frac{3}{8}\right), \qquad s = \frac{d\ln q}{d\ln r} \qquad \Rightarrow \qquad n_e(\psi) \to n_0\left[1 - \frac{4}{3R}\int_0^r \left(s + \frac{3}{8}\right) dr'\right]$$

# Thermodiffusion

- Quasilinear (QL) gyrokinetic (GK) theory: Consider GK equation for nonadiabatic part of the distribution function

$$iv_{\parallel}\nabla_{\parallel}g_a + (\omega - \omega_{da})g_a - C(g_a) = \frac{e\phi}{T_a}\left(Z_a\omega - \omega_{*a}^{T}\right)f_{a0}, \qquad \omega_{da} = \mathbf{k}_{\perp}\cdot\mathbf{v}_{da}, \qquad \omega_{*a}^{T} = \frac{T_a}{eB^2}\mathbf{k}_{\perp}\times\mathbf{B}\cdot\nabla\ln f_{a0}$$

and work out the quasilinear flux from the non-adiabatic density response

$$\langle \mathbf{\Gamma}_a\cdot\nabla\psi\rangle = -Z_a e\left\langle R\tilde{E}_{\varphi}^{*}\int g_a d^3v\right\rangle$$

- Using Krook $C(f)$ gives pinch under certain conditions (Terry, 1989; Waltz and Dominguez, 1989)
  - Depends on type of mode (ITG, TEM) and parameter regime, particularly collisionality.

- Beyond QL theory (Terry, 2006): nonlinear inward pinch in TEM turbulence

# Fluid approach (1)

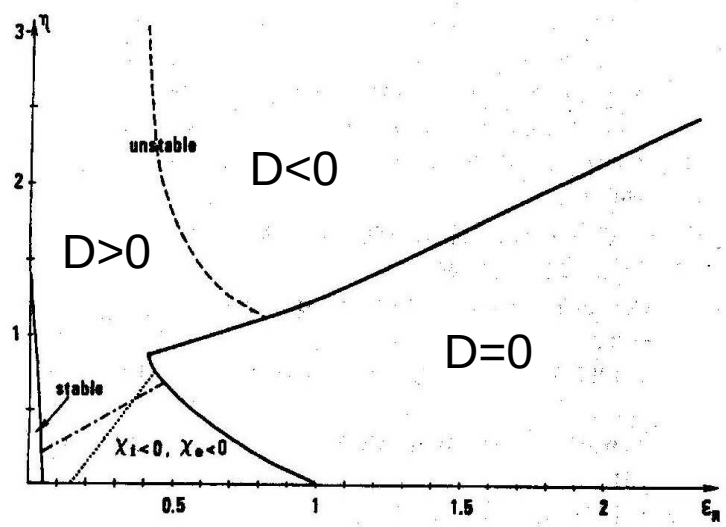
- Coppi and Spright (1978) considered QL drift wave turbulence in a resistive plasma (Braginskii eqs, with thermal force and collisional parallel heat conduction), and found

$$\frac{\tilde{n}_e}{n_e} = \frac{e\tilde{\phi}}{T_e}\left[1 + \frac{1.71\,i}{\chi_{\parallel} k_{\parallel}^2}\underbrace{\left(\omega + \left(\frac{3\eta_e}{2} - 1\right)\omega_{*e}\right)}_{>0 \;\Rightarrow\; \text{inward flux}}\right]$$

- For low collisionality, the fluid truncation of Weiland, Jarmen and Nordman (1989)
  - no viscosity or friction in momentum equation
  - only diamagnetic heat flux

  results in
  - Particle pinch
  - Heat pinch
  - Curvature pinch

$\eta = L_n/L_T$




$2\,L_n/R$

# Fluid approach (2)

- Garbet et al (2003, 2005) derive refined fluid equations for $\omega << \omega_{bounce}$, recognising that J is conserved for trapped particles, taking moments of the kinetic equation

$$\frac{\partial f}{\partial t} + \dot{\psi}\left(\frac{\partial f}{\partial \psi}\right)_{\mu,J} + \dot{\alpha}\left(\frac{\partial f}{\partial \alpha}\right)_{\mu,J} = 0, \qquad \alpha = \varphi - q\theta$$

and assuming that f is Maxwellian (but different on each field line), giving

$$\frac{dn}{dt} = -n\nabla\cdot\mathbf{v}_{\perp} = -2\underbrace{\left(\frac{1}{4}+\frac{2s}{3}\right)}_{=1 \text{ in Weiland's model}}\frac{\mathbf{B}\times\nabla B}{B^3}\cdot\left(n\nabla\phi + \frac{\nabla p}{Ze}\right)$$

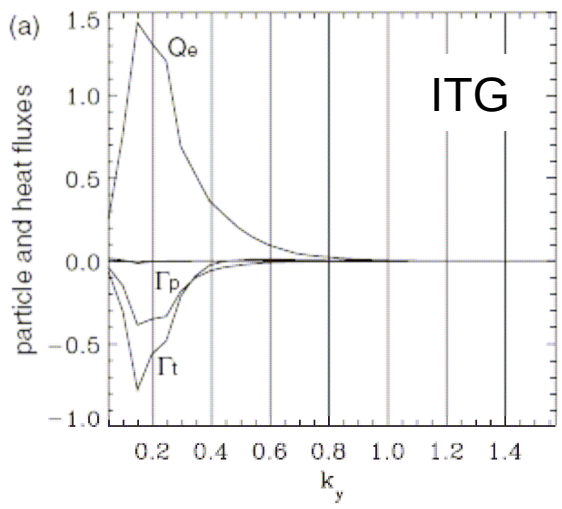
- If temperature fluctuations are small this result can be written

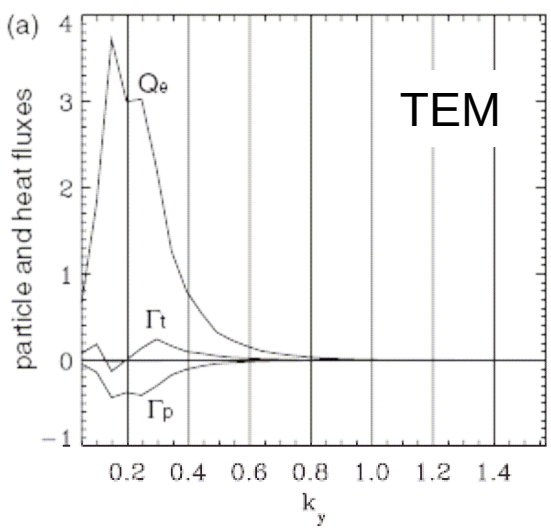
$$\frac{d}{dt}\left(\frac{n(r)}{n_{can}(r)}\right) = 0, \qquad n_{can}(r) = 1 - \frac{4}{3R}\int_0^r\left(s+\frac{3}{8}\right)dr'$$

- In the QL theory, the transport is proportional to $d(n/n_{can})/dr$ rather than $dn/dr$.

# Trapped vs circulating electrons (1)

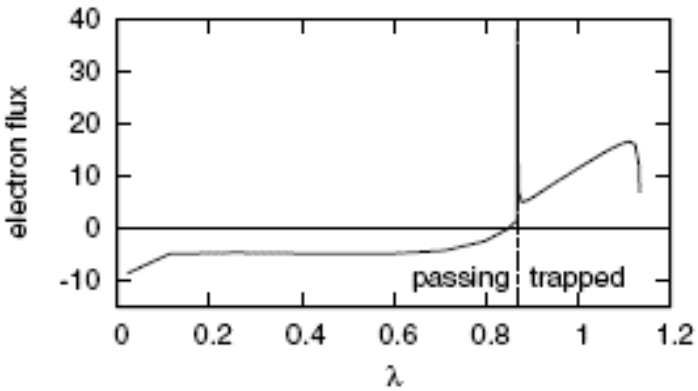
- Jenko, Dannert and Angioni (EPS 2005) report different transport properties for trapped and circulating electrons in collisionless GK simulations:
- ITG: passing electrons nearly adiabatic, trapped ones go inward.
- TEM: passing electrons go inward, trapped ones outward, but different k can drive transport in different directions (also found by Kinsey, 2006). Transport suppressed at high collisionality (Romanelli, 2006)



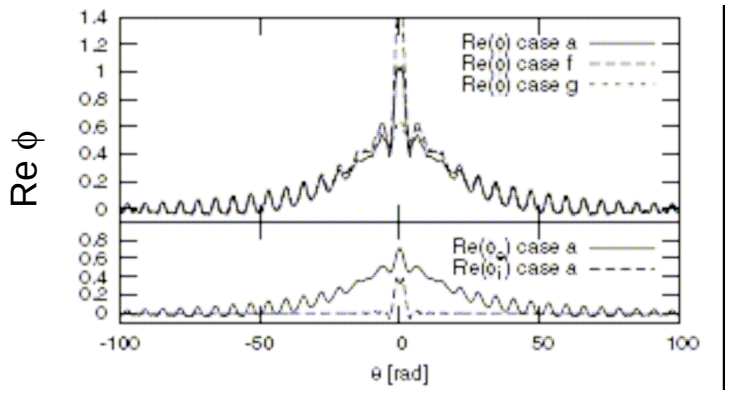



# Trapped vs circulating electrons (2)

- Hallatschek and Dorland (2005) find in nonlinear GK simulations
  - Trapped electrons go out.
  - Passing ones go in, giving a net inward pinch.
  - This pinch is small: $\Gamma T_e/Q_e \sim 10^{-2}$.
  - Attributed to long parallel "tail" in the fluctuation spectrum, violating $\omega << k_{\|} v_{Te}$
  - Confirmed by Ernst (2004), but disputed by GYRO group.




pitch angle variable

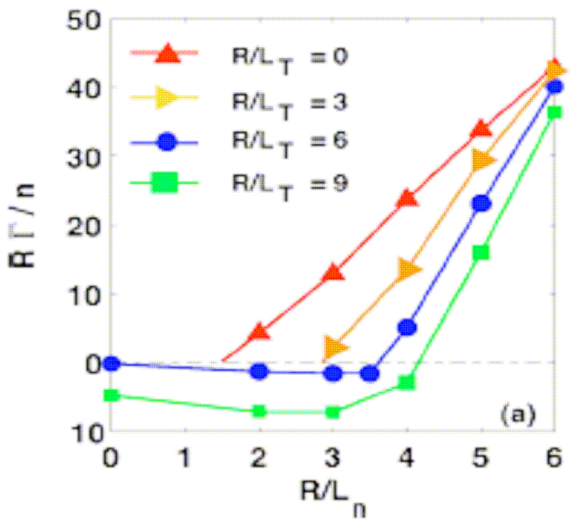


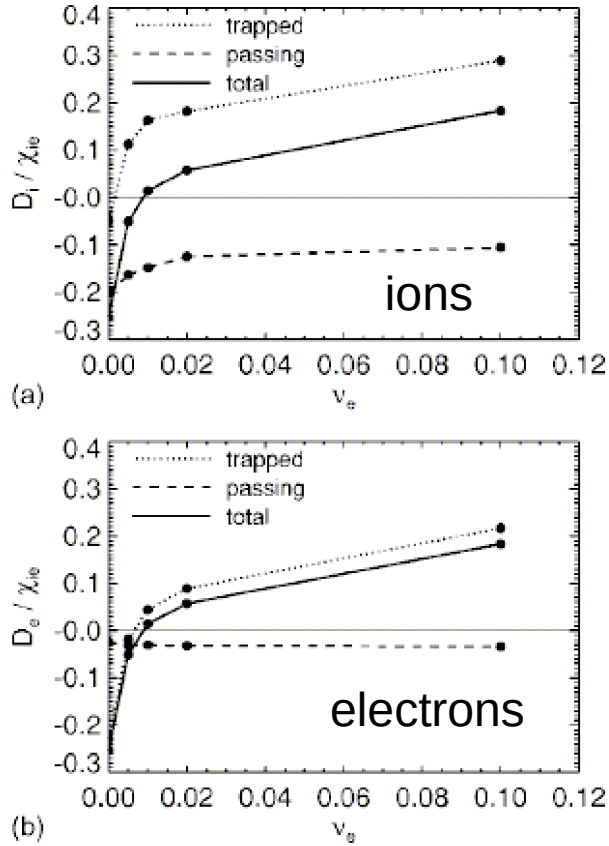

poloidal angle

# Role of trapping and collisionality

- Estrada-Mila, Candy & Waltz (2005), Kinsey (2006): nonlinear GK simulations
  - Trapped particles go out, except at low collisionality
  - Passing ones to in
  - Total flux inward only at low collisionality
  - Agreement with observed peaked density profile in DIII-D if gradients are adjusted slightly
  - Qualitatively similar for electrons and ions
  - Pinch attributed to temperature gradient rather than magnetic curvature, but depends on magnetic shear

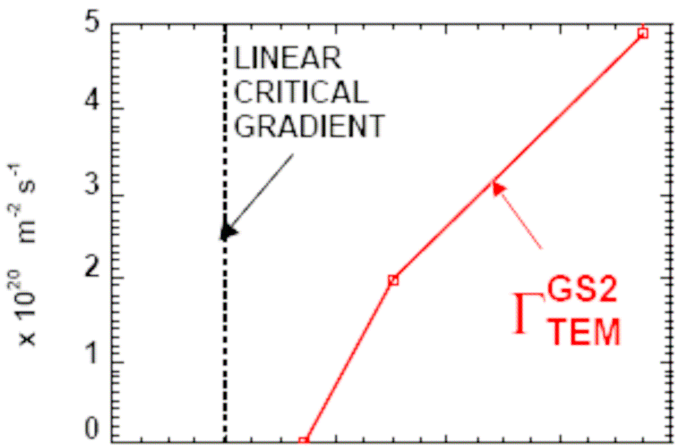
- Angioni et al (2005)
  - QL GK simulations
  - Pinch if dT/dr large



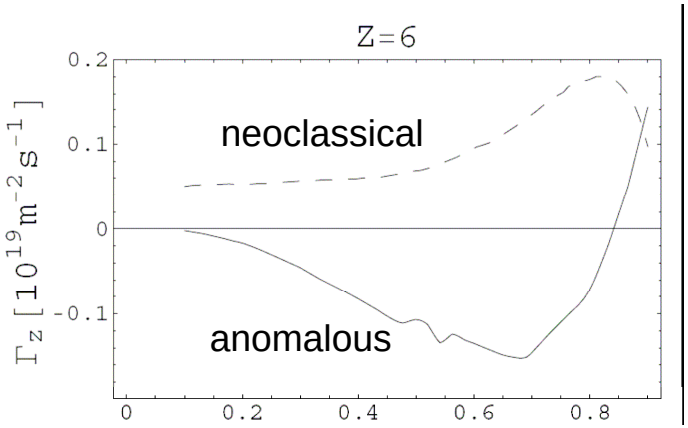
# Nonlinear TEM particle transport

- Ernst (2004) finds in nonlinear GK simulations
  - Ware pinch sufficient to explain ITB formation in Alcator C-mod.
  - Ware flux balanced by outward particle flux driven by TEM turbulence, exhibiting “Dimits shift”, attributed to zonal flow generation.



# Impurity transport

- Fülöp and Weiland (2006)
  - Neoclassical transport + Weiland model
  - ITER: inward anomalous flux and outward nc transport due to temperature screening




- Angioni and Peeters (2006)
  - Similar model, sign of flux depends on rotation direction of the tubulent fluctuations
  - Explains the dependence of impurity transport on ion and electron temperature gradients

- Estrada-Mila, Candy and Waltz (2005)
  - In a 50-50 DT plasma, T flows inward relative to D
  - Inward He flow at moderate He density gradient
  - Both these NL GK results are reproduced by QL theory in limit of small $k_\theta \rho_a$ and $\omega_d/\omega$

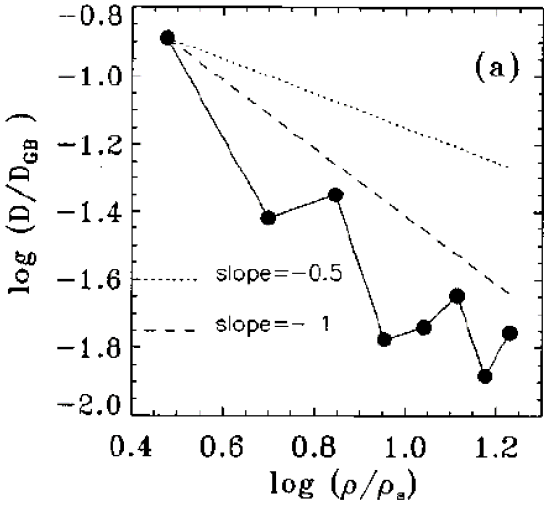
# α-particle transport (1)

- Transport reduced because of FLR averaging
  - Analytically predicted
  - Numerically confirmed

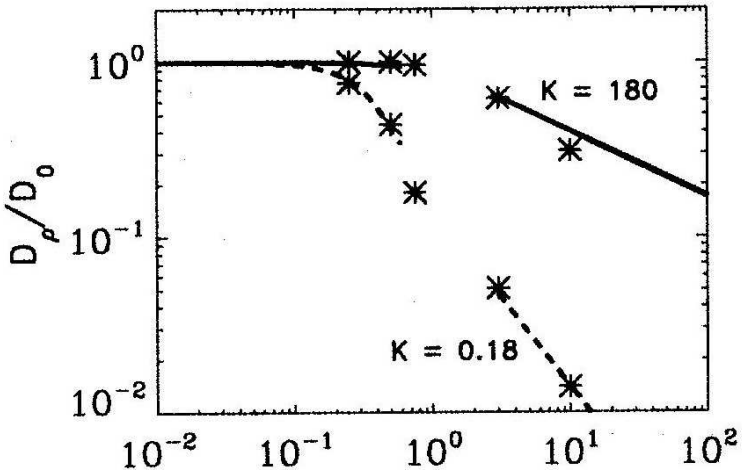
- Recent controversy about scaling of fast-ion transport in prescribed random ExB flow

  - Decorrelation Trajectory Method (M. Vlad, Spineanu et al, 2005) suggests transport increasing with $\rho_\alpha$ at high Kubo numbers $K = V_{ExB}\,\tau_{corr}/\lambda_{corr}$

  - Disputed by Hauff and Jenko (2006), who find transport almost independent of $\rho_\alpha$ at high K
    - Modified Decorrelation Trajectory Method
    - Supported by direct numerical simulations

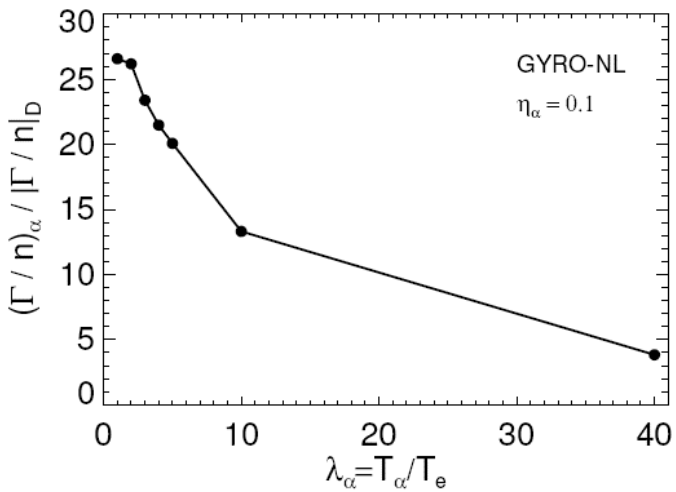



Manfredi and Dendy (1997)



# $\alpha$-particle transport (2)

- However, in practice $k_\theta \rho_\alpha$~0.5 (Estrada-Mila, Candy and Waltz, 2006)
  - Nonlinear GK simulation with Maxwellian trace alphas ($T_\alpha >> T_{e,i}$) suggests <u>larger</u> flux per alpha particle than for thermals
  - Alpha particle confinement time still much longer than the slowing-down time, but
  - Some broadening (~15%) of the alpha particle density profile




Estrada-Mila, Candy and Waltz (2006)

# Summary

- Classical mechanisms for density peaking:
  - Ware pinch – important at low temperature
  - NBI pinch – fuelling occurs one beam-ion banana width inside or outside deposition radius
  - RF pinch – smaller than Ware pinch if resonant electrons are suprathermal
  - Fuelling – plays important role in present-day experiments
- Turbulence-driven particle pinches:
  - Dominant at low collisionality or large device size
  - Thermodiffusion inward only at low enough collisionality
  - Trapped and circulating ions/electrons have different transport properties
- Impurity transport:
  - Neoclassical transport important – temperature screening prevents central impurity accumulation in banana regime
  - Anomalous flux inward or outward
- Alpha particle transport:
  - Reduction because of gyro-averaging
  - Surprisingly large, but confinement still satisfactory

# Open questions

- When and why is the thermodiffusion flux inward/outward?
- What is the role of collisionality, magnetic shear, flux-surface geometry etc?
- Is thermodiffusion more or less important than the curvature pinch?
- Why is the core density profile more peaked in L-mode than in H-mode?
- What is the cause of density transport barriers?
- How much impurity accumulation can we expect in ITER?
  - Standard operation
  - Advanced regimes
- Will D-T separation be a problem?
- Will the ash be removed easily?
- Can we be sure that alpha particles will be confined well enough?

# Core fuelling

- Neutral atoms diffuse into the plasma through charge exchange. Diffusion velocity of short-mean-free path neutrals:

$$\mathbf{V}_n = -D\left(\nabla \ln n_n + 0.76 \nabla \ln T_i\right), \qquad D = \frac{T_i}{m_i \nu_x}, \quad \nu_x = 2.93 n_i \left(T_i / m_i\right)^{1/2} \sigma_x$$

- Continuity equation

$$\frac{1}{r}\frac{d}{dr} r \frac{d(n_n V_{nr})}{dr} = \nu_z n_n$$

has WKB solution

$$\frac{n_n(r)}{n_n(a)} \sim \frac{1}{\sqrt{rT_i}} \left(\frac{n_i}{\nu_z}\right)^{1/4} \exp\left(-\int_r^a \frac{dr'}{\lambda(r')}\right)$$

$$\lambda^2 = \lambda_x \lambda_z = \frac{\sqrt{T_i / m_i}}{2.93 n_i \nu_z \sigma_x}$$