



# **Large transport induced operational limits in tokamak plasmas**

**P. N. GUZDAR**

**INSTITUTE FOR RESEARCH IN ELECTRONICS AND APPLIED  
PHYSICS (IREAP)  
UNIVERSITY OF MARYLAND**

**11<sup>TH</sup> EU-US TTF Workshop,  
Sofitel Vieux Port  
Marseille, France  
4-7 September 2006**

**IN COLLABORATION WITH R.G. KLEVA , UMD, USA  
P. K. KAW AND R. SINGH, IPR GANDHINAGAR, INDIA  
B. LABOMBARD AND M. GREENWALD, MIT, USA**



# OUTLINE OF THE TALK

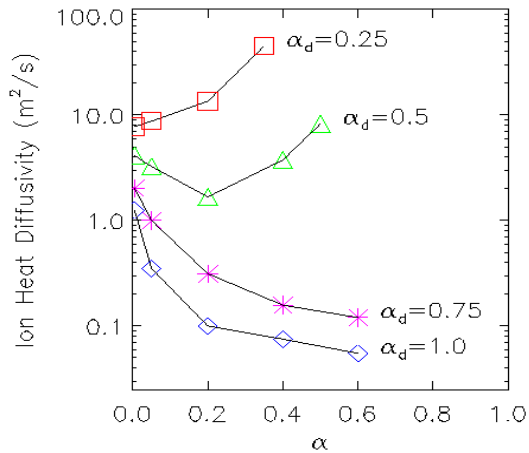
---

- MOTIVATION
- THEORETICAL/NUMERICAL MODEL
- COMPARISON WITH EXPERIMENTS
- CONCLUSIONS/FUTURE WORK



# RECENT COMPARISON ON C-MOD WITH SIMULATIONS/THEORY

Rogers, Drake and Zeiler, PRL, 81, 4396, 1998  
 Guzdar, Kleva, Das and Kaw, PRL 2001  
 L-H transition criterion  $\alpha_{MHD} \alpha_D^2 = 0.08-0.12$



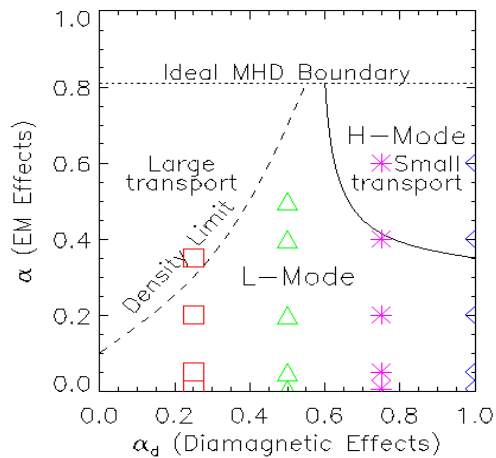
$$\alpha_D = \frac{\rho_s c_s t_0}{L_0 L_n (1 + \tau)}$$

$$t_0 = \frac{(R L_n / 2)^{1/2}}{c_s}$$

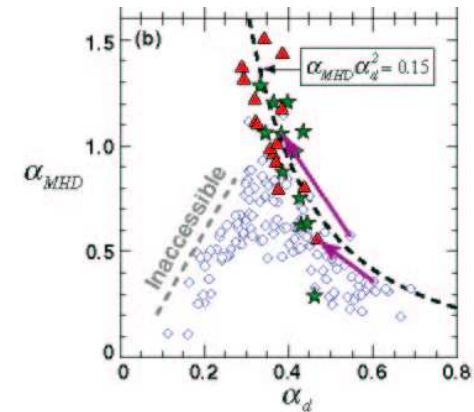
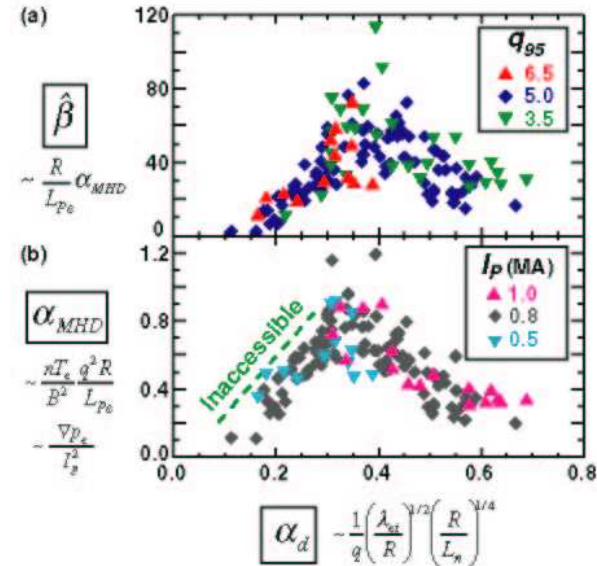
$$\alpha_{MHD} = \frac{q^2 R \beta}{L_p}$$

$$\hat{\beta} = \frac{q^2 R^2 \beta}{4 L_p^2} = \alpha \frac{R}{4 L_p}$$

$$L_0 = 2\pi q \left( \frac{v_{ei} R \rho_s}{2\Omega_e} \right)^{1/2} \left( \frac{2R}{L_n} \right)^{1/4}$$



LaBombard et al NF 45,1568,(2005)





# TABLE 1

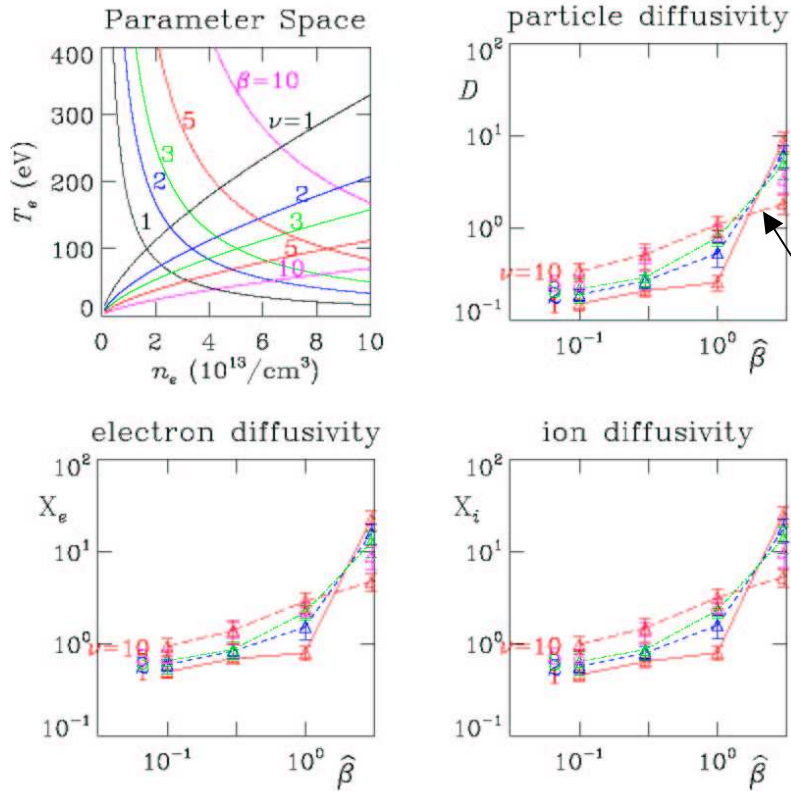
LaBombard et al NF 45,1568,(2005)

	RDZ	Scott
<b>Definitions and Normalizations</b>		
Time $\frac{\partial}{\partial t} \sim \frac{1}{t_0}$	Ideal Ballooning Time $t_0 = \left(\frac{RL_n}{2}\right)^{1/2} \frac{1}{C_s}$	Drift Scaling $t_0 = \frac{L_{pe}}{C_s}$
Perpendicular $\nabla_{\perp} \sim \frac{1}{L}$	Linear Resistive Ballooning Scale $L = L_0 = 2\pi q \left(\frac{\rho_s \rho_e R}{\lambda_{ei}}\right)^{1/2} \left(\frac{2R}{L_n}\right)^{1/4}$	$L = \rho_s$
Parallel $\nabla_{\parallel} \sim \frac{1}{L_{\parallel}}$	$L_{\parallel} = 2\pi q R$	$L_{\parallel} = qR$
Background Gradients	$\frac{\nabla_{\perp} n_0}{n_0} \sim \frac{1}{L_n}$ ; $\frac{\nabla_{\perp} T_{e0}}{T_{e0}} \sim \frac{1}{L_{Te}}$ ; $\frac{\nabla_{\perp} T_{i0}}{T_{i0}} \sim \frac{1}{L_{Ti}}$ ; $\frac{\nabla_{\perp} p_{e0}}{p_{e0}} \sim \frac{1}{L_{pe}}$	
<b>Key Control Parameters</b>		
Poloidal Beta Gradient	MHD Ballooning Parameter $\alpha_{MHD} = \frac{q^2 R}{L_{pe}} \beta$ ; $\beta = \frac{4\mu_0 p_{e0}}{B^2}$	$\hat{\beta} = \frac{q^2 R}{L_{pe}} \beta \left(\frac{R}{4L_{pe}}\right)$
Collisionality	Drift Frequency/Ballooning Time $\alpha_d = \frac{T_{e0} t_0}{eBL_n L_0}$ $\alpha_d = \left(\frac{\lambda_{ei}}{q^2 R}\right)^{1/2} \left(\frac{2R}{L_n}\right)^{1/4} \frac{1}{8\pi} \left(\frac{M_i}{m_e}\right)^{1/4}$	Normalized Collision Frequency $C_0 = \frac{m_e}{M_i} \left(\frac{qR}{L_{pe}}\right)^2 \frac{\nu_{ei} L_{pe}}{C_s}$ $C_0 = \left(\frac{q^2 R}{\lambda_{ei}}\right) \left(\frac{R}{L_{pe}}\right) \left(\frac{m_e}{M_i}\right)^{1/2}$
<b>Relationships Between Control Parameters</b>		
$\alpha_{MHD} = \hat{\beta} \frac{4L_{pe}}{R}$ ; $\alpha_d = \frac{C_0^{-1/2}}{4\pi} \left(\frac{R}{2L_n}\right)^{1/4} \left(\frac{R}{2L_{pe}}\right)^{1/2}$		

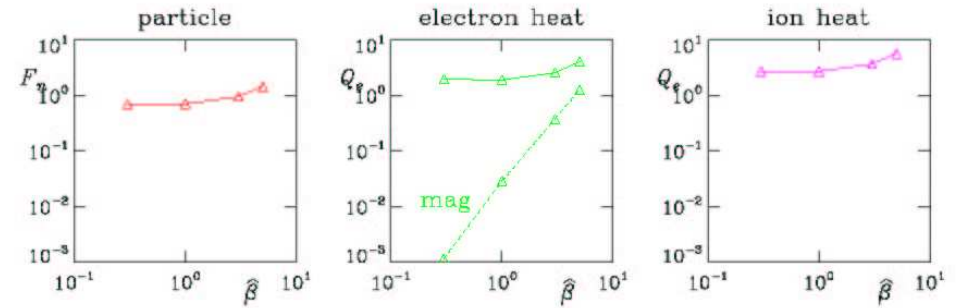


Scott NJP, 4, 52.1-52.30, (2002)  
Braginskii Equations

Scott IAEA-11-S7, (2005)  
Gyrokinetic Edge Code



$L_p = 4.25 \text{ cm}, R=169 \text{ cm}, L_n = 2L_T$   
 $\alpha_{\text{MHD}} = 0.1\hat{\beta} \quad \alpha_D = 0.4\nu^{-1/2}$



$\frac{R}{L_T} = 30, L_n = 2L_T, L_p = \frac{2}{3}L_T$   
 $\alpha_{\text{MHD}} = 0.09\hat{\beta} \quad \alpha_D = 0.46$

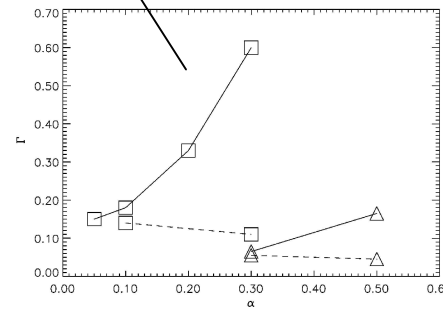
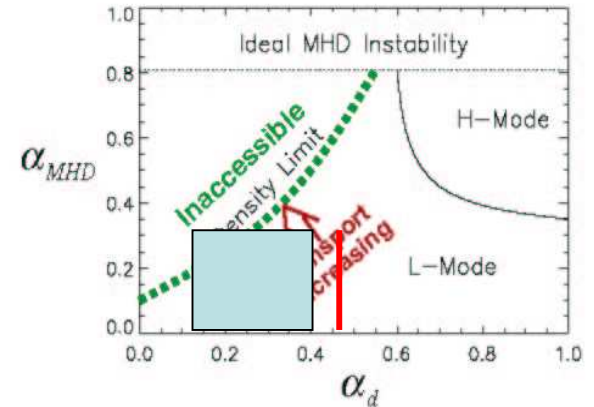


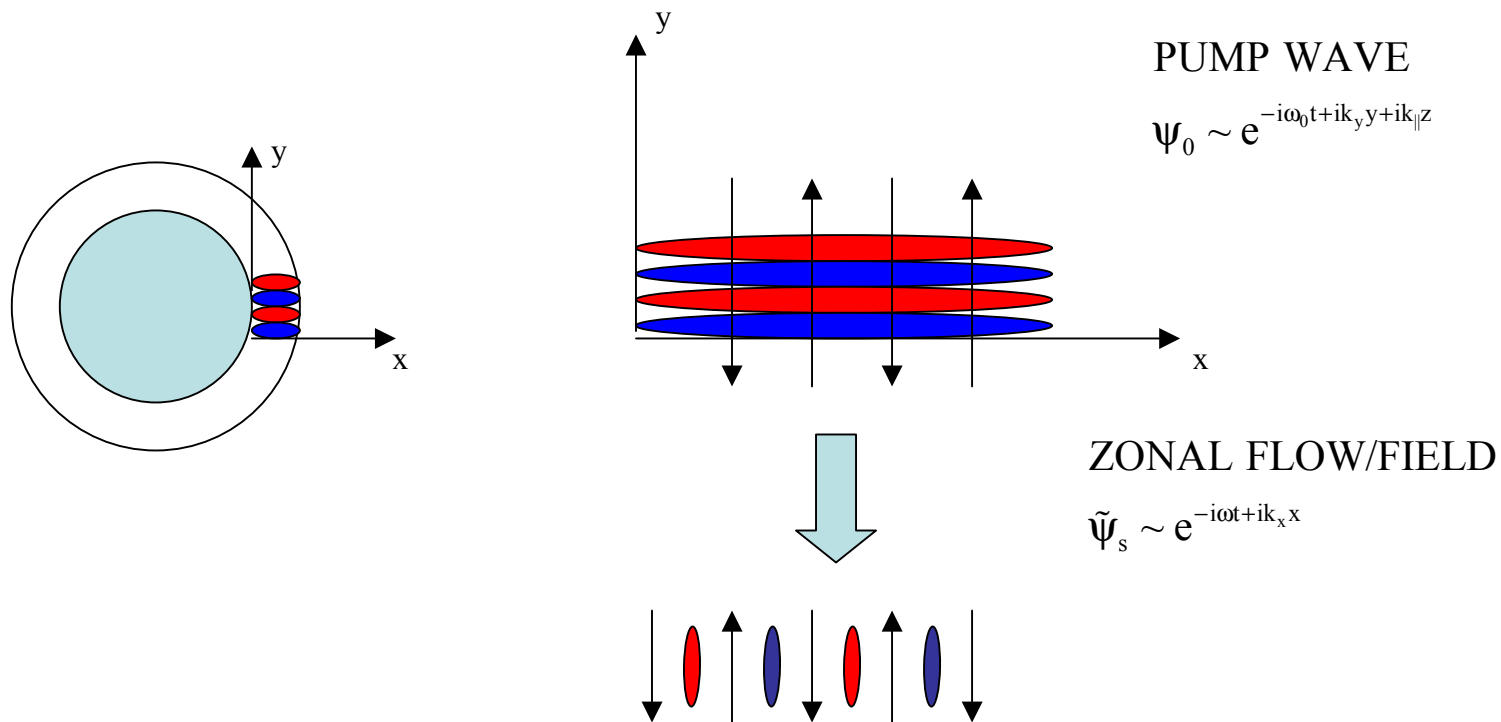
FIG. 3. Normalized particle flux  $\Gamma$  vs  $\alpha$ .





# BASIC IDEA FOR PRESENT MODEL-1

- BASIC INSTABILITIES IN THE EDGE REGION OF TOKAMAKS ARE FINITE  $\beta$  DRIFT WAVES AND DRIFT RESISTIVE BALLOONING MODES
- “MODULATIONAL” INSTABILITY GENERATES ZONAL FLOWS WHICH SATURATE FINITE  $\beta$  DRIFT WAVE/RESISTIVE BALLOONING TURBULENCE



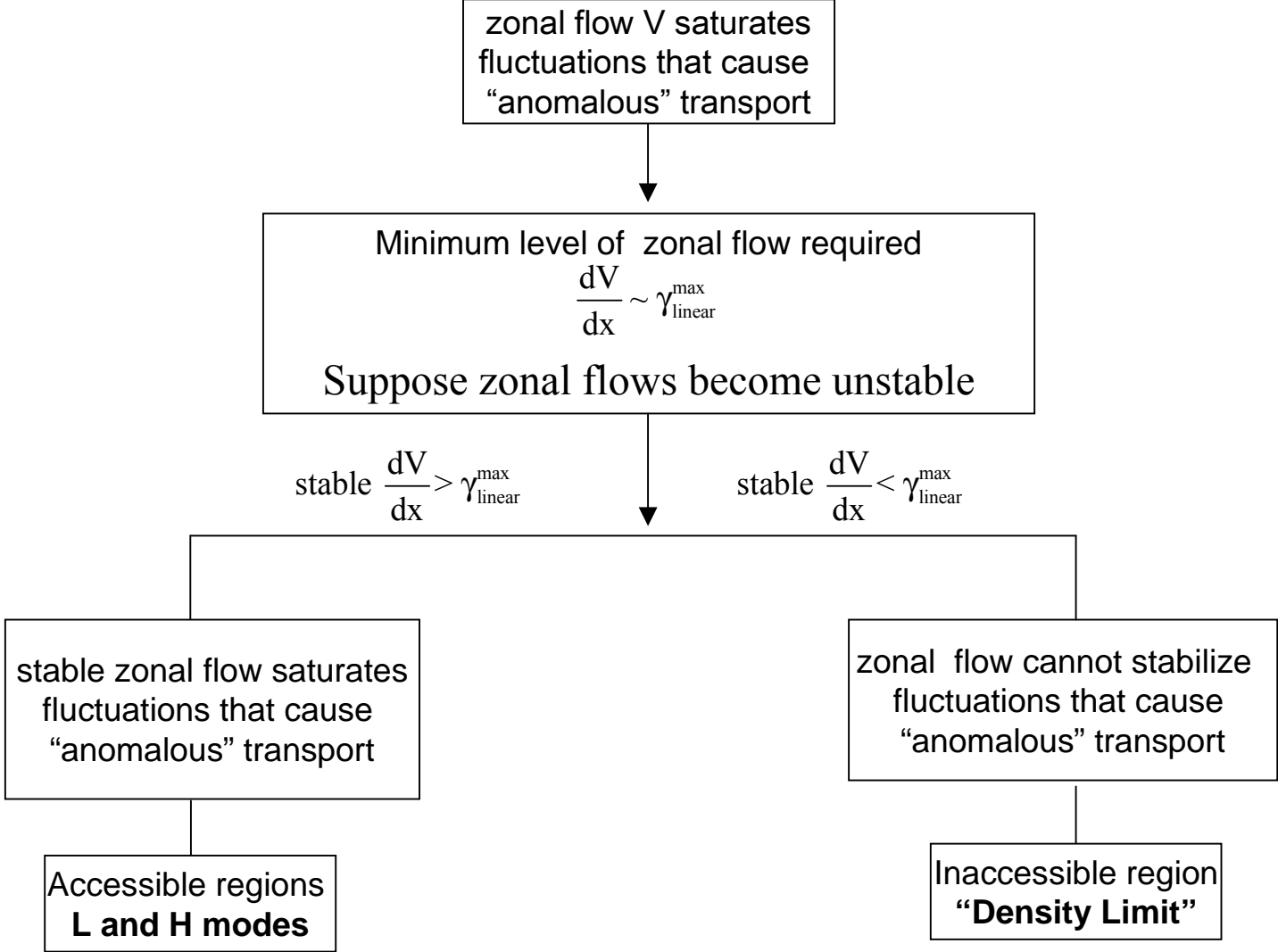
INACCESSIBLE REGION DUE TO VERY LARGE EDGE TRANSPORT

-GREENWALD PPCF 44, R27-R53 2002

WHAT PHYSICAL PROCESS LEADS TO VERY LARGE TRANSPORT ?



# SCENARIO FOR LARGE EDGE TRANSPORT





# STABILITY OF ZONAL (SHEAR) FLOWS

B. N. Rogers and W. Dorland PoP **12**, 062511 2005

Marginal stability (for linear velocity and magnetic shear)

$$\underbrace{k_y V'_E}_{\text{Kelvin-Helmholtz drive}} = \frac{k_y \hat{s}_{eff}}{\underbrace{\sqrt{\alpha_{MHD}}}_{\text{fieldline bending}}}$$

With ion diamagnetic effects

$$V'_E = \left[ \frac{(\hat{s} - f \alpha_{MHD})^2}{\alpha_{MHD}} + \frac{\alpha_D^2}{4x_L^2} \right]^{1/2}$$

$x_L \sim 1$ , *localization – width*





# LINEARIZED EQUATIONS FOR UNSTABLE MODES IN THE EDGE REGION OF TOKAMAKS

Rogers, Drake and Zeiler, PRL, **81**, 4396, 1998,  
Hastie Ramos and Porcelli PoP, **10**, 4405, 2001

$$\frac{d}{d\theta} [f_1(\theta)\Psi] + i\alpha_{MHD} [\Omega f_1(\theta)\Phi + f_2(\theta)N] = 0$$

$$\frac{d}{d\theta} [\Phi - (\tau + 1)\hat{m}\alpha_D N] - \frac{\hat{m}^2 f_1(\theta)}{4\pi^2} \Psi + i(\Omega - \hat{m}\alpha_D)\Psi = 0$$

$$[1 - 2\varepsilon_n f_2(\theta)]\Phi - \tau\hat{m}\alpha_D N - \Omega \left[ N + \frac{\hat{m}\hat{\rho}_s^2}{\alpha_D} f_1(\theta)\Phi \right] = 0$$

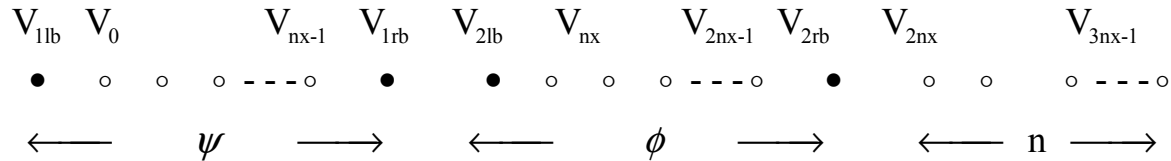
$$f_1(\theta) = 1 + [s\theta - \alpha_{MHD} \text{Sin}(\theta)]^2$$

$$f_2(\theta) = \text{Cos}(\theta) + [s\theta - \alpha_{MHD} \text{Sin}(\theta)] \text{Sin}(\theta)$$



# NUMERICAL METHOD

- Finite difference the equations
- Create vector  $V=(\Psi, \Phi, n)$



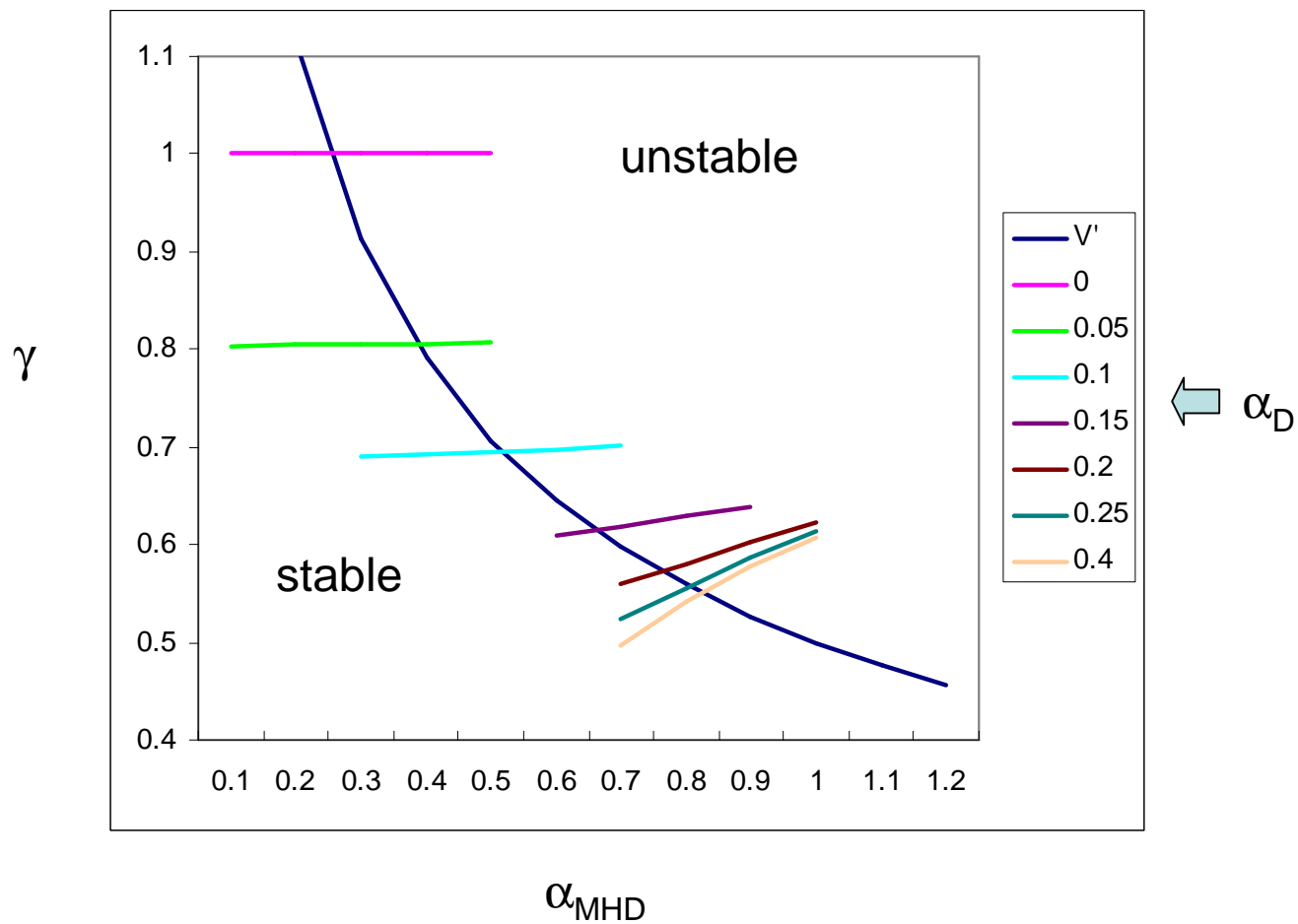
- Reduce three coupled equations to eigenvalue equation
- A and B are matrices

$$\tilde{A}\tilde{V} = \Omega\tilde{B}\tilde{V}$$

Leads to determination of unstable eigenmodes from “all” branches, finite  $\beta$  dissipative drift waves and drift resistive ballooning modes

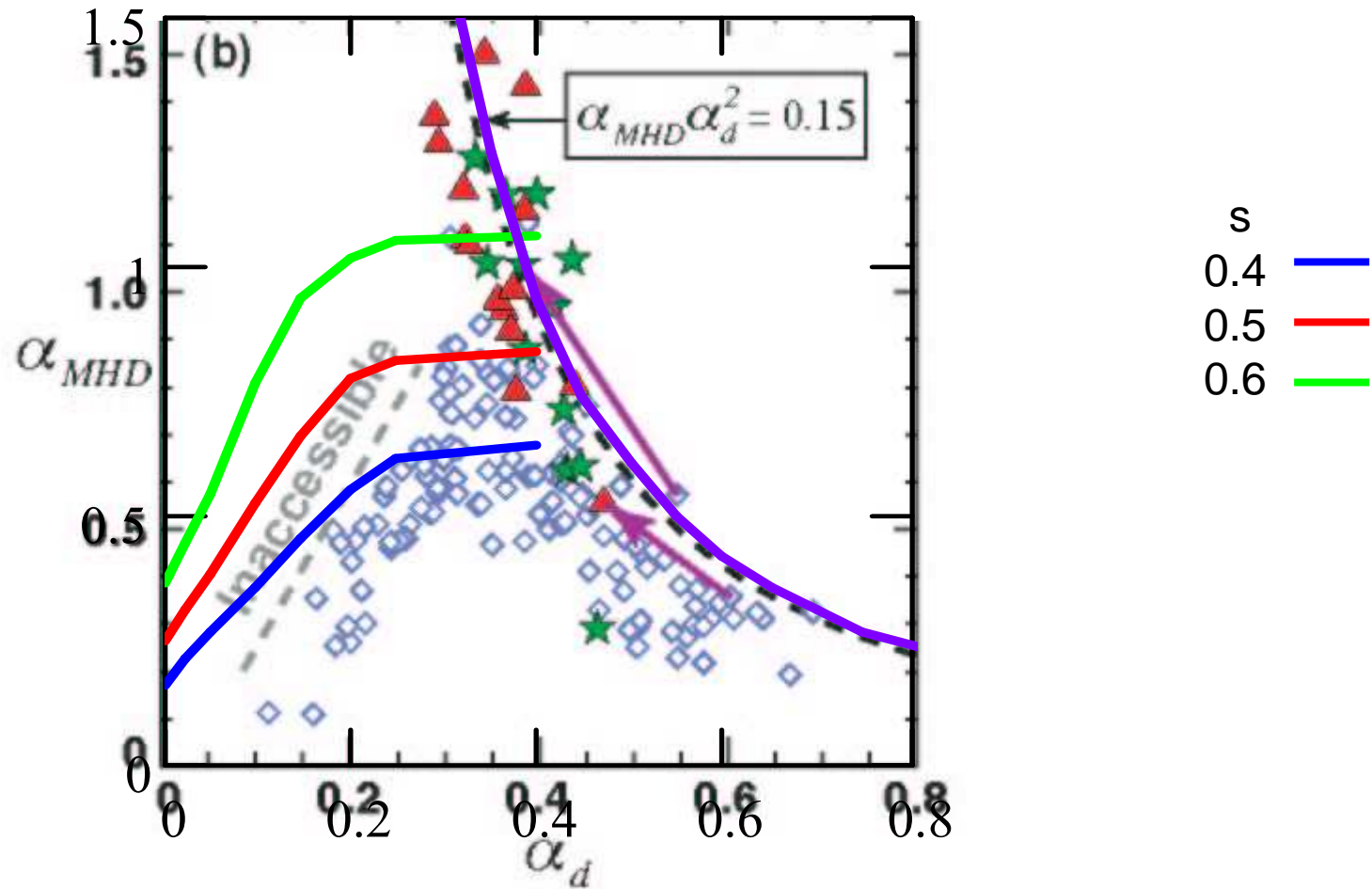


# NUMERICAL RESULTS



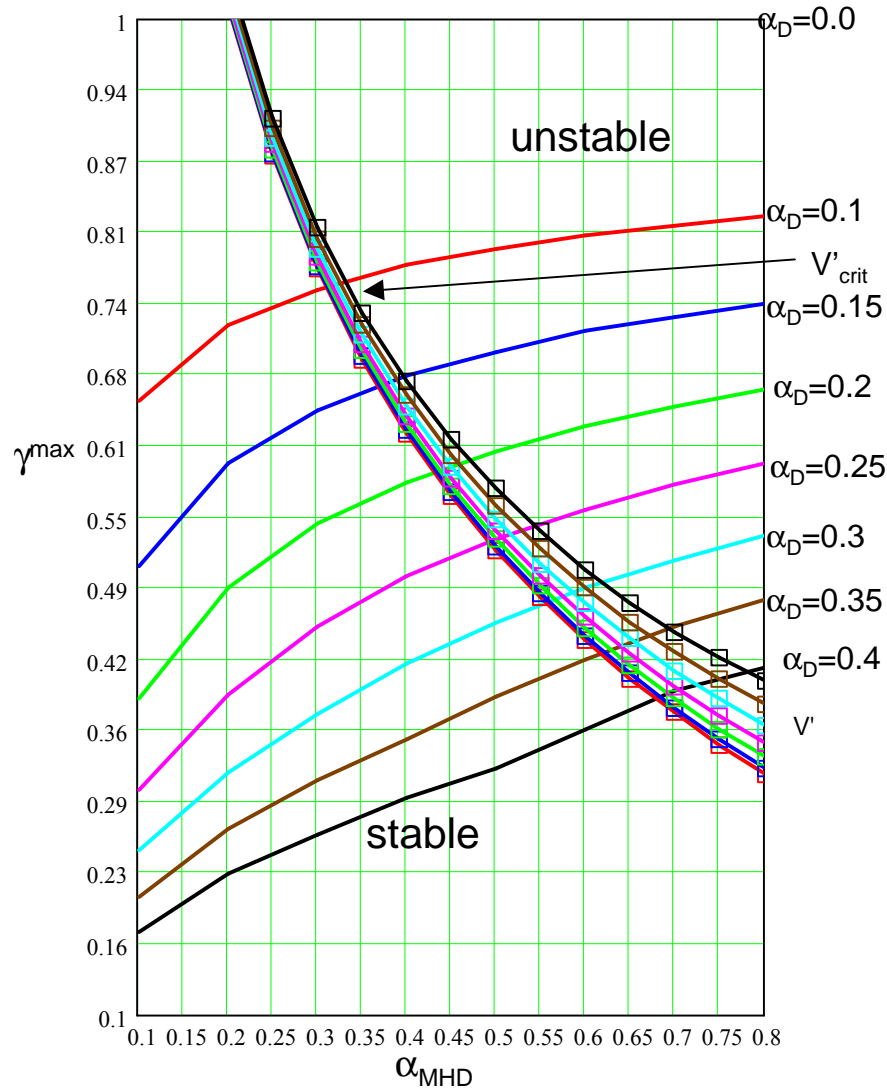


# NUMERICAL RESULTS

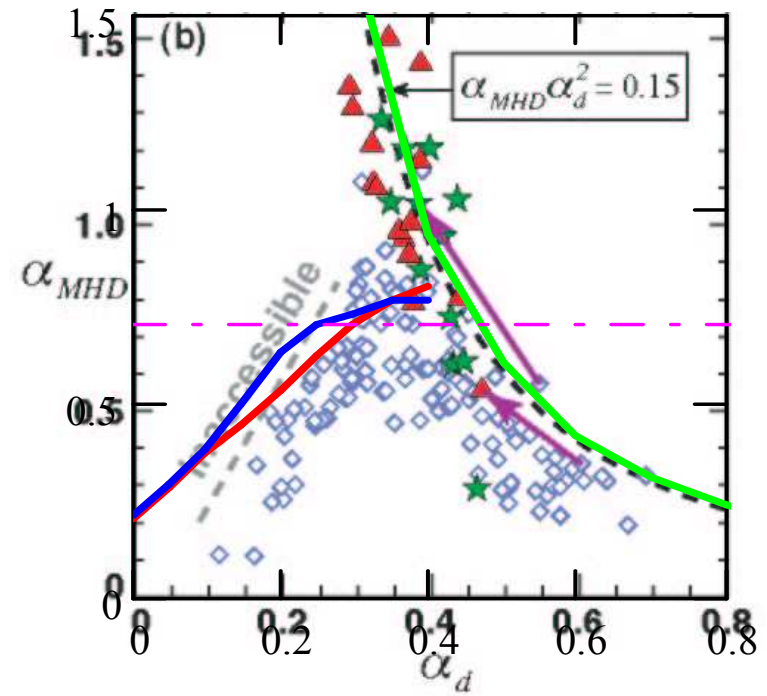




# NUMERICAL RESULTS



$s=0.5$   $f=0.0$   
 $s=0.5$   $f=0.5$





# NUMERICAL RESULTS

“DENSITY LIMIT” CONDITION  
NOT GREENWALD LIMIT

From

$$\alpha_{\text{MHD}} = 3f(s)\alpha_{\text{D}} \quad f(s) \sim 1$$

$$\sqrt{n} \frac{a^2}{I_p} = 2 \frac{R^{7/12} L_n^{1/4} f(s)}{B^{1/3} Z_{\text{eff}}^{1/6}}$$

$n(10^{20} / \text{m}^3)$ ,  $a(\text{m})$ ,  $R(\text{m})$ ,  $I_p(\text{MA})$ ,  $B(\text{T})$



# CONCLUSIONS

- Investigated stability of zonal flows
- Criterion for inaccessible boundary stable  $dV/dx < \text{Max}(\gamma_{\text{linear}})$
- Gives qualitative boundary consistent with observations and Rogers' et al. simulations
- Present model suggests weak magnetic shear in edge for quantitative comparison with C-MOD data

## FUTURE WORK

- Improve estimate of stability condition for zonal flow
- Incorporate shaping effects in model for threshold for KH instability of zonal flows and finite beta drift wave/DRBM modes
- Explore the “interaction” region between the stability boundary for KH and LH to identify type of ELMS. ELMS are interplay between ballooning type modes and zonal and/or shear flow
- Extend study to full two dimensional eigenvalue stability of modes with shear to determine stability boundary more accurately
- Explore the connection of the “operational limit” boundary in  $(\alpha_{\text{MHD}}, \alpha_{\text{D}})$  space to the Greenwald limit