The common ground of the density pinch and the transition to high confinement regimes in tokamak

Florin Spineanu and Madalina Vlad
Association EURATOM-MEdC Romania
National Institute of Laser, Plasma and Radiation Physics, Bucharest

Overview

There is a fundamental dynamics of the ideal plasma/neutral fluid leading to evolution toward regular structures and coherent flows. It exists in the background of the instabilities and turbulence.

First part : why is acceptable to say that the vorticity self-organises

- The coherent structures arising at relaxation can be seen as the result of organisation of the vorticity field. The model of point-like vortices interacting via a potential allows the representation of the vorticity as a self-organising fluid, which acts as a drive for passive fields like density.
- The property of the vorticity field to self-evolve to organised states is supported by fundamental properties of the discrete point-like vortices model:

- the ensemble of point-like vortices interacting in plane via a short-range or long-range potential is a statistical system with **negative temperature**

$$\frac{1}{T} = \frac{a^2}{4\pi n e^2} \left[\exp \left(-\frac{E}{Ne^2} \right) - 1 \right]$$

- the continuum limit of the system of point-like vortices can be described as the equations of continuity and momentum conservation for an ideal compressible fluid; this fluid has **negative pressure**.
- the ideal fluid with negative pressure can be derived from a very fundamental physical model: a line evolving in plane, under the condition that the area of the world-surface is minimum
- the model of point-like vortices directly leads to the sinh-Poisson equation describing the stationary states of organised vorticity;
- the same sinh-Poisson equation is the Gauss-Codazzi equation for the world-surface from which derives the model of ideal fluid with

negative pressure

- Conclusion of the first part: the vorticity self-organises; in the Euler fluid (point-like vortices with long-range, logarithmic interaction) the density of the fluid is constant, the fluid is incompressible. In the Hasegawa-Mima plasma (point-like vortices with short-range K_0 interaction) the density follows the vorticity

Second part : density reaction is via the effective Larmor radius

- The vorticity builds up a central cluster surrounded by a ring of vorticity of opposite sign. The density must follow the vorticity (Ertel's theorem). This is only possible due to the polarisation drift, compressible, which implies (but does not explicitely uses) the third dimension.
- The pinch of vorticity is at the origin of the pinch of density; the density develops gradients and in consequence, diamagnetic flow

- The combination of diamagnetic flow v_d and of rotation speed u induces a change in a basic parameter: ρ_s is replaced by an *effective* Larmor radius

$$\frac{1}{\left(\rho_s^{eff}\right)^2} = \frac{1}{\rho_s^2}\left(1 - \frac{v_d}{u}\right)$$

- When the density dragged by the vorticity develops higher gradients, the effective Larmor radius becomes very large $\rho_{eff} \to \infty$. The model of point-like vortices with short-range interaction (Hasegawa-Mima) evolves to the point-like vortices with long-range interaction (Euler fluid). The density decouples from the vorticity;
- (Actually the dynamics is slowed down)

Third part : the structure of the space of stationary solutions

The equation is:

$$\Delta\psi + \frac{1}{2p^2}\sinh\psi\left(\cosh\psi - p\right) = 0$$

There are two exact solutions :

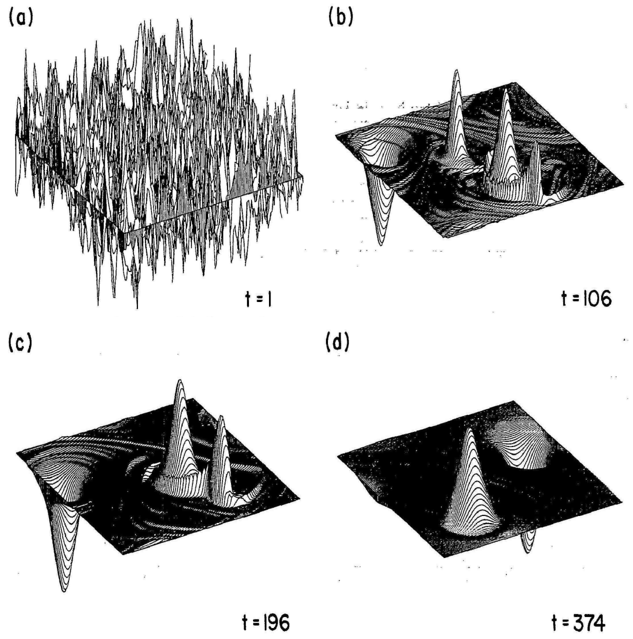
- smooth symmetric monopolar vortex
- singular vortex

We find a string of quasi-solution in-between. Under a weak physical factor the system may slide from the smooth vortex to the singular one, entraining the density.

A simple functional measure shows that there is a line of minimum error in the parameter space, along which these quasi-solutions are found.

A special class of solutions is determined, consisting of strongly sheared v_θ in the edge region. These states are connected via a string of quasi-solutions with states of regular (of the type Hasegawa-Wakatani) potential profiles. There are differences in both energy and total vorticity between the two classes of solutions. This shows that a transition to a better confinement should also require a minimum amount of vorticity input.

First part : What is behind the evolution of vorticity to coherent structures

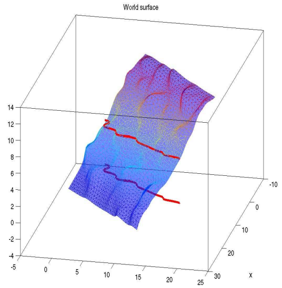


Euler equation. D. Montgomery et al., Phys. Fluids A4 (1992) 3.

There are two fundamental models :

- the line evolving under Nambu-Goto action leads to a fluid with negative pressure and strange polytropic (Chaplygin gas).
- a fluid/plasma with vorticity is equivalent with a system of point-like vortices interacting in plane via a potential
  - long-range (Coulombian, logarithmic) : Euler fluid. No density ($\rho =$ const) The stationary states *sinh*-Poisson equation. Exactly integrable.
  - short-range (screened, K_0) : Charney-Hasegawa-Mima plasma/atmosphere. The third direction is implicitely present. The density is not constant.

How are they connected: both reduce to the *sinh*-Poisson equation, along two ways: at self-duality and as condition of emdedding in $\mathbf{R}^3$ according to Gauss-Codazzi eqs.



Line moving in space

More generally: a d-dimensional object evolves in a $d+1$ dimensional space. Internal coordinates: $\left(\phi^0, \phi^1, \ldots, \phi^d\right)$ (where ϕ^0 is the time). External coordinates: $x^\mu \equiv \left(x^0, x^1, \ldots, x^d, x^{d+1}\right)$ (where x^0 is the time).

A line describes a **World Surface**. The *action* is the area

$$I_d = -\int d\phi^0 d\phi^1 \ldots d\phi^d \sqrt{G}\ ,\ G_{\alpha\beta} \equiv (-1)^d \det\{\frac{\partial x^\mu}{\partial \phi^\alpha}\frac{\partial x_\mu}{\partial \phi^\beta}\}$$

$$\begin{aligned} \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}(\rho v_i) &= 0 \\ \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial p}{\partial x_i} \end{aligned}$$

for an ideal fluid with the pressure $p = -2\lambda/\rho$

Dynamics in plane. Good for the density pinch in Tokamak

A line moving in plane in cylindrical geometry: *absolute* coordinates (x^0, x^1, x^2) : $x^0 \equiv$ time, $x^1 \equiv$ azimuthal length $x^2 \equiv$ radial length; *internal* coordinates (ϕ^0, ϕ^1), $\phi^0 \equiv$ time, $\phi^1 \equiv$ length along the line.

$$g = \left(\frac{dx^1}{d\phi^1}\right)^2 = \frac{1}{\rho^2}\ , \ \frac{dx^1}{d\tau} = \nabla\theta = v$$

The velocity is directed toward the current centre. The solution is a constant current $\mathbf{j} = \rho\mathbf{v} = 2\lambda$

The existence of the world-surface generated by the line is possible if the Gauss-Codazzi eq. is verified. This leads to

$$\Delta\psi + \sinh\psi = 0$$

The same equation is derived from the system of point-like vortices interacting via long-range (logarithm) potential. This connection explains why the latter has a *negative pressure* and the former *negative temperature*.

The fact that the density follows the vorticity is due to the existence of an intrinsic finite length in $2D$ plasma: the Larmor radius

The Charney-Hasegawa-Mima equation

The equation (CHM) derived for the two-dimensional plasma drift waves and for Rossby waves in meteorology is:

$$\left(1 - \nabla_\perp^2\right) \frac{\partial \phi}{\partial t} - \kappa \frac{\partial \phi}{\partial y} - \left[\left(-\nabla_\perp \phi \times \widehat{\mathbf{n}} \right) \cdot \nabla_\perp \right] \nabla_\perp^2 \phi = 0 \qquad (1)$$

This is the equation governing the stationary states of the CHM eq.

$$\Delta \psi + \sinh \psi \left(\cosh \psi - 1 \right) = 0$$

Second part. How the vorticity evolves to a coherent structure

The conservation laws are less useful here. We need *dynamical* equations

- the aggregation-coagulation statistical process (Phase I, at the beginning of the discharge when a random distribution of vortices is generated. The merging takes place at a rate $t^{-3/4}$ and the result is a central vortex and possibly a ring of opposite vorticity).
- the equations of motion of the field theoretical model (Phase II)
- the equations of the ideal fluid with negative pressure
- the Manton's method : dynamics on the parameter manifold, close to self-duality

$$\mathcal{L} = -\kappa\varepsilon^{\mu\nu\rho}\mathrm{tr}\left(\partial_\mu A_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho\right) \tag{2}$$
$$-\mathrm{tr}\left[(D^\mu\phi)^\dagger(D_\mu\phi)\right]$$
$$-V\left(\phi,\phi^\dagger\right)$$

Sixth order potential

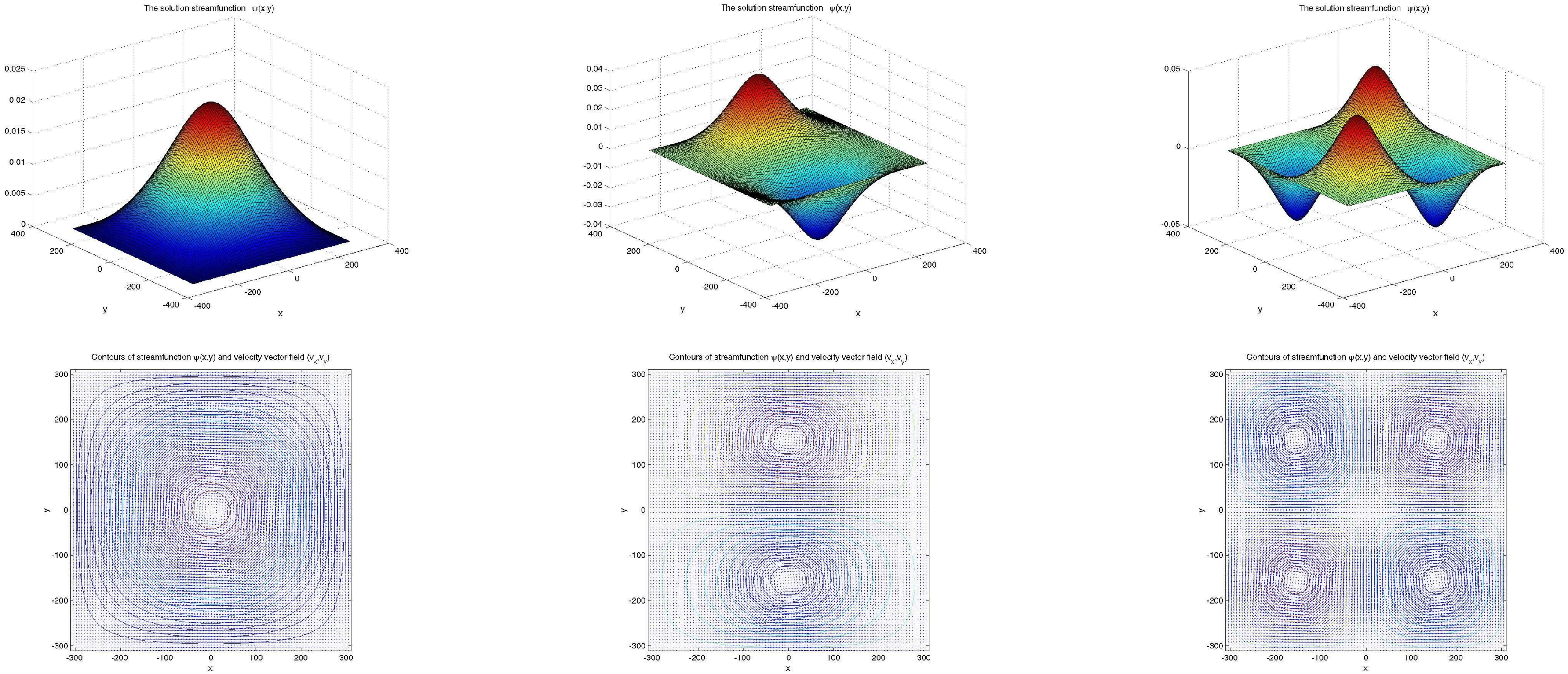
$$V\left(\phi,\phi^\dagger\right) = \frac{1}{4\kappa^2}\mathrm{tr}\left[\left(\left[\left[\phi,\phi^\dagger\right],\phi\right] - v^2\phi\right)^\dagger\left(\left[\left[\phi,\phi^\dagger\right],\phi\right] - v^2\phi\right)\right]. \tag{3}$$

The Euler Lagrange equations are

$$D_\mu D^\mu\phi = \frac{\partial V}{\partial\phi^\dagger} \tag{4}$$

$$-\kappa\varepsilon^{\nu\mu\rho}F_{\mu\rho} = iJ^\nu \tag{5}$$

Numerical solution for $L = 307$: mono- and multipolar vortex



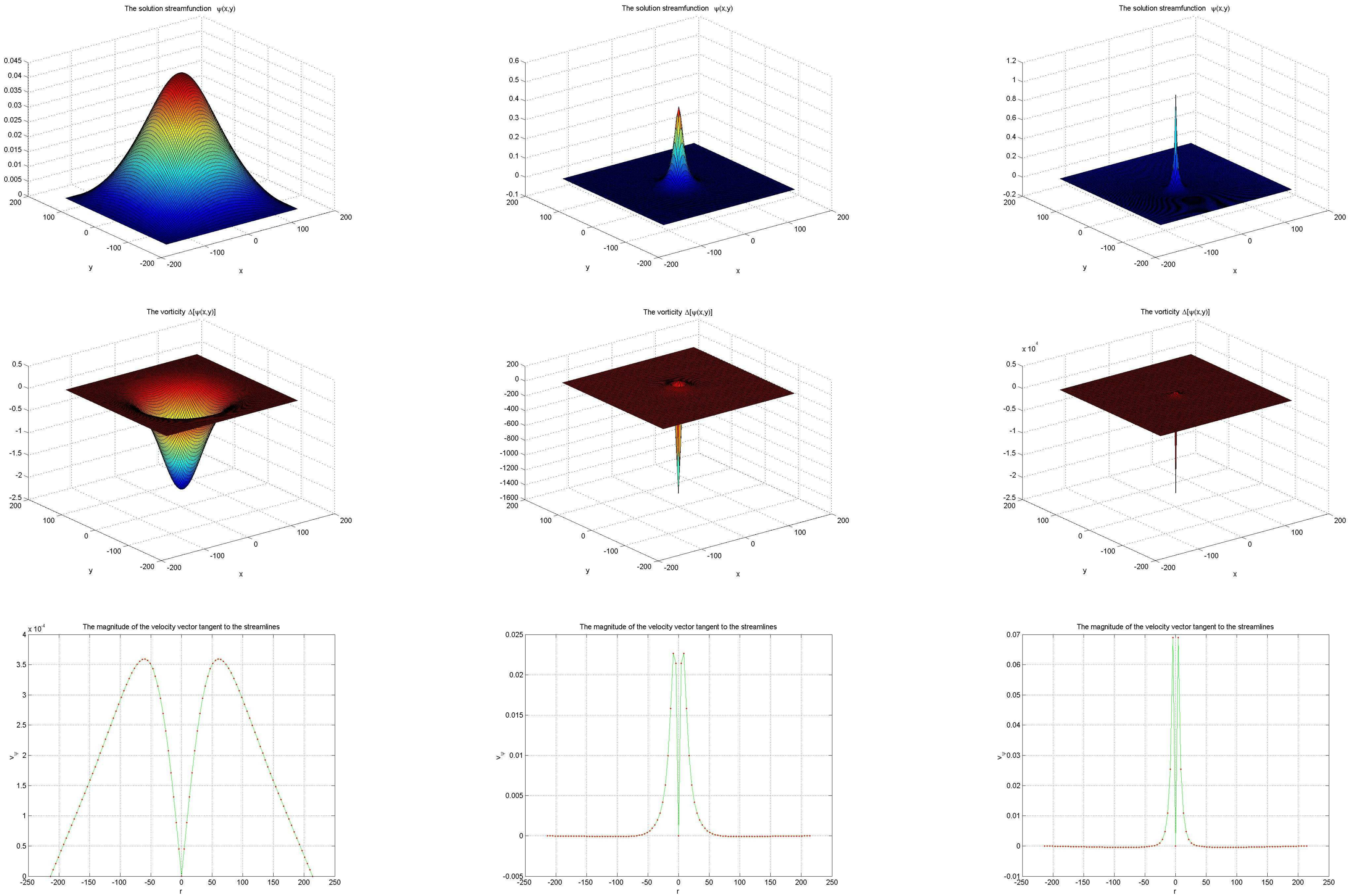
Third part. The space of solutions and the possibility of transitions.

The numerical study of the equation reveals the existance of three type of states

- smooth vortex, stable
- strongly localised (narrow) vortex, a physical approximation of the singular vortex
- a class of intermediate quasi-solutions, organized along a line of minimum departure from the action extremum.

The smooth vortices are accessible from a wide range of initial conditions. The narrow vortices are only accessible from a subset of initial functions.

There are differences: for narrow vortices the maximal vorticity is much higher compared to the smooth vortex. There is a difference in both energy and vorticity between them.


The solution streamfunction ψ(x,y)
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The vorticity Δ[ψ(x,y)]
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x
y
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Comparison with the self-organisation of the vorticity obtained in a statistical approach (HW)

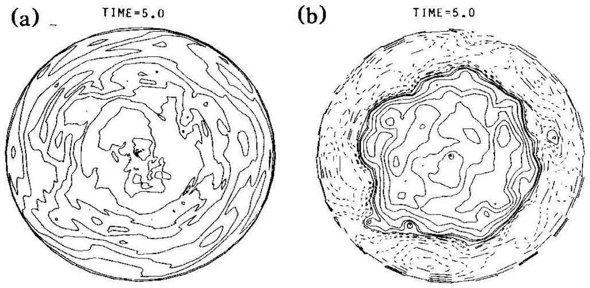


FIG. 1. (a) The density contour and (b) the potential contour from the three-dimensional computer simulation of electrostatic plasma turbulence in a cylindrical plasma with magnetic curvature and shear. In (b) the solid (dashed) lines are for the positive (negative) potential contours. Note the development of closed potential contours near the $\phi \simeq 0$ surface.

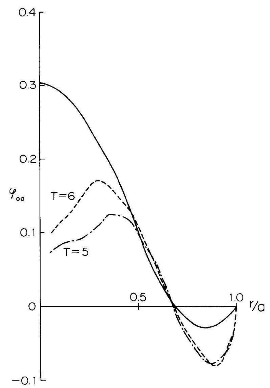


FIG. 2. Profiles of $\phi(r)$ for $m=0, n=0$ mode at two different time steps (dashed and dash-dotted lines) as compared with the predicted profile (solid line) based on the self-organization conjecture. The predicted curve is fitted at $r/a=0.5$.

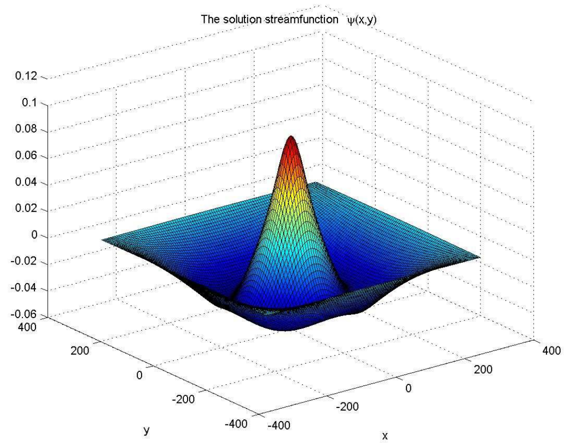
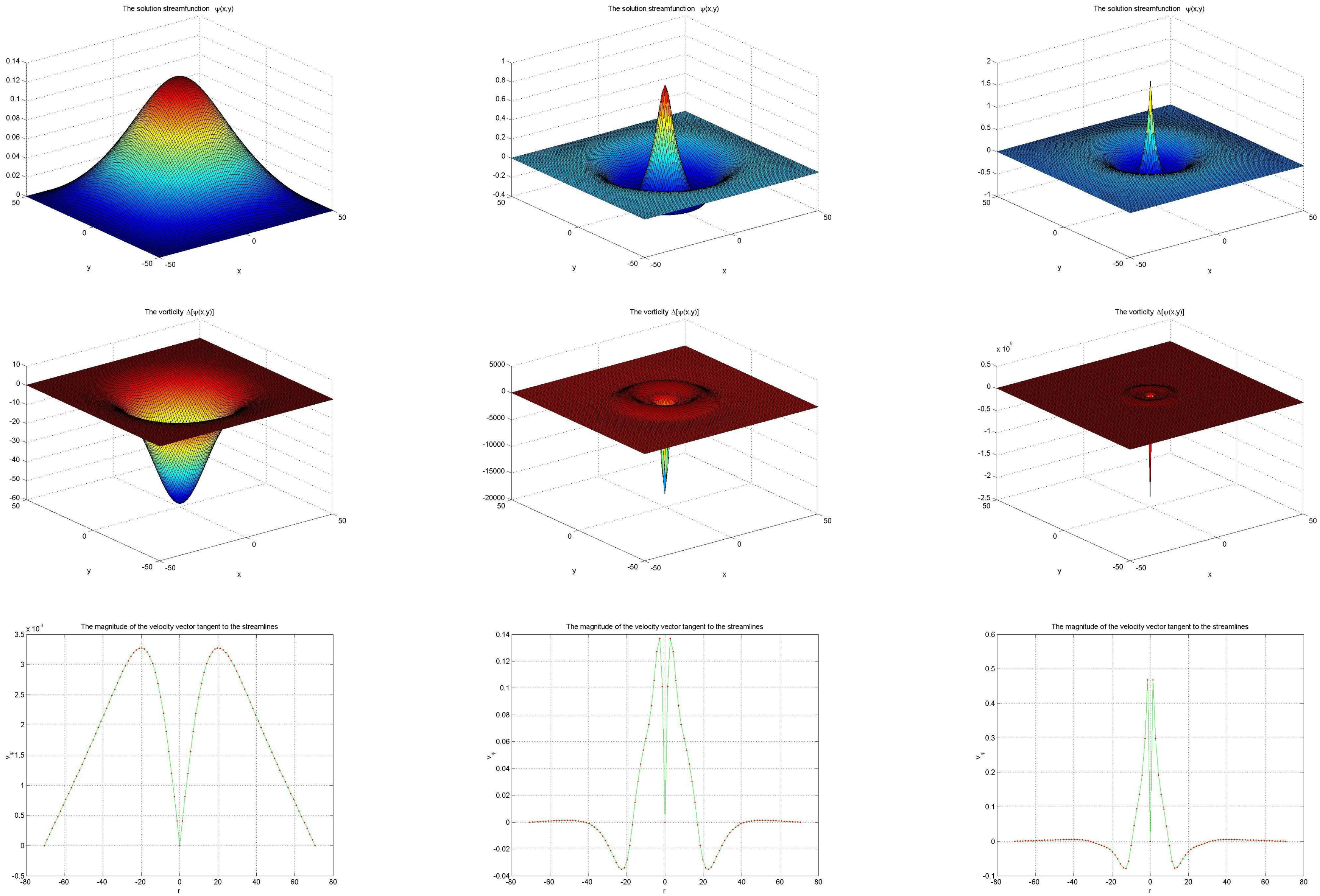


Figure 1: A theory of self-organization of drift turbulence (Hasegawa-Wakatani) leads to this radial profile for the potential. Actually their model does not explicitely need excitation of drift waves.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x
y
r
v_ψ

Transition from *smooth* to *narrow* requires an input of kinetic energy and localised vorticity

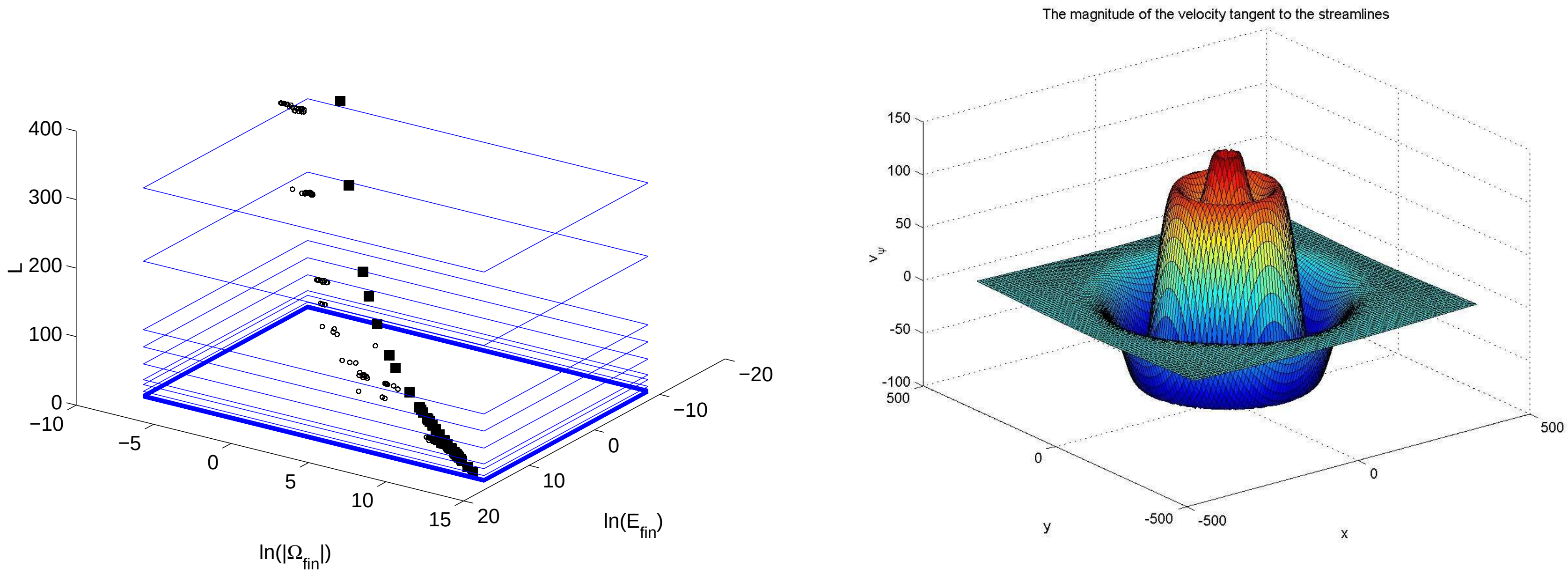

Figure 2: The squares are *smooth* and the dots are *narrow* vortices. The plot presents the difference in both kinetic energy and vorticity. At right is a qs. for $L = 401$ with slow intermediate variation of v_θ.

Scatterplot (Energy, Vorticity) of final results for *pinch* and for *sheared velocity* v_θ

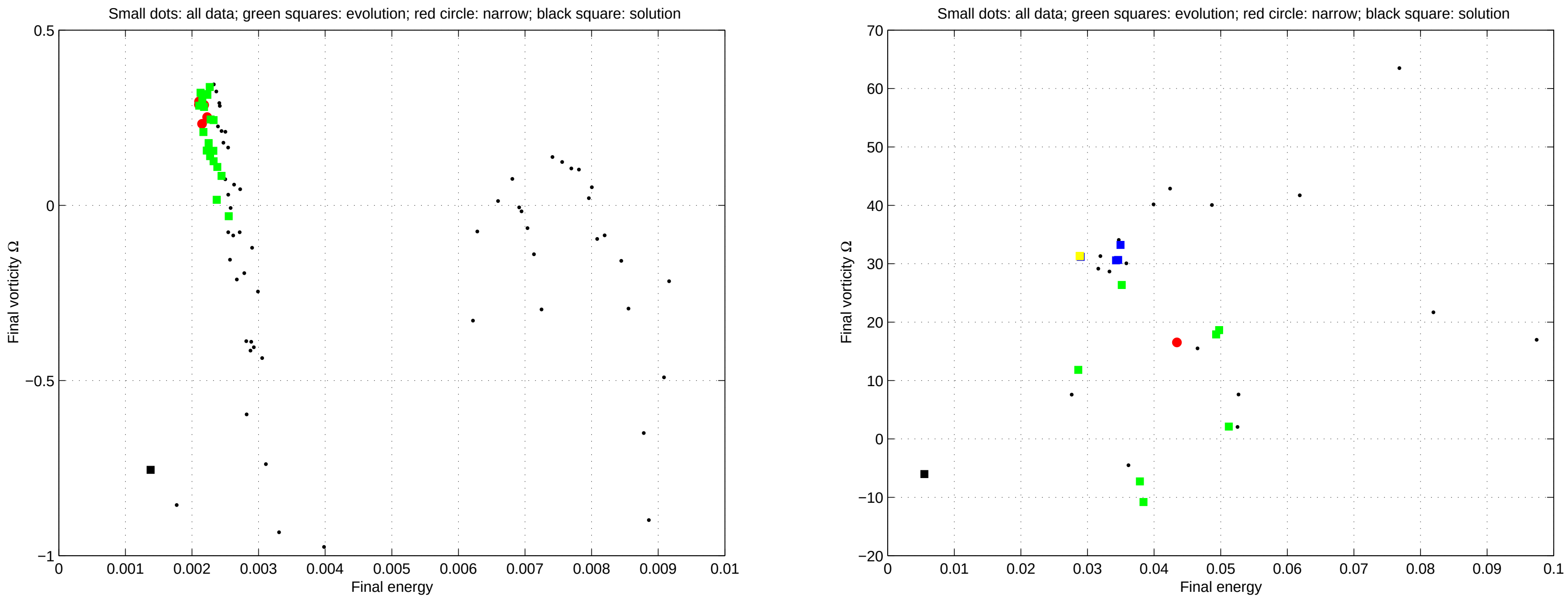

Figure 3: Black squares: smooth solutions; green: intermediate quasi-solutions; red: singular solutions. In addition, for barriers: yellow: stable qs. with ring; blue: intermediate forms evolving to stable ring qs.

The role of the effective Larmor radius

v_d/u	ρ_s^{eff}/ρ_s	$L = a/\rho_s^{eff}$	$\lvert v_{\theta bottom}\rvert$	$\left\lvert v_{\theta bottom}^{phys}\right\rvert (m/s)$	$\lvert E_r\rvert\,(kV/m)$
0.2	1.118	273	1×10^{-3}	979	2.45
0.4	1.29	236	1×10^{-3}	979	2.45
0.85	2.58	95	2.2×10^{-3}	2152	5.38
0.95	4.47	63	0.02	19580	48.95

Conclusions As part of the phenomenology of structure generation in fluids and plasma it appears convenient to take the vorticity as the active factor of self-organization. This is supported by the model of point-like vortices interacting in plane, model that has provided us with the description of the coherent stationary states. Examining the solutions of the coherent vortical type and their neighborhood in the function space it is found that there are states with properties similar to the physical states of $2D$ plasma. The pinch of density is the evolution of the system along a string of quasi-solutions, between a smooth vortex and a singular vortex. The increase of the effective Larmor radius due to the increase of the density gradient slows down this process without suppressing it. The transition between the L to H states appears as the transition between the smooth vortex and a state with high v_θ shear (a quasi-solution which is close to the extremum of action), of which it is separated by a gap in energy *and* vorticity.