Quasi-linear theory of turbulent generation of "spontaneous" toroidal rotation

Ö. D. Gürcan, P. H. Diamond

Center for Astrophysics and Space Sciences, University of California, San Diego

T.S. Hahm

Princeton Plasma Physics Laboratory

Acknowledgments: X. Garbet, J Rice and A. Peeters

Outline

- Momentum?
- Mechanism:
  - Symmetry breaking and its origins
- Model
  - Basis: More than just a pinch...
- Numerics
- Conclusion

All in 10 minutes!

Momentum?

- Motivation
  - Toroidal rotation, without NBI. [1]
- Momentum:

$$\frac{\partial}{\partial t}\langle n v_\phi \rangle + \overline{n}\nabla_r \langle \tilde{v}_{Er}\tilde{v}_\phi \rangle + \overline{v}_\phi \nabla_r \langle \tilde{v}_{Er}\tilde{n} \rangle = \nu \nabla_r^2 \langle n v_\phi \rangle$$

Reynolds stress drive Convective transport

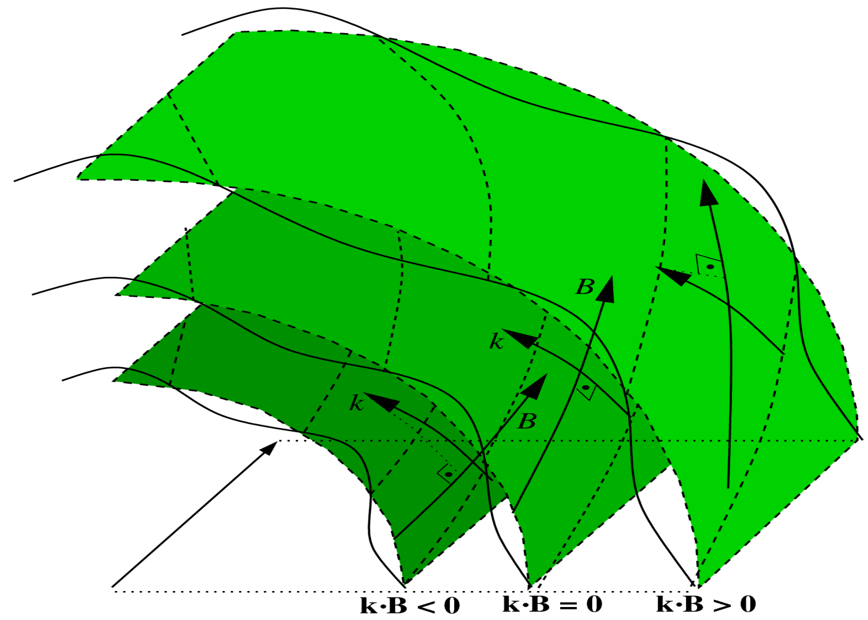
- Hence:
  - Particle transport can drive momentum transport convectively.
  - Particle pinch may drive inward momentum flux when the transported particles posess average momentum (convective transport).
  - Parallel velocity moment contains both.

[1] Rice J., *et. al. Nucl. Fusion* **44** 379 (2004).

Mechanism

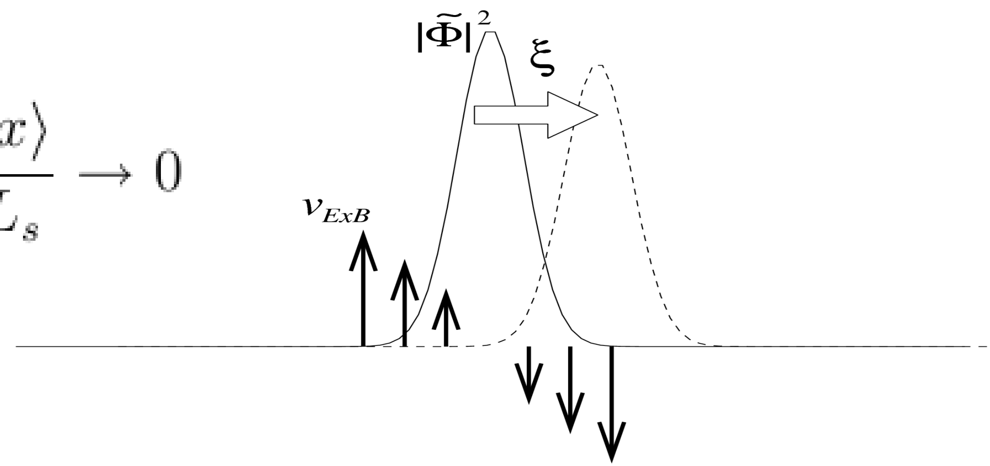
- Parallel equation of motion (say ITG):

$$\frac{\partial}{\partial t}\langle v_\phi\rangle + \overline{n}\frac{\partial}{\partial r}\langle \widetilde{v}_{Er}\widetilde{v}_\phi\rangle = \nu\nabla_r^2\langle v_\parallel\rangle$$



- For symmetrical spectra[2]:

$$\langle \widetilde{v}_{Er}\widetilde{v}_\phi\rangle = -\nu_t\nabla_r\overline{v}_\phi \qquad \langle k_\parallel k_y\rangle = k_y^2\frac{\langle x\rangle}{L_s}\rightarrow 0$$



- using quasi-linear theory (similar to [3] except the first term is not missing):

$$\langle \widetilde{v}_{Er}\widetilde{v}_\phi\rangle = \sum_k ic_s^2\left[\frac{c_s\rho_s k_\parallel k_y}{\omega}\left(1-\frac{c_s k_y}{\omega}\frac{\rho_s}{P_e}\frac{d\overline{P}}{dx}\right)-\frac{\rho_s^2 k_y^2}{\omega}\frac{d\overline{v}_\phi}{dx}\right]\left|\frac{e\widetilde{\Phi}}{T_e}\right|^2$$

[2] Mattor N. and Diamond P. H., *Phys Fluids* **31**, 1180 (1988)
[3] Coppi B., *Nucl. Fusion,* **42**, 1 (2002)

Mechanism

- Shift of the mode off of resonant surface creates imbalance in populations of sound waves going clockwise and counter-clockwise around the torus.
  - This is origin of torque density.

- Shift is caused by ExB flow shear

$$\frac{\langle x \rangle}{L_s} = -\alpha \frac{d\overline{v}_{Ey}}{dx}$$

Model

$$\langle \tilde{v}_{Er}\tilde{v}_{\phi}\rangle = -\sum_k i c_s^2 \left[\frac{\rho_s^2 k_y^2}{\omega}\frac{d\overline{v}_{\phi}}{dx} - \alpha \frac{c_s \rho_s k_y^2}{\omega}\left(\frac{c_s k_y}{\omega}\frac{\rho_s}{P_e}\frac{d\overline{P}}{dx} - 1 \right) \frac{\partial}{\partial x}\overline{v}_{Ey} \right] \left| \frac{e\tilde{\Phi}}{T_e} \right|^2$$

- Gives the model (similar to [4])

$$\frac{\partial}{\partial t}\overline{v}_{\phi} - \frac{\partial}{\partial x}\left((\nu_{neo} + \varepsilon)\frac{\partial}{\partial x}\overline{v}_{\phi} - \alpha\varepsilon \frac{\partial}{\partial x}\overline{v}'_{Ey} \right) = F_{\parallel}$$

off diagonal momentum flux.

  - Where ExB flow is determined by the radial force balance:

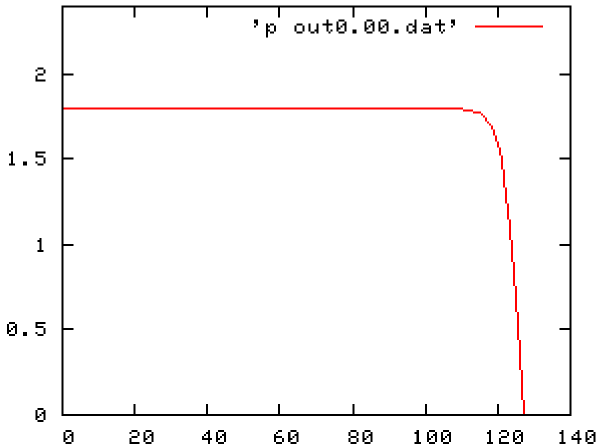
$$\frac{\partial}{\partial x}\overline{v}_{Ey} = -\underbrace{\left(\frac{\Omega_i}{\overline{n}T_e}\frac{\partial^2 \overline{P}}{\partial x^2} - \frac{\Omega_i}{\overline{n}^2 T_e}\frac{\partial \overline{P}}{\partial x}\frac{\partial \overline{n}}{\partial x} \right)}_{\text{Torque due to pressure profile}} - \frac{B_\theta}{B}\frac{\partial \overline{v}_{\phi}}{\partial x} - \frac{\overline{v}_{\phi}}{B}\frac{\partial B_\theta}{\partial x}$$

True momentum “pinch”
co vs. counter different.

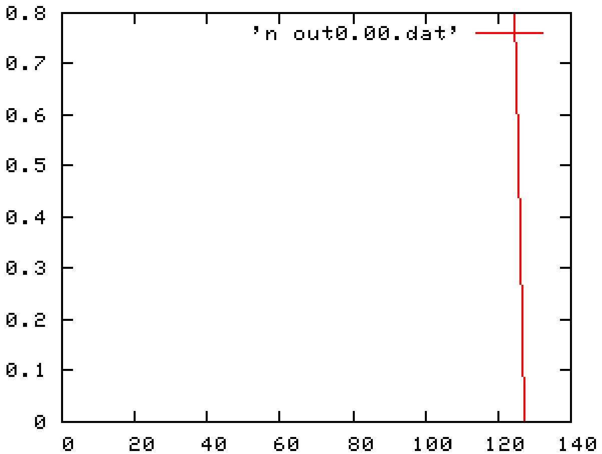
- In addition to the usual density and pressure evolution.

[4] Diamond P. H. *et. al.*, *Proceedings of IAEA*, (1994)

Numerics

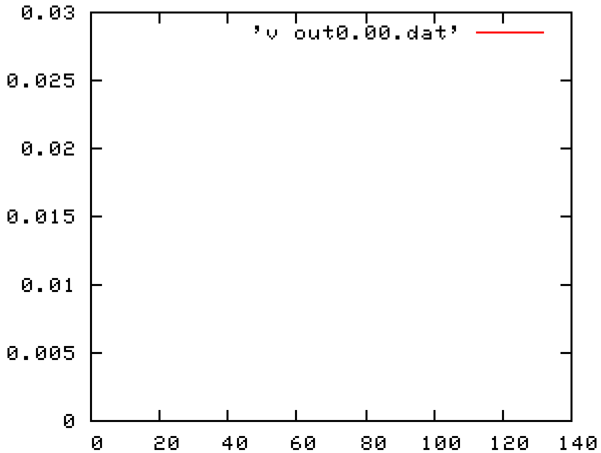


Pressure Profile



Density Profile

V rather small.



Momentum Profile

Conclusions

- Elements missing in previous modelling attempts:
  - Seperation of the convective transport of momentum and the effect of particle pinch.
  - Important role of ExB shear for symmetry breaking.
    - Scaling $x \propto \rho_s (L_s / L_n)^2$.
    - i.e. Proportional to ρ_* , but BIG due to $(L_s / L_n)^2$
  - Elements of turbulent flux beyond the “pinch”:
    - Torque density, pinch, correction to diffusion.
- Self-consistency is essential.
  - ExB shear creates anisotropy and causes conversion.
  - but also depletes turbulence
    - Hence depletes the torque. But less strongly!
- Momentum source at the edge needs to be understood.
  - Flows at the SOL?