MHD instabilities and fast particles*

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Outline

- Historic Background.
- Overview of shear Alfvén Spectra.
- Eigenmodes vs. Resonant Modes.
- Nonlinear Dynamics Aspects.
- 3D Hybrid MHD-Gyrokinetic Simulations.
- Transition from weak to strong energetic particle transport: Avalanches.
- Conclusions.

Historic Background

- Possible detrimental effect of Shear Alfvén (SA) instabilities on energetic ions recognized theoretically before experimental evidence was clear.

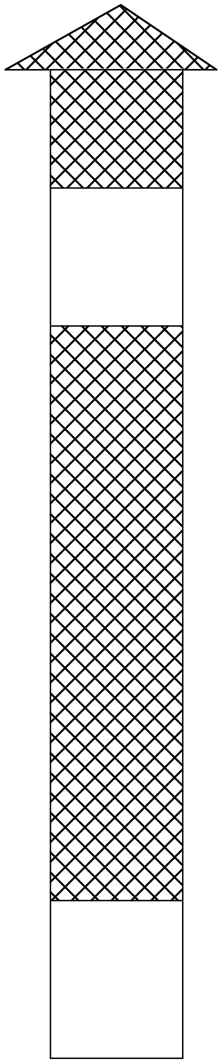
- Mikhailowskii, Sov. Phys. JETP, **41**, 890, (1975)
  Rosenbluth and Rutherford, PRL **34**, 1428, (1975)
  $\Rightarrow$ resonant wave particle interaction of $\approx$ MeV ions with SA inst. due to $v_E \approx v_A$ $(k_\parallel v_A \approx \omega_E)$

- Experimental observation of fishbones on PDX – McGuire et al., PRL **50**, 891, (1983) – fast $\perp$ injected ion losses ...
  ...followed by numerical simulation of mode-particle pumping loss mechanism – White et al., Phys. Fluids **26**, 2958, (1983)
  ...and by theoretical explanation of internal kink excitation –
  Chen, White, Rosenbluth, PRL **52**, 1122, (1984)
  Coppi, Porcelli, PRL **57**, 2272, (1986)

- Existence of gaps in the SA continuous spectrum (due to lattice symmetry breaking) $\omega \approx v_A/2qR$ – Kieras and Tataronis, J. Pl. Phy. **28**, 395, (1982)
- Existence of discrete modes (TAE) in the toroidal gaps – Cheng, Chen, Chance, Ann. Phys. **161**, 21, (1985)
- Possible excitations of TAE by energetic particles . . .
  Chen, "Theory of Fusion Plasmas, p.327, (1989)
  Fu, Van Dam, Phys. Fluids B **1**, 1949, (1989).
- KBM excitation by fast ions: Biglari, Chen, PRL **67**, 3681, (1991)
- Experimental evidence . . .
  Wong et al., PRL **66**, 1874, (1991)
  Heidbrink et al, Nucl. Fusion **31**, 1635, (1991)
  Heidbrink et al, PRL **71**, 855, (1993) $\Rightarrow$ BAE $\omega \approx \omega_{ti} \approx \omega_{*pi}$

- TAE modes are predicted to have small saturation levels and yield negligible transport unless stochastization threshold in phase space is reached: H.L. Berk and B.N. Breizman, Phys. Fluids B **2**, 2246, (1990) and D.J. Sigmar, C.T. Hsu, R.B. White and C.Z. Cheng, Phys. Fluids B, **4**, 1506, (1992).
- Excitation of Energetic particle Modes (EPM), at characteristic frequencies of energetic particles when free energy source overcomes continuum damping L. Chen, Phys. Plasmas **1**, 1519, (1994). [...also RTAE excitation C.Z. Cheng, N.N. Gorelenkov, C.T. Hsu, Nucl. Fusion **35**, 1639, (1995).]
- Strong energetic particle redistributions are predicted to occur above the EPM excitation threshold in 3D Hybrid MHD-Gyrokinetic simulations: S. Briguglio, F. Zonca and G. Vlad, Phys. Plasmas **5**, 1321, (1998).

Overview of shear Alfvén spectra



Energetic Particle Modes (EPM) : forced oscillations

TAE – KTAE ⇒ Transition to EPM

Energetic Particle Modes (EPM) : forced oscillations

Beta induced Alfvén Eigenmodes (BAE)

MHD!!! Kinetic Ballooning Modes (KBM)

KBM ⊕ BAE ⇒ Alfvén ITG (AITG)

Eigenmodes vs. Resonant Modes

- Fundamental difference in mode dynamics and particle transports is to be attributed to mode excitation and particle phase space motion
- Use Secular Perturbation Theory in nonlinear Hamiltonian dynamics ...
- Extended Phase Space to treat explicit time dependencies: $2N \Rightarrow (2N+2)$-dim.; for low frequency modes $(\omega \ll \omega_{ci})$ the resulting 8-dim phase space reduces (μ and $\mathcal{H} \equiv H(\mu, P_\phi, J_\parallel) - H$ are conserved) to 6-dim phase space, i.e. the general problem is equivalent to an autonomous Hamiltonian with 3 degrees of freedom
- Use Secular Perturbation Theory is a method for locally removing a single resonance: what happens in the multiple resonance case ???

$$H = H_0(\mathbf{J}) + \epsilon H_1(\mathbf{J}, \theta)$$

$$\omega_1 = \frac{\partial H_0}{\partial J_1} \qquad \omega_2 = \frac{\partial H_0}{\partial J_2} \qquad \frac{\omega_2}{\omega_1} = \frac{h}{k} \quad h, k \in \mathbb{Z}$$

- Canonical transformation with generating function $F_2 = (h\theta_1 - k\theta_2)\hat{J}_1 + \theta_2 \hat{J}_2$

$$J_1 = \frac{\partial F_2}{\partial \theta_1} = h\hat{J}_1 \qquad J_2 = \frac{\partial F_2}{\partial \theta_2} = \hat{J}_2 - k\hat{J}_1$$

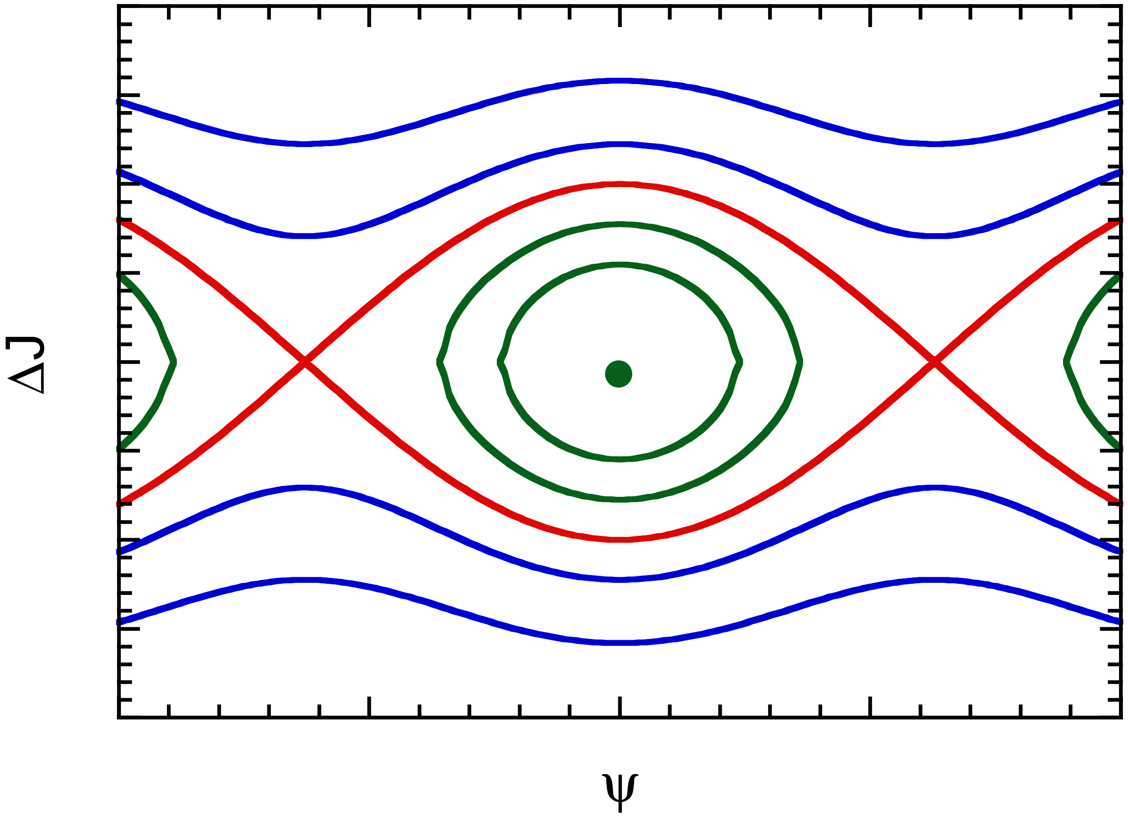
$$\hat{\theta}_1 = \frac{\partial F_2}{\partial \hat{J}_1} = h\theta_1 - k\theta_2 \qquad \hat{\theta}_2 = \frac{\partial F_2}{\partial \hat{J}_2} = \theta_2$$

- After averaging on $\hat{\theta}_2$ (near resonance)

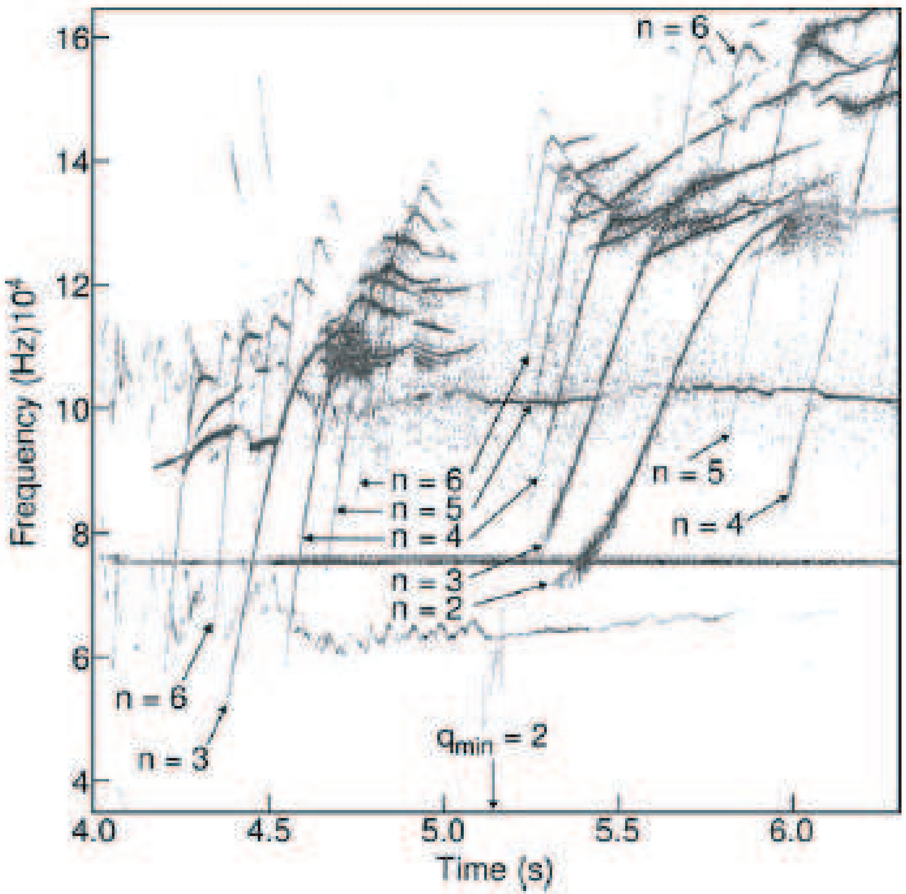
$$\bar{H} = \bar{H}_0(\hat{\mathbf{J}}) + \epsilon \bar{H}_1(\hat{\mathbf{J}}, \hat{\theta}_1) = \bar{H}_0(\hat{\mathbf{J}}_\mathbf{0}) + \Delta \bar{H}$$

□ Standard Hamiltonian

$$\Delta\bar{H} \simeq (1/2)F(\Delta\hat{J}_1)^2 - G\cos\theta_1 \qquad F = \partial^2\bar{H}_0/\partial\hat{J}_{10}^2 \qquad G\cos\theta_1 \simeq -\epsilon\bar{H}_1$$



□ Further complications: mode frequency often sweeps: fast vs. slow sweeping
From S.E. Sharapov *et al.*, Phys. Lett. A **289**, 127, (2001)



Theoretical interpretation by H.L. Berk *et al.*, PRL **87**, 185002, (2001)

- Qualitative description in terms of frequency sweeping, (H.L. Berk and B.N. Breizman, Comm. PPCF **17** 145 (1996).)

$$\frac{\dot{\omega}}{\omega} \sim \gamma_L \ll \omega_B^2/\omega; \qquad \text{adiabatic(TAE)}$$

$$\frac{\dot{\omega}}{\omega} \sim \gamma_L \gtrsim \omega_B^2/\omega; \qquad \text{fast(EPM)}$$

- Adiabatic (TAE) case: quasilinear flattening is dominant and, in the absence of externally imposed adiabatic frequency chirping (e.g., via equilibrium changes), saturation is either at $\omega_B \approx \gamma_L$ or it occurs via other mode-mode coupling mechanisms

- Fast (EPM) case: there no time for the distribution to flatten and the mode should freely grow $\Rightarrow$ particle convection/mode particle pumping ??? Saturation should occur at $\omega_B \approx (\omega\gamma_L)^{1/2}$.

(Weak) Modes in the GAP: nonlinear

- NL Saturation via mode-mode coupling (also Thyagaraja et al., Proc. EPS-97, vol 1, p 277, 1997):
  - Saturation via "ion Compton scattering" at $\delta B_r/B \approx \epsilon^2(\gamma_L/\omega)^{1/2}$ (Hahm and Chen, PRL **74**, 266, (1995))
  - Saturation via $\delta\mathbf{E}^* \times \delta\mathbf{B}$ at $\delta B_r/B \approx \epsilon^{5/2}/nq$ (Zonca et al., PRL **74**, 698, (1995))
  - Saturation via $\delta n/n$ at $\delta B_r/B \approx \epsilon^{3/2}\beta^{1/2}$ (Chen et al., PPCF **40**, 1823, (1998))
- NL Saturation via phase-space nonlinearities (wave-particle trapping):
  - Steady-state: $(\delta B_r/B)^{1/2} \approx \omega_B \approx \gamma_L(\nu_{eff}/\gamma_d)$ for $\gamma_d \ll \nu_{eff} \sim \nu(\omega/\omega_b)^2$ (Berk, Breizman, Phys. Fluids B **2**, 2246, (1990))

- TAE pulsations: $(\delta B_r/B)^{1/2} \approx \omega_B \approx \gamma_L$ for $\gamma_d \gg \nu_{eff0} \sim \nu(\omega/\gamma_L)^2$ (Berk et al, PRL **68**, 3563, (1992))

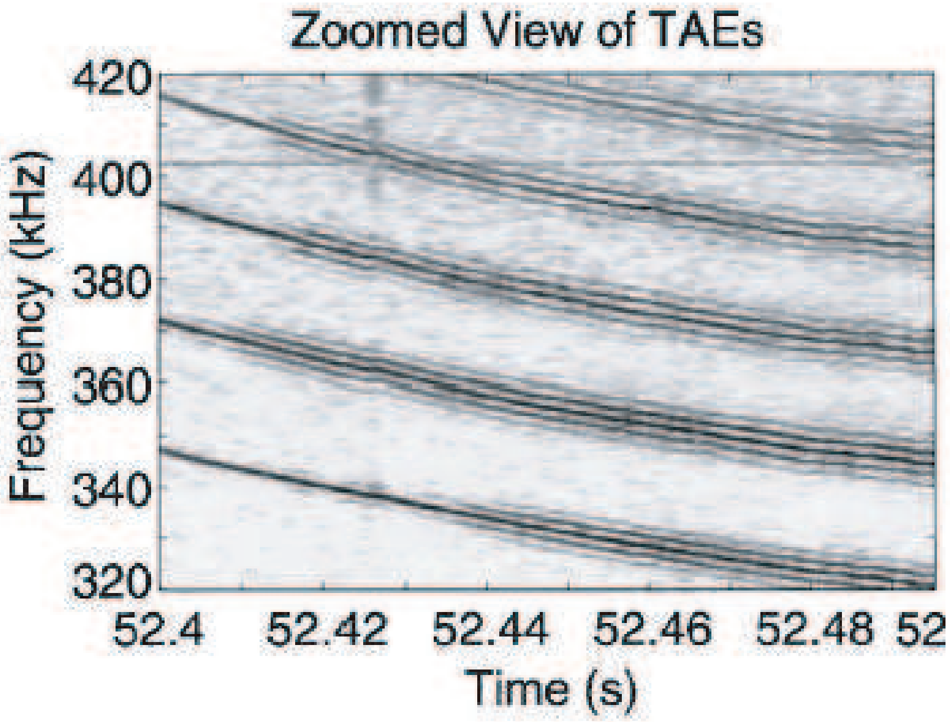
- Both steady-state and TAE pulsations yield negligible losses unless phase-space stochasticity is reached, possibly via domino effect (Berk et al, Nuc. Fus. **35**, 1661, (1995); Heeter et al,PRL **85**, 3177 (2000).)

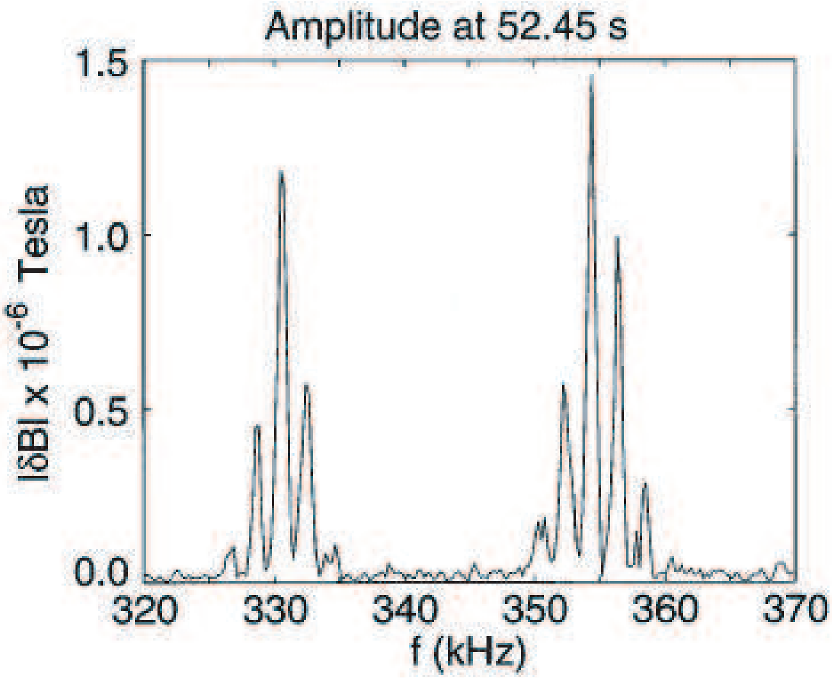
- In the case of a single mode, spontaneous formation near threshold of hole-clump pair in phase space (Berk et al., Phys. Lett. A, **234**, 213, (1997); Phys. Plasmas **6**, 3102 (1999).) may yield to frequency chirping and/or pitchfork splitting of mode-frequency

- Theory seems to explain pitchfork splitting of TAE lines observed in JET (Fasoli, IAEA-TCM-97); however, $\delta\omega \sim \gamma_L(\gamma_d/\gamma_L)^{1/2}(\gamma_L/\nu_{eff})^{3/2}$; thus, large chirping requires very small ν_{eff}.

Pitchfork splitting of TAE in JET

Fasoli, Phys. Rev. Lett. **81**, 5564, (1998)





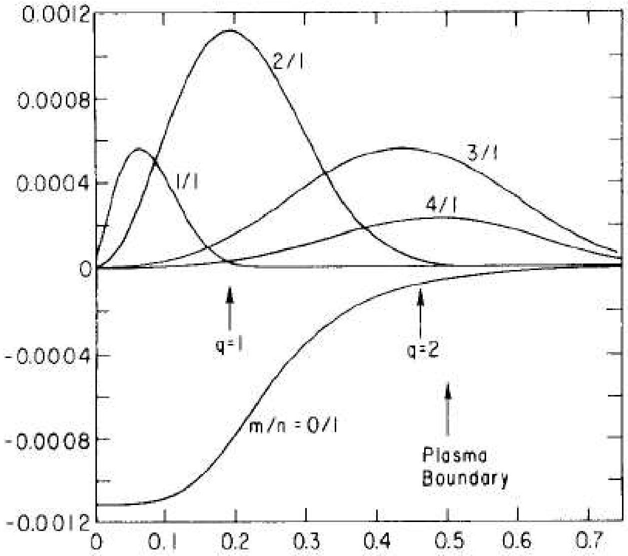
High resolution MHD spectroscopy: Pinches et al, PPCF **46**, S47, (2004)

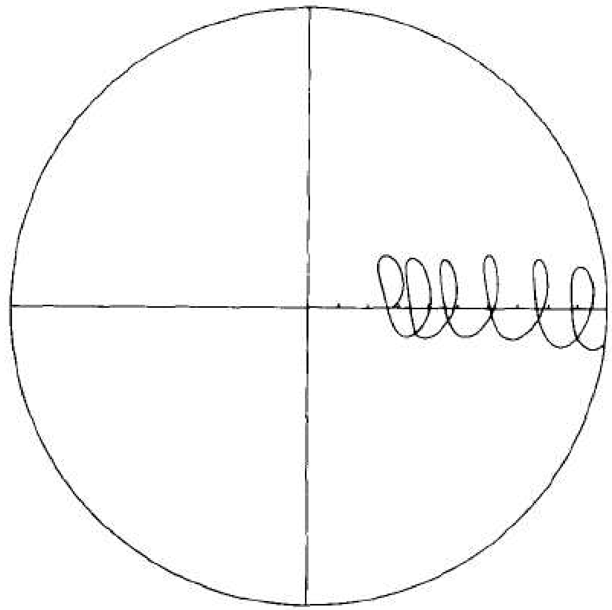
Nonlinear dynamics issues

- Role of nonlinear dynamics near marginal stability:
  - Explosive instabilities: (Berk et al., Phys. Plasmas **6**, 3102 (1999).)
  - Phase space stochastization: (Heeter et al,PRL **85**, 3177 (2000).)

- Alfvèn Eigenmodes are very inefficient in tapping plasma expansion free energy (fast particle kinetic energy):
  - Fraction $\Delta W/W \propto \delta B^2/B^2 \propto (\gamma/\omega)^4$: (H.L. Berk and B.N. Breizman, Comm. PPCF **17** 145 (1996).)
  - Free energy build up, except in a few selected regions of phase space (near resonances). Complex behaviors in phase space.

- Mode-particle pumping: (White et al., Phys. Fluids **26**, 2958, (1983))
  - Coexistence of chaotic regions and regular structures.
  - Typical fast particle trajectories are made of regular segments, corresponding to "sticking" to the regular structures, and erratic behaviors due to wanderings in the chaotic sea.





- Why fast particle losses do not get (radially) trapped in the wave?

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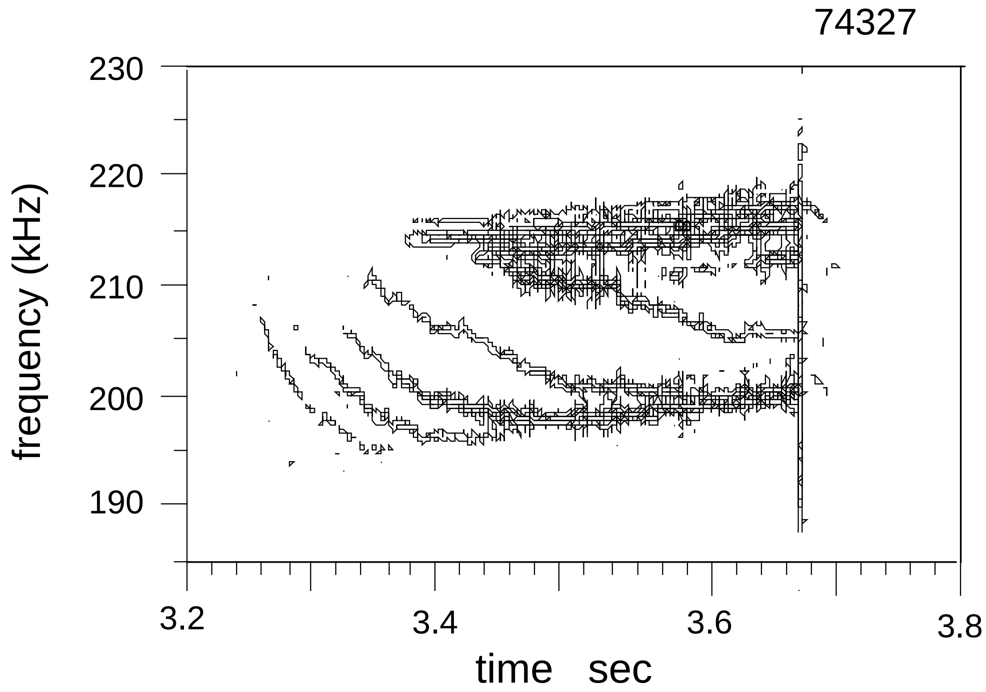
- Presence of multiple resonances
- Fluctuations appear in bursts and with variable frequency or a broad spectrum

- Convective amplification and Avalanches:
  - Transient (bursty) onset of Alfvèn Eigenmodes (e.g. at Sawtooth crashes). two possible examples in TFTR and JET
  - Propagation of unstable fronts and ballistic transport.
- Zonal Flows, Fields and resonant particle behaviors:
  - Zonal dynamics can influence the modes via $\mathbf{E} \times \mathbf{B}$ shearing as well as via fast particle source modulation (modulational instability).
  - Crucial role of resonant particles

ICRF Experiments on TFTR

- EPM and TAE excitations, by ICRF induced fast minority ion tails on TFTR. From **Bernabei** *et al.*, Phys. Plasmas **6**,1880, (1999).

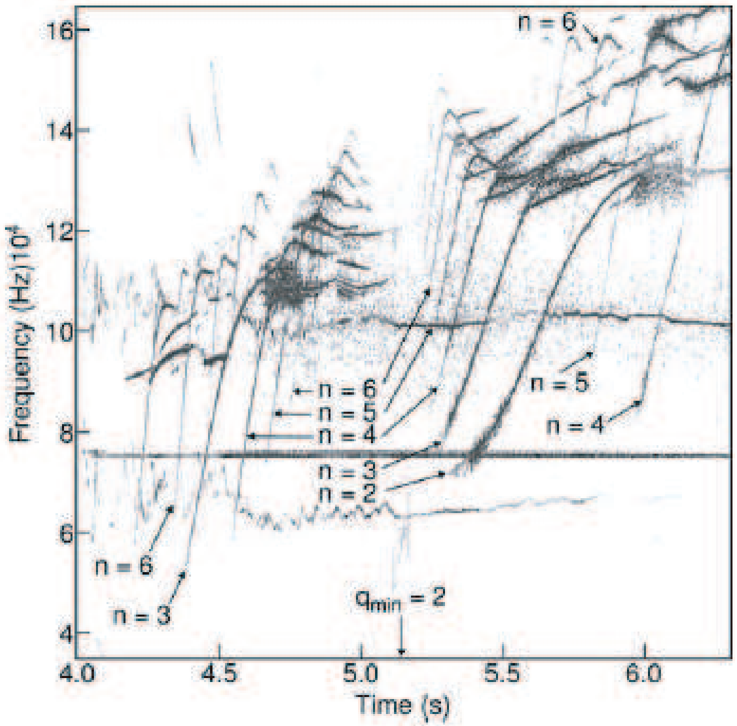


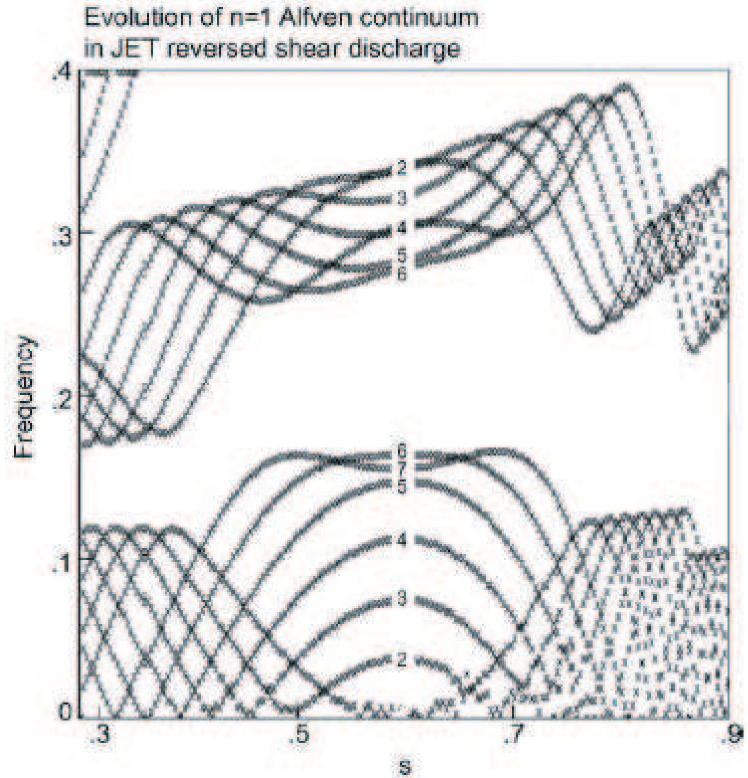
- Energetic Particle Modes*, appear to be excited in the plasma core, due to strong energy source due to ICRF tail ions
- Energetic Particle Modes*, are forced oscillations at $\omega \simeq \bar{\omega}_{dE}$. They chirp downward in frequency because $\bar{\omega}_{dE} = k_\theta \rho_{LE} v_{thE}/R_0$.
- TAE Modes, are eventually excited at the plasma edge, due to ICRF tail ions transported outward by EPM.
- Fast Ion Losses, seem associated with TAE's, and appear to be related to EPM frequency chirping.
- Detailed Numerical simulations of TAE bursts on TFTR:
  - Y. Todo et al., NF **41** 1153 (2001)
  - Y. Todo et al., PoP **10** 2888 (2003)

*L. Chen, Phys. Plasmas **1**, 1519, (1994).

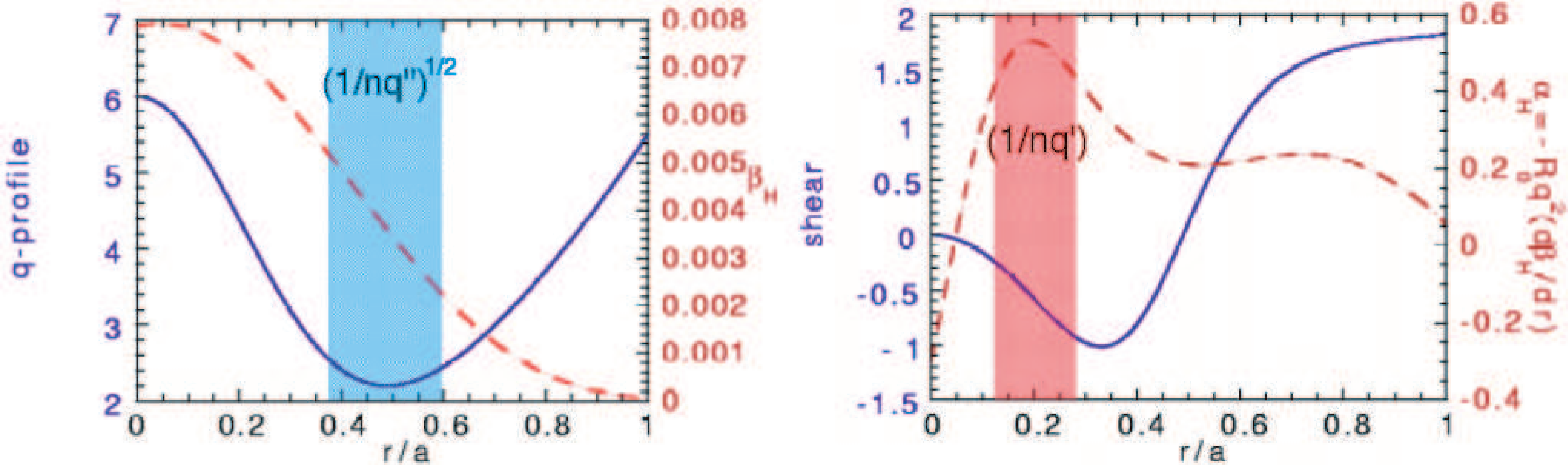
ICRF Experiments on JET

- EPM and TAE excitations, by ICRF induced fast minority ion tails on JET. From S.E. Sharapov *et al.*, Phys. Lett. A **289**, 127, (2001). **Alfvén Cascades** in reversed-q equilibria (advanced tokamak)





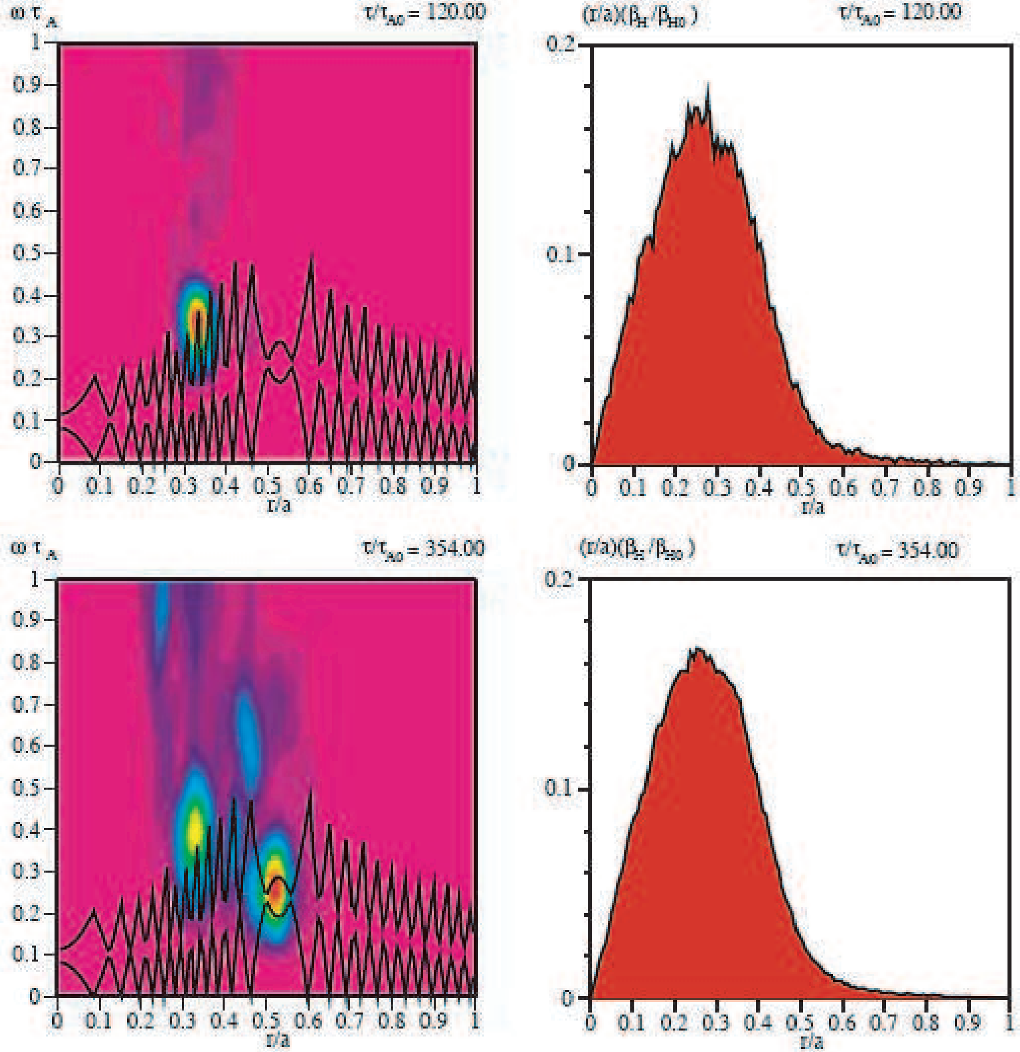
EPMs are excited at different radial locations



- Strong resonant excitation of EPMs should occur at $r/a \approx 0.2$, where α_H is maximum.L. Chen, Phys. Plasmas **1**, 1519, (1994).
- Natural gap in the Alfvén continuum appears at q_{min} Berk *et al.*, Phys. Rev. Lett., **87**, 185002, (2001) $\Rightarrow$ EPM Gap Modes.

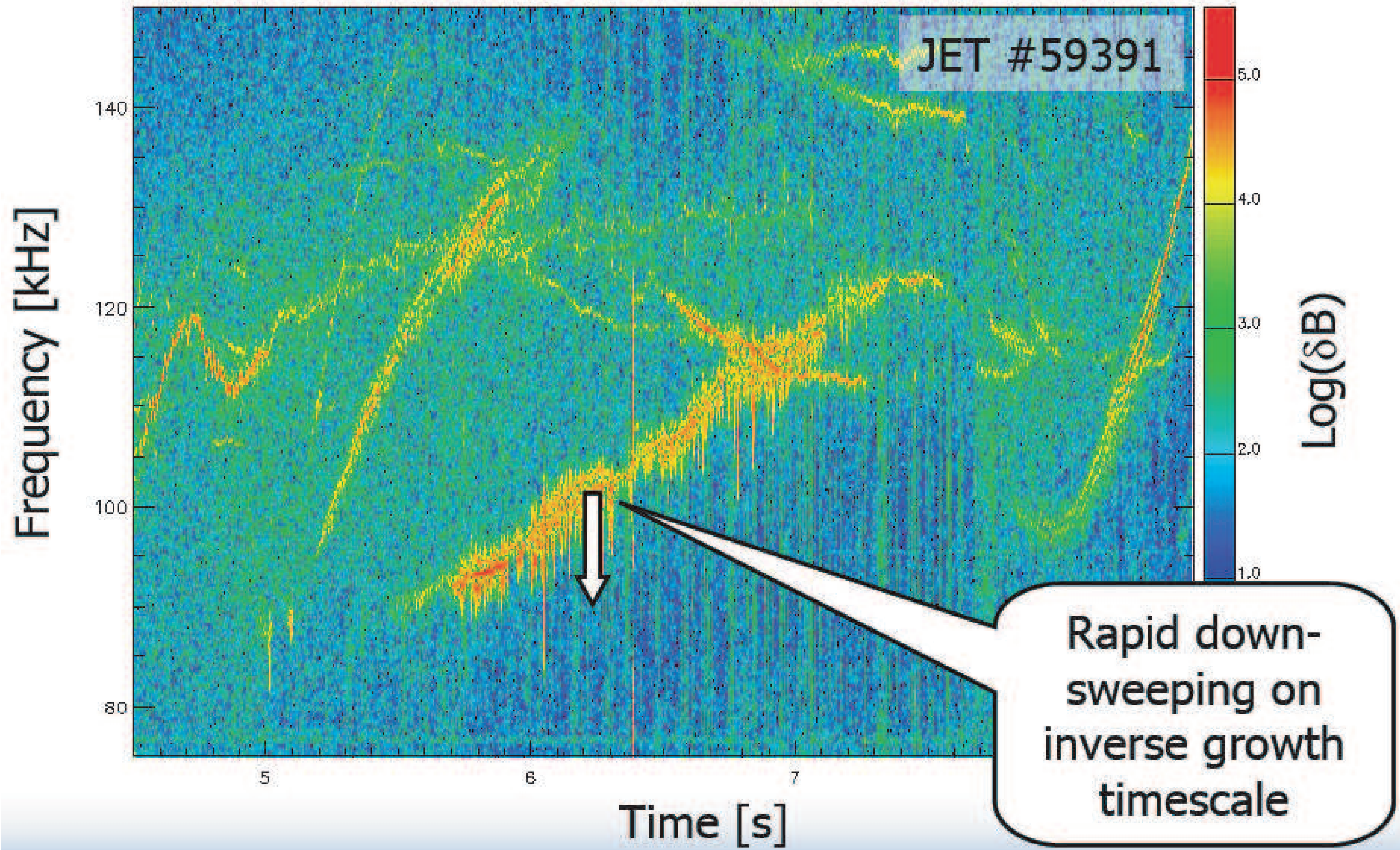
3D Hybrid MHD-GK simulation of EPM (Phys. Plasmas. **9**, 4939, (2002))

Weak Drive, $\beta_{H0} = 0.01$, and hollow q profile



Possible EPMs during Alfvén Cascades in JET

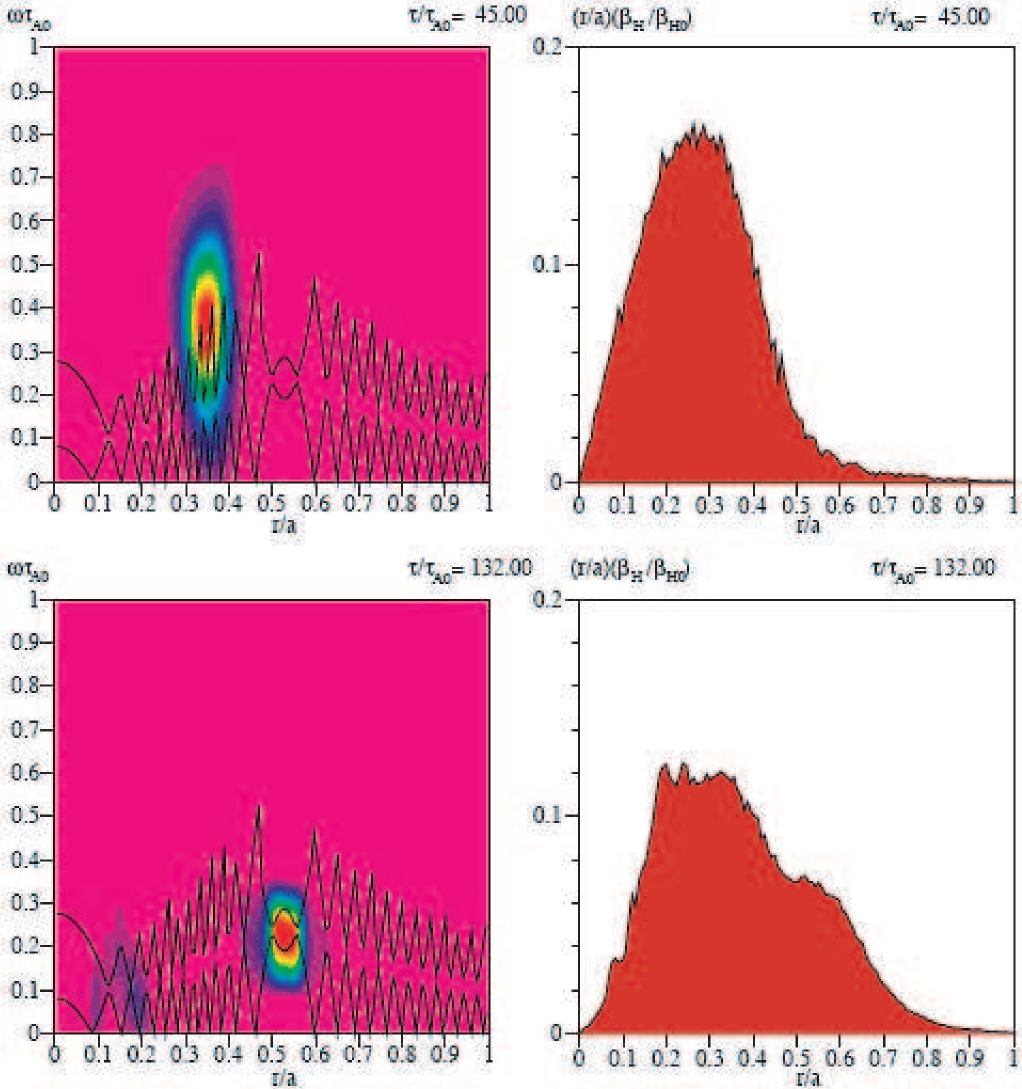
Courtesy of S.D. Pinches and JET-EFDA



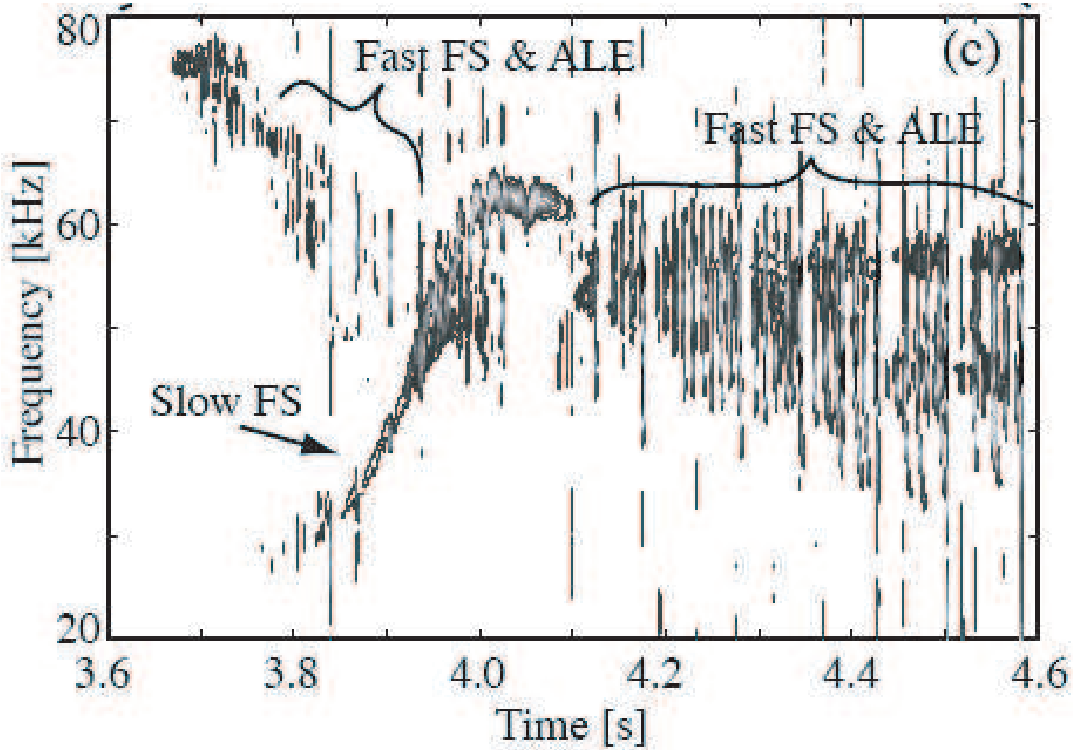
Simon Pinches, 31st EPS Conference on Plasma Physics, London, 2004

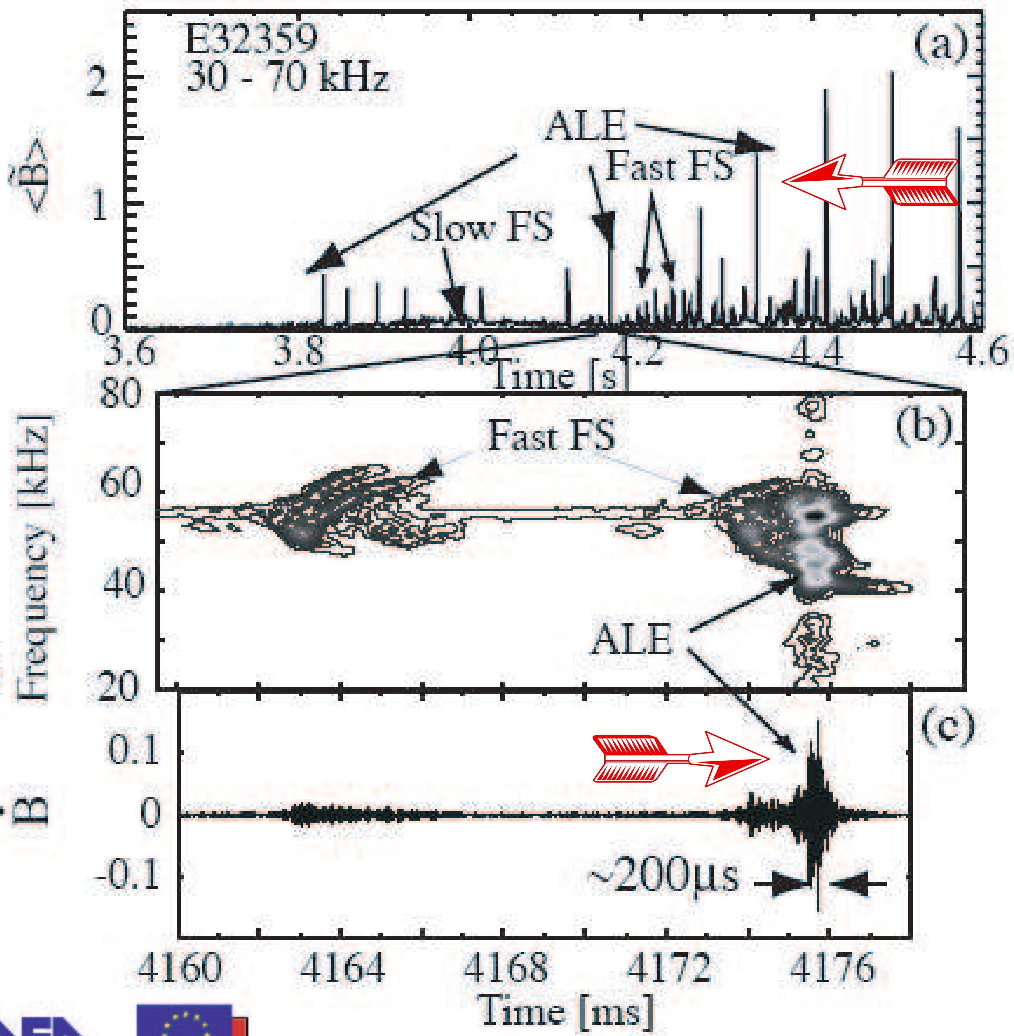
3D Hybrid MHD-GK simulation of EPM (Phys. Plasmas. **9**, 4939, (2002))

Strong Drive, $\beta_{H0} = 0.025$, and hollow q profile



ALE on JT-60U (K. Shinohara, *et al.*, Nucl. Fus. 41, 603, (2001))



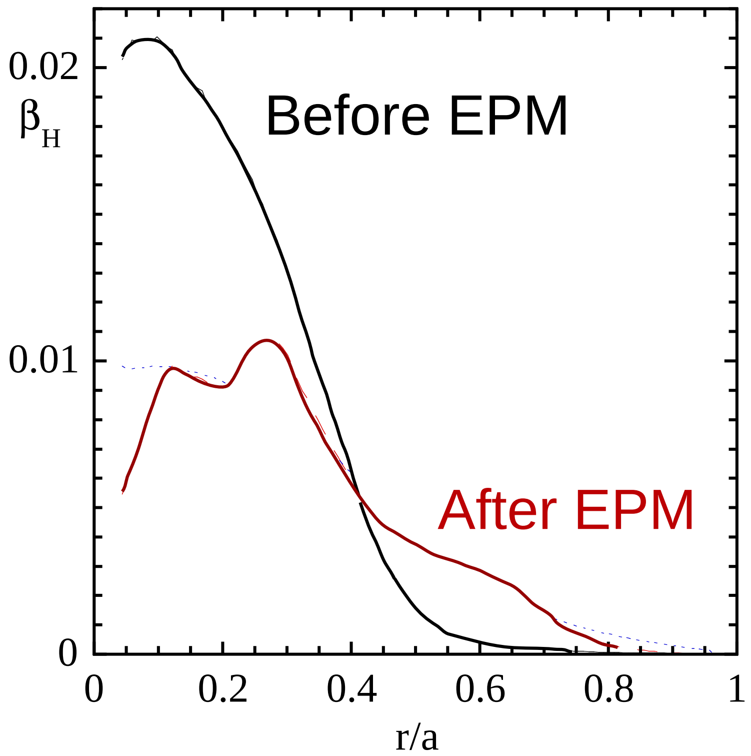


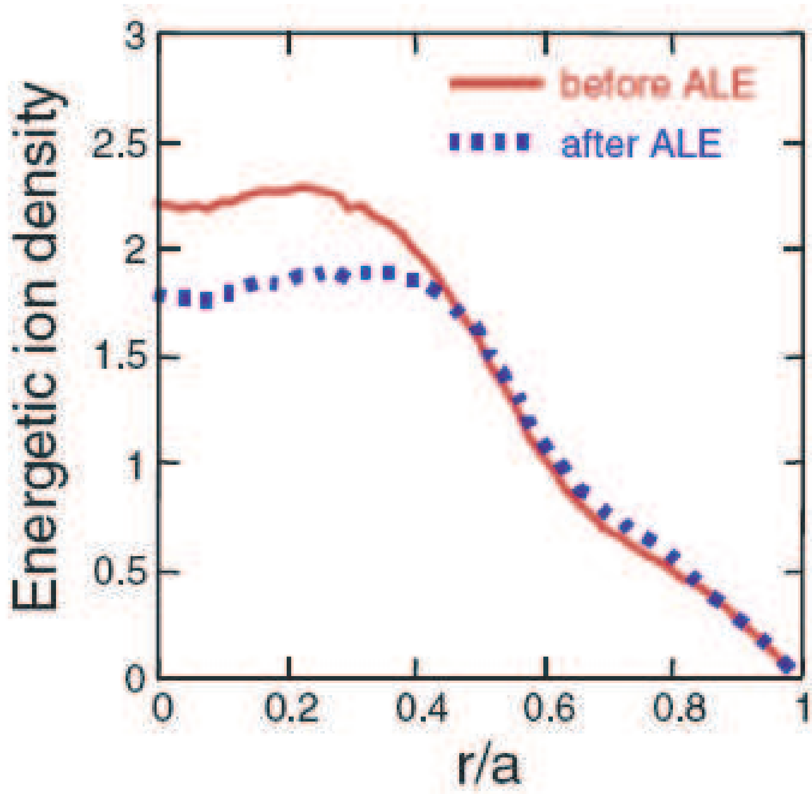
ALE = Abrupt Large amplitude Event

Courtesy of K. Shinohara and JT-60U

Fast ion transport: simulation and experiment

- Numerical simulations show fast ion radial redistributions, qualitatively similar to those by ALE on JT-60U.



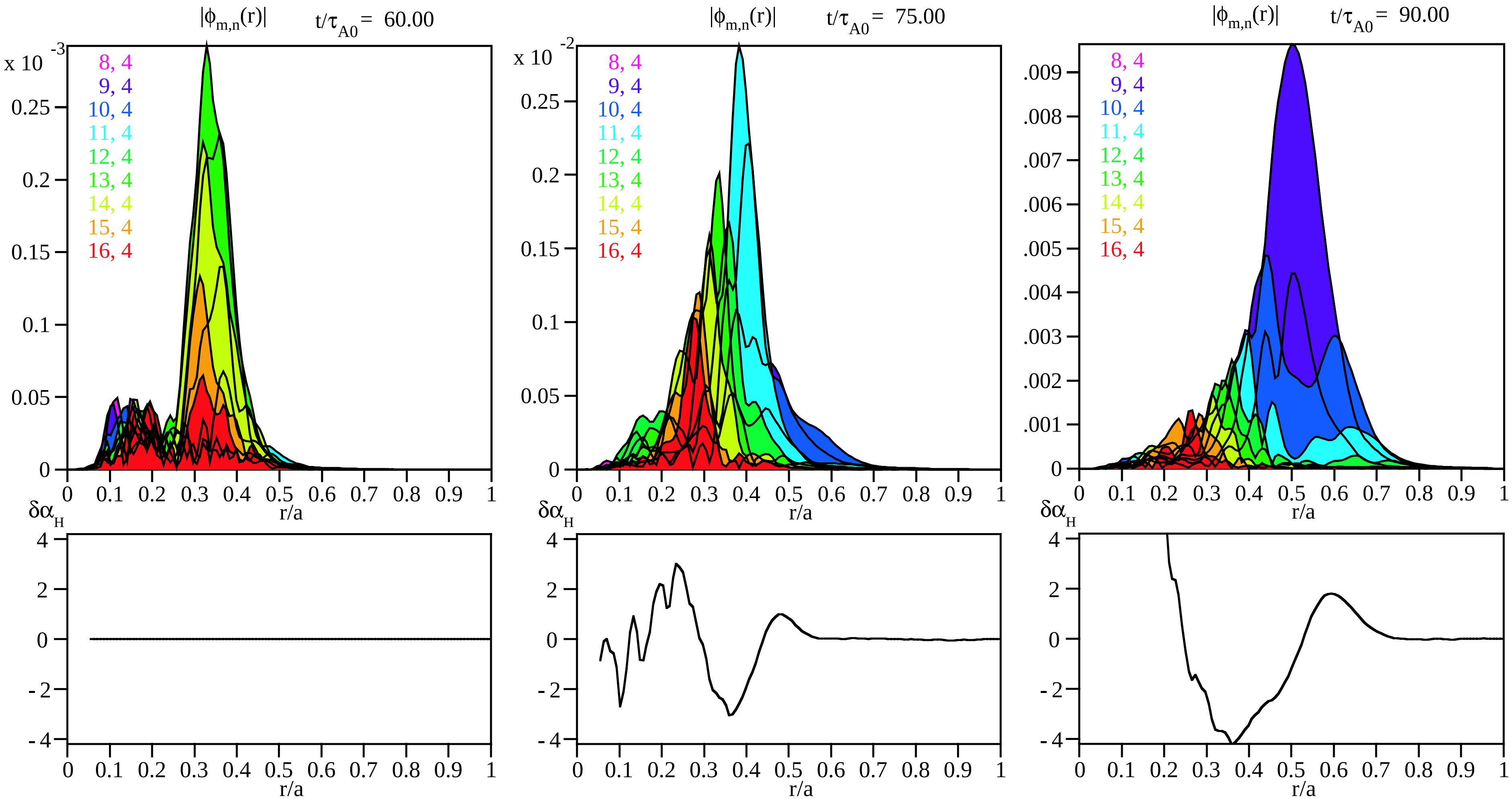


G. Vlad *et al.*, EPS02, ECA 26B, P-4.088, (2002).

K. Shinohara *etal* PPCF 46, S31 (2004)

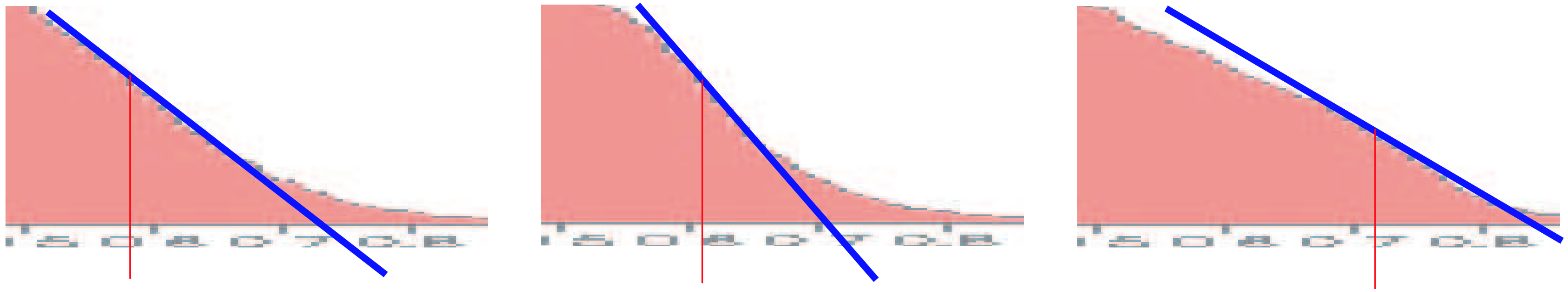
Courtesy of M. Ishikawa and JT-60U

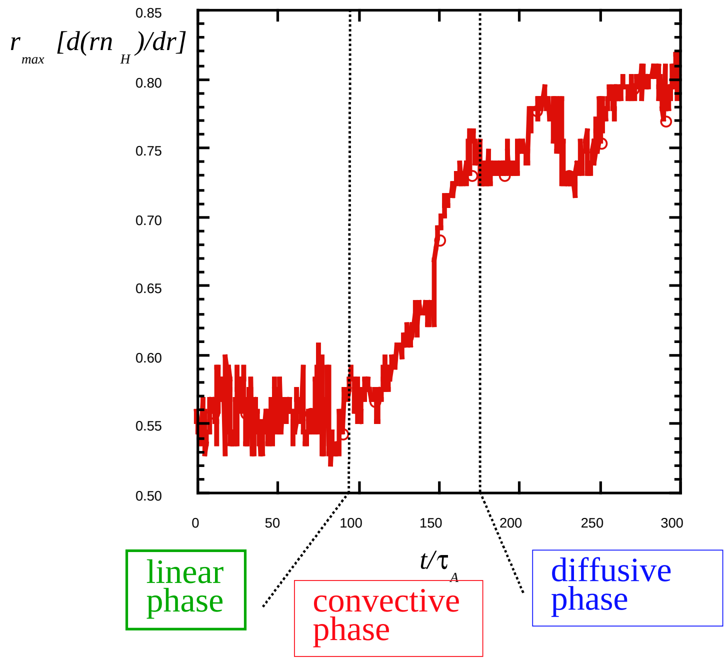
Avalanches and NL EPM dynamics (IAEA 02)

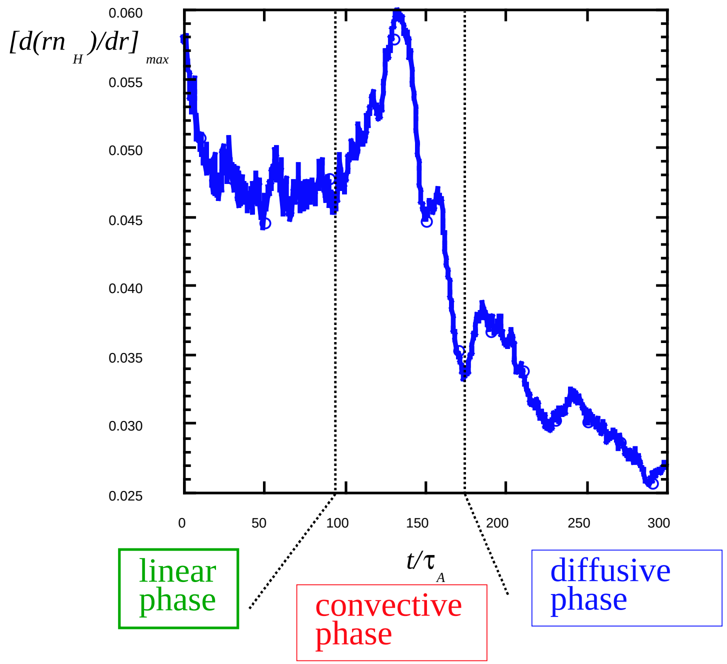


Vlad et al. IAEA-TCM, (2003)

Propagation of the unstable front







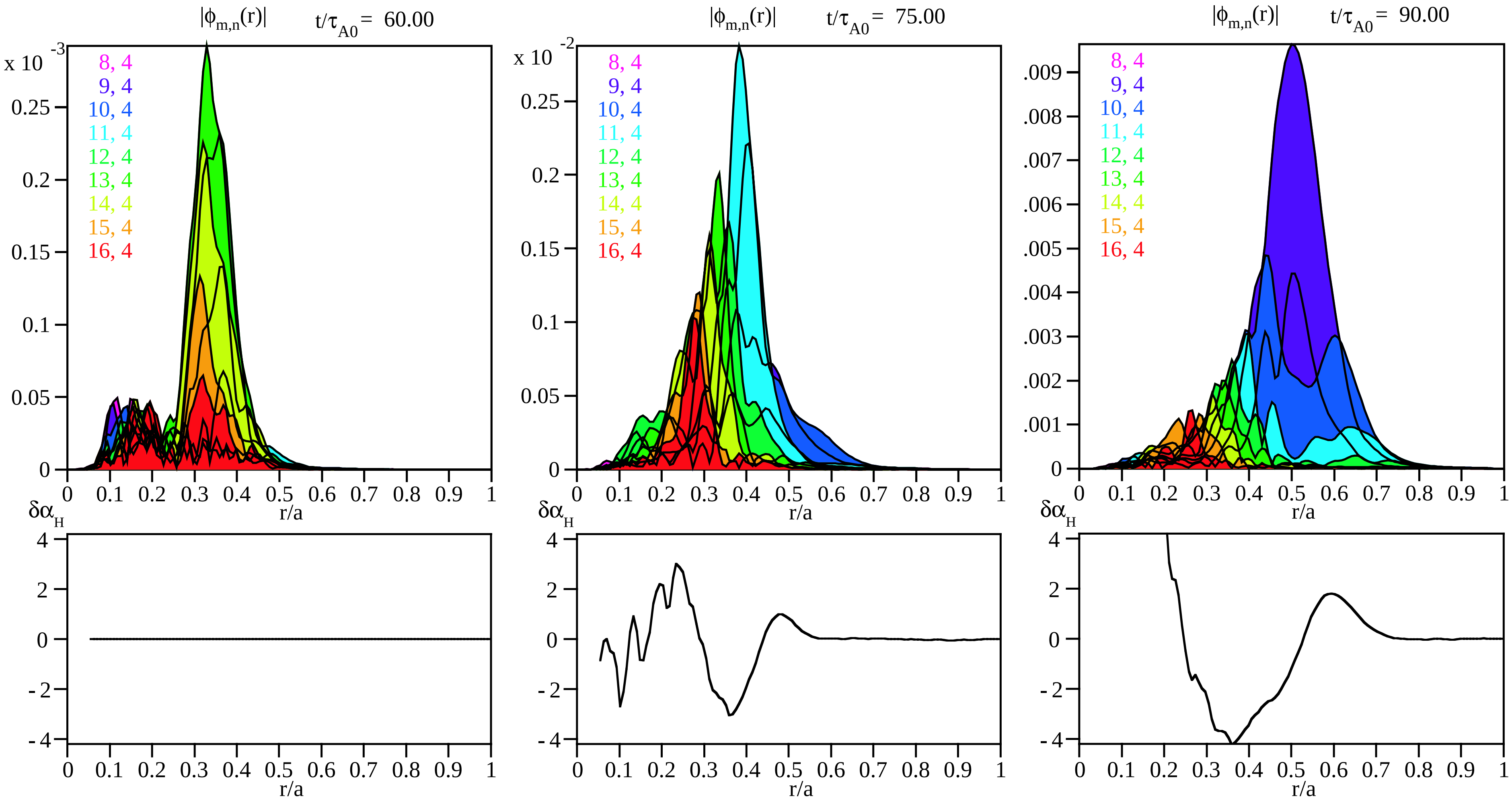
- Assume isotropic slowing-down and EPM NL dynamics dominated by precession resonance.

$$\left[D_R(\omega,\theta_k;s,\alpha)+iD_I(\omega,\theta_k;s,\alpha)\right]\partial_t A_0 = \frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}\alpha_H\left[1+\frac{\omega}{\bar{\omega}_{dF}}\ln\left(\frac{\bar{\omega}_{dF}}{\omega}-1\right)\right.$$
$$\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A_0+\underbrace{i\pi\frac{\omega}{\bar{\omega}_{dF}}A_0\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H\left|A_0\right|^2\right)} .$$

DECREASES DRIVE@ MAX $|A_0|$

INCREASES DRIVE NEARBY

Avalanches and NL EPM dynamics (IAEA 02)



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$$\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A_0 + \underbrace{i\pi\frac{\omega}{\bar{\omega}_{dF}}A_0\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H\left|A_0\right|^2\right)} \quad .$$

DECREASES DRIVE@ MAX $|A_0|$

INCREASES DRIVE NEARBY

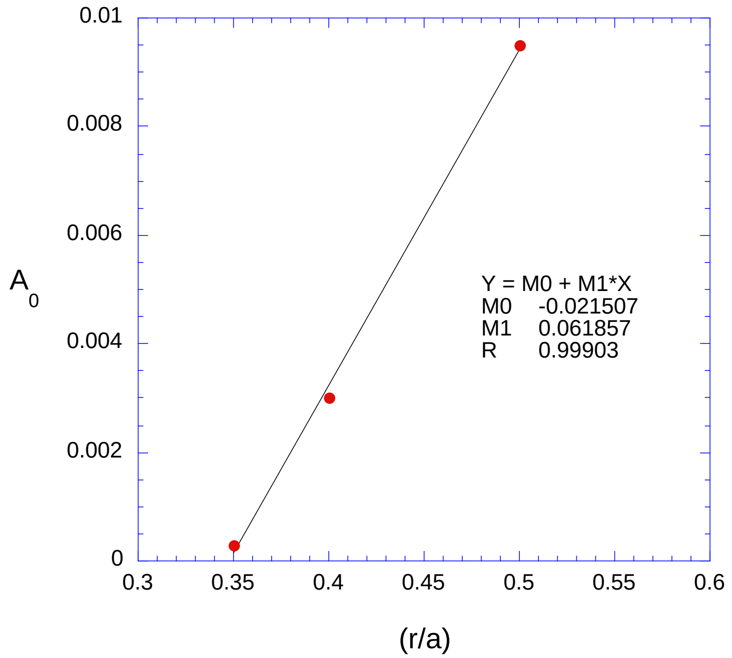
- Assume localized fast ion drive, $\alpha_H = -R_0q^2\beta'_H = \alpha_{H0}\exp(-x^2/L_p^2) \simeq \alpha_{H0}(1-x^2/L_p^2)$, with $x=(r-r_0)$.

- In order to maximize the drive the EPM radial structure is nonlinearly displaced by

$$(x_0/L_p) = \gamma_L^{-1}k_\theta\rho_H\left(T_H/M_H\right)^{1/2}\left(|A_0|/W_0\right) \quad ,$$

- x_0 is the radial position of the max EPM amplitude and W_0 indicating the typical EPM radial width in the NL regime

- During convective amplification, radial position of unstable front scales linearly with EPM amplitude.



- Real frequency chirping accompanies convective EPM amplification in order to keep $\omega \propto \bar{\omega}_d$

$$\Delta\omega = (s-1) \left.\bar{\omega}_{dF}\right|_{x_0} (x_0/r) \left(\omega_0 / \left.\bar{\omega}_{dF}\right|_{x=0}\right) \quad .$$

Conclusions

- Linear Theory: sound and well understood. However, most codes still do not include nonperturbative particle dynamics
- Nonlinear Theory: Partially understood
  - Theory of Non-linear phase space dynamics (single mode) seems to explain a number of experimentally observed phenomena: saturation levels, pitch-fork splitting of spectral lines, chirping . . . (possibly)
  - NL GK-MHD simulations of EPM's indicate saturation via source redistribution rather than $\omega_b \approx \gamma_L$; fast ion radial convection
  - What happens in the multiple (m, n) case??? and for a strong source???
  - Chirping is a very complex phenomenon, observed in most tokamaks with intense hot particle tails: due to equilibrium variations??? and/or Nonlinear dynamics???

- Prediction and interpretation of particle losses is still lacking: domino effect (phase space stochasticity) . . . and/or mode-particle pumping (particle convection)???

• Nonlinear Hamiltonian Dynamics: Strong mathematical methods exist . . . but what about solving the self-consistent problem???

• Experimental investigations: Understanding local transport , using . . . high power density sources seems the key for a crucial progress and physics insights (. . . similar to thermal plasma transport problem . . .)