# Particle acceleration and resonant transport*

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# Outline

- The concept of resonant transpot.
- Historic paradigm: Mode Particle Pumping (secular radial motion).
- An example involving fast electrons: electron fishbones.
- Mode structures, Nonlinear Dynamics and relevant linear time scales.
- Analysis of one example of self-consistent avalanche dynamics.
- Conclusions.

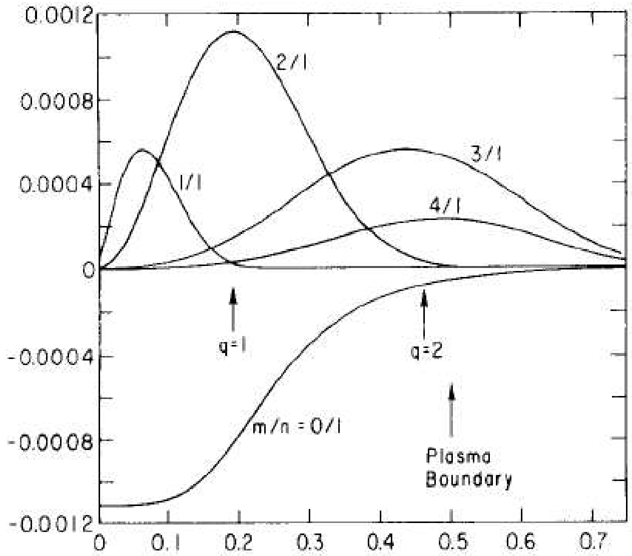
- In burning plasmas, charged fusion products ($\alpha$-particles), as well as energetic ions due to additional heating and current drive (ICRH, NBI), must be confined in order to transfer their energy via Coulomb collisions to the thermal plasma and sustain ignition.
- Some energetic particles (a few) have unconfined orbits, and are lost in the plasma equilibrium configuration: e.g. ripple losses. Losses depend on control of plasma equilibrium.
- Fast particle losses are most dangerous when associated with fluctuations (instabilities). Collective effects, often appearing in bursts, can cause significant losses and severe first wall damage, besides quenching the ignition process.
- Transport is always involving resonant particles. But are there special classes of particles participating to the transport process?
  Resonant transport.
- Refer to July 11th tutorial for a historic review of collective modes.

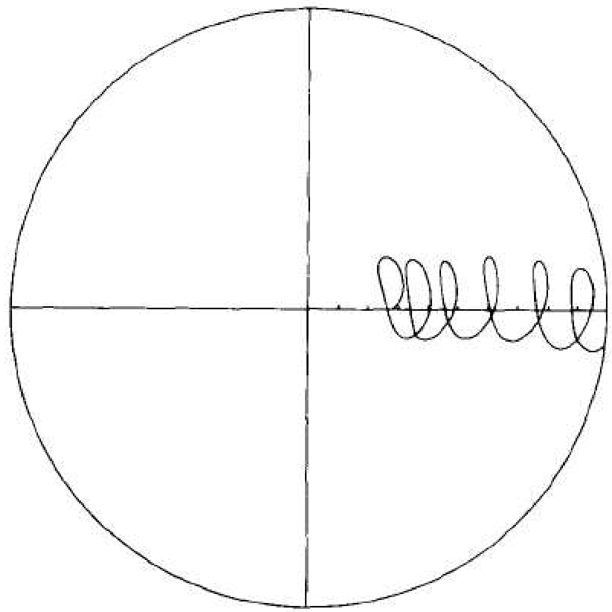
- Mode-particle pumping: (White et al., Phys. Fluids **26**, 2958, (1983))
  MHD $(\delta\phi, \delta A_\parallel)$ with $\delta\phi = \delta\phi_0(r)\sin(n\varphi - m\theta - \omega t + \psi)$

$$r \simeq r_0 + \frac{v_{D0}}{\omega_B}\theta_B\cos(\omega_B t) + \langle\Delta r\rangle \qquad \langle\dot{\Delta r}\rangle = \frac{c}{B}\frac{m}{r_0}\left[\frac{N\omega_B + (m/q)\bar\omega_D}{N\omega_B + (m/q)\bar\omega_D}\right]\delta\phi_0 J_N(m\theta_B)\cos\psi$$

$$\theta \simeq -\theta_B\sin(\omega_B t) \qquad \dot\psi = \frac{\langle\Delta r\rangle}{r_0}(s-1)n\bar\omega_D$$

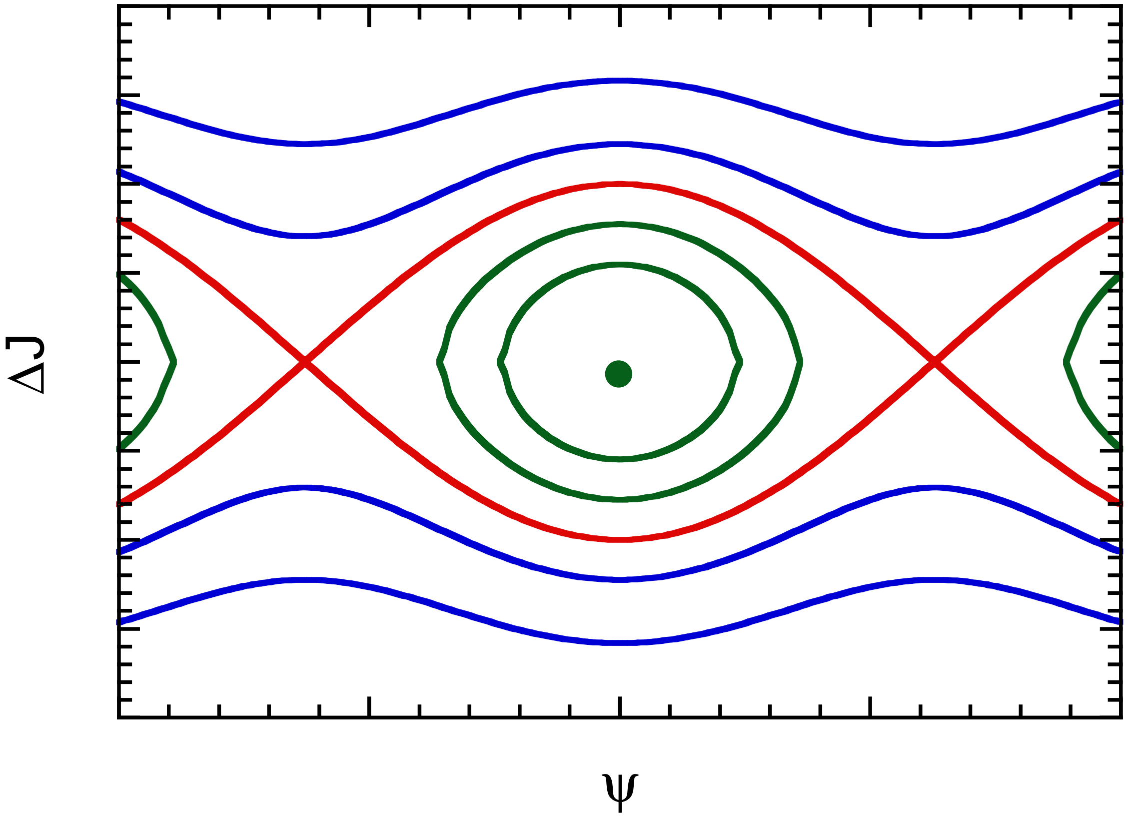
$$\omega = n\bar\omega_D + N\omega_B$$



□ Standard Hamiltonian

$$\Delta \bar{H} \simeq (1/2) F (\Delta \hat{J}_1)^2 - G \cos\theta_1 \qquad F = \partial^2 \bar{H}_0 / \partial \hat{J}_{10}^2 \qquad G \cos\theta_1 \simeq -\epsilon \bar{H}_1$$



- Why fast particles do not get (radially) trapped in the wave and are eventually lost?

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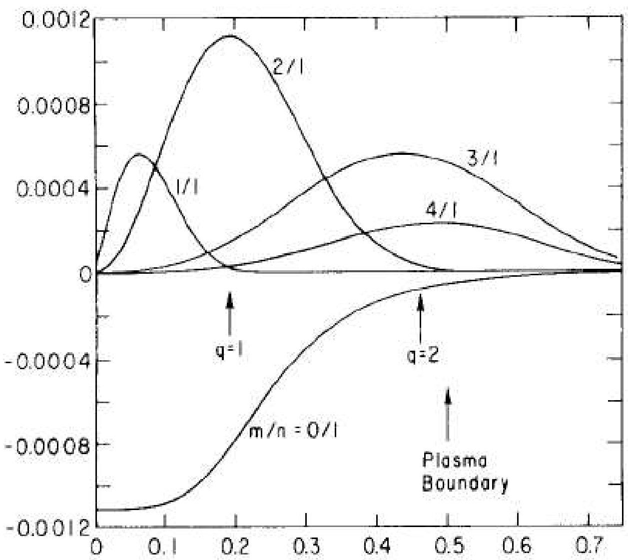


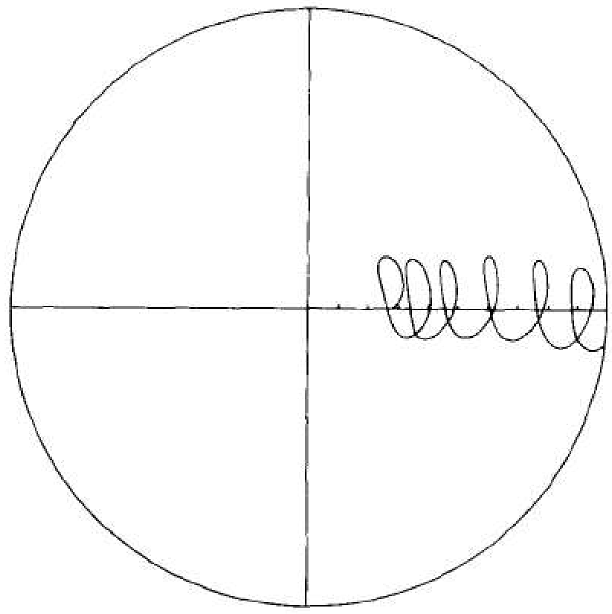

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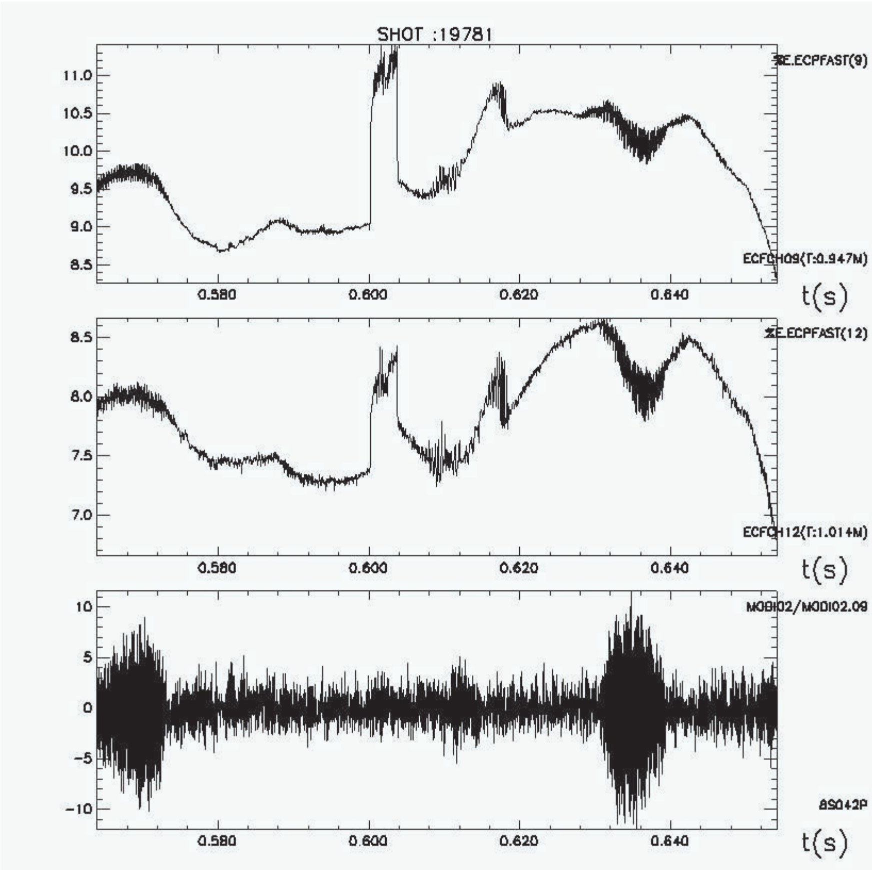
- Fluctuations appear in bursts and with variable frequency or a broad spectrum
- Refer to July 11th tutorial for examples of burst observations in connection with particle losses

# Observations: Electron Fishbones on FTU I

- Lower Hybrid Related fishbones connected with $T_e$ fluctuations

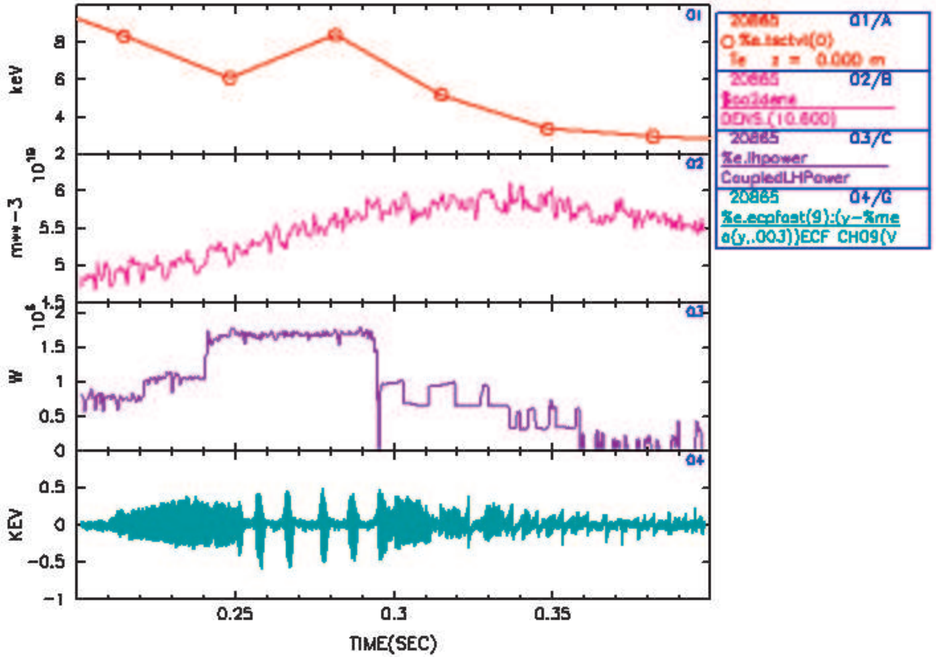



These fishbones can also be seen on the ECE diagnostic. Figure shows the time traces of 2 ECE channels near the plasma center together with a Mirnov coil signal. Two fishbones appear followed by a precursor to a disruption

P.Smeulders, *et al.*, ECA **26B**, D-5.016 (2002)

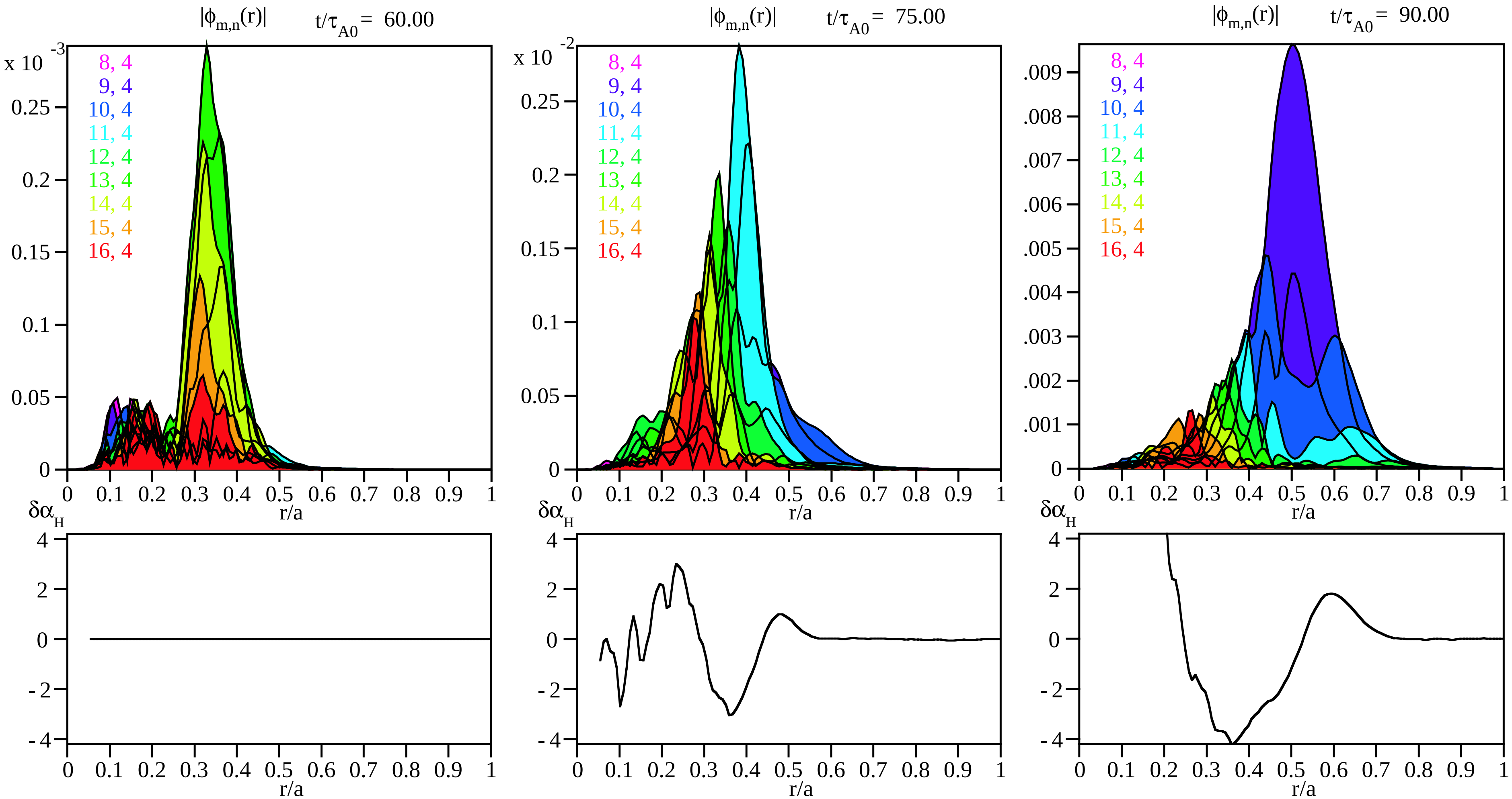
# Observations: Electron Fishbones FTU II

- Electron fishbones observed on FTU are strongly excited with LH. Similar to Tore Supra is the presence of an inverted $q$ profile in the center.




Fishbones are visible with only when LH power is on P.Smeulders, *et al.*, ECA **26B**, D-5.016 (2002)

# Avalanches and NL EPM dynamics (IAEA 02)



# Analyzing mode structures in 2D

- Typical space time scales of low frequency plasma waves.
  - Ballooning Formalism...
  - PSF: A mode structure decomposition approach
  - How does an eigenmode form in 2D
- Extension to weakly nonlinear problems.
  - Nonlinear dynamics and relevant time scales
  - Analysis of one example of self-consistent avalanche dynamics

# Typical space time scales of low frequency plasma waves

- Consider a magnetized plasma with a sheared magnetic field: 2D equilibrium
- Magnetic shear $\Rightarrow k_{\parallel} = k_{\parallel}(\psi_p)$; $\psi_p \equiv$ magnetic flux.
- In order to minimize kinetic damping mechanisms, compression and field line bending effects $\lambda_{\parallel} \approx L$, with $L$ the system size
- Perpendicular wavelength $\lambda_{\perp} \approx L_p/n$ can be significantly shorter than the characteristic scale length of the equilibrium profile $L_p$ for sufficiently high mode number $n$.
- Using the ordering $k_{\parallel}/k_{\perp} \ll 1$ and $k_{\perp} L_p \gg 1$, the 2D problem of plasma wave propagation can be cast into the form of two nested 1D wave equations: parallel mode structure $\oplus$ radial wave envelope.

# Ballooning Formalism...

- Ballooning Formalism (BF): Using asymptotic techniques based on scale separation.
- BF introduced by a number of authors in the late 70's (Coppi PRL77, Lee PFBW 77, Glasser PFBW 77, Pegoraro IAEA 78, Connor PRL 78, Dewar NF81) to conveniently treat linear stability problems on the basis of solution of double periodicity problem with magnetic shear (Connor 75)
- Fourier decomposition of scalar potential fluctuations:

$$\delta\phi = e^{in\zeta} \sum_m e^{-im\theta} \delta\phi_m(r,t)$$

- $(r, \theta, \zeta)$ are field-aligned flux coordinates, with $r$ the radial (flux) variable, $\theta$ the poloidal angle and the equilibrium $\mathbf{B}$ field given by the Clebsch representation $\mathbf{B} = \nabla(\zeta - q\theta) \times \nabla\psi_p$ and $q(r) \equiv \mathbf{B} \cdot \nabla\zeta / \mathbf{B} \cdot \nabla\theta$

- Fourier harmonics $\delta\phi_m(r,t)$ have two scale structures:
  - $\approx (nq')^{-1}$ due to $-1 \lesssim k_\parallel qR = (nq-m) \lesssim 1$: $\parallel$ mode-structure
  - $\approx L_A \ll L_p$ due to equilibrium variation: radial envelope
- Multiple scale structure of Fourier harmonics:

$$\delta\phi_m(r,t) = \underbrace{A(r,t)}_{\text{envelope}} \underbrace{\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}(nq-m)\eta}\delta\Phi(\eta,r,t)d\kappa}_{\text{parallel mode structure}}$$

$$= \exp \mathrm{i} \int nq'\theta_k dr \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}(nq-m)\eta}\delta\Phi(\eta,r,t)d\kappa$$

$$\theta_k = -\mathrm{i}\frac{1}{nq'}\frac{\partial}{\partial r} \quad \text{(Dewar ; NF81)}$$

- Mapping $(r,\theta)$ into $(r,\eta)$: the problem remains 2D

- Eikonal Ansatz for the radial envelope make it possible to solve the 2D problem of plasma wave propagation in the form of two nested 1D wave equations: provided

$$\left| \frac{nq'\theta_k'}{(nq'\theta_k)^2} \right| \ll 1$$

$$\begin{aligned}
\text{2D ODE} \qquad & \overbrace{L(\partial_t, \partial_r, \partial_\theta; r, \theta)}^{\text{symmetric}}\delta\phi &=& \; 0 \\
&& \Downarrow & \\
\text{1D ODE} \qquad & \overbrace{\mathcal{L}(\partial_t, \partial_\eta, \theta_k; r, \eta)}^{\text{symmetric}} A(r,t)\delta\Phi(\eta, r, t) &=& \; 0 \\
&& \Downarrow & \\
& \int_{-\infty}^{\infty} \delta\Phi(\eta, r, t)\mathcal{L}(\partial_t, \partial_\eta, \theta_k; r, \eta) A(r,t)\delta\Phi(\eta, r, t) d\eta &=& \; 0 \\
&& \Downarrow & \\
\text{1D } \Psi\text{DE} \qquad & D(\partial_t, \theta_k; r)A(r,t) &=& \; 0
\end{aligned}$$

# PSF: A mode structure decomposition approach

- The Poisson Summation Formula (PSF) provides a more general (square integrable functions) and elegant derivation of mode structure decomposition (MSD), which reduces to BF in special cases
- The PSF can be put in the form of a periodization operator

$$\delta\phi = e^{in\zeta}\sum_m e^{-im\theta}\delta\phi_m(r,t) = e^{in\zeta}\sum_m e^{-im\theta}\delta\hat{\phi}(m;r) = 2\pi e^{in\zeta}\sum_m \delta\bar{\phi}(\theta+2\pi m;r) \ .$$

- A completely equivalent form is via Fourier Integral representation:

$$\delta\phi = e^{in\zeta}\sum_m e^{-im\theta}\delta\hat{\phi}(m;r) = e^{in\zeta}\sum_m e^{-im\theta}\int_{-\infty}^{+\infty} e^{im\eta}\delta\bar{\phi}(\eta;r)d\eta \ .$$

- More details in Zonca et al., Theory of Fusion Plasmas, Varenna (2004).

# Nonlinear dynamics and relevant linear time scales

- A variety of NL behaviors can be understood in terms of relative importance with respect to different linear time scales. The MSD approach indicates three of them.

- Time for a complete rotation/libration in phase space between a TP pair of the radial envelope $(r_{T1}, r_{T2})$: time scale for global linear eigenmode:

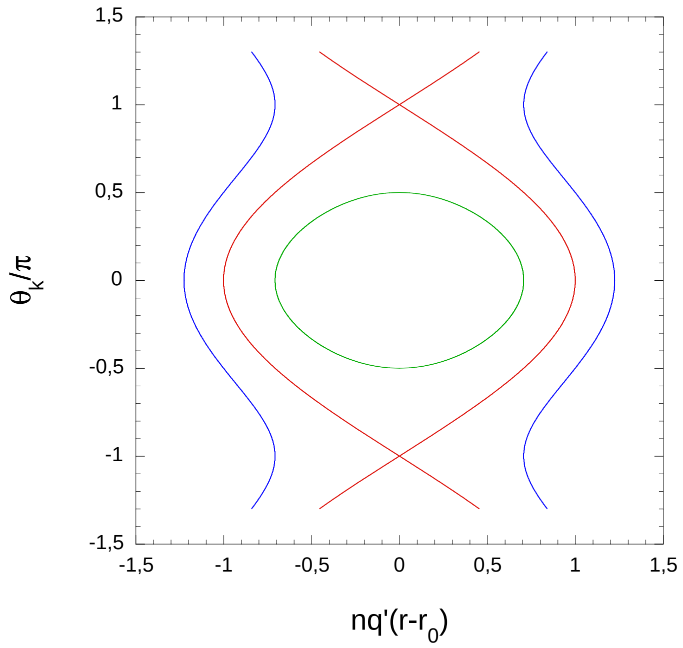
$$\tau_A = 2 \int_{r_{T1}}^{r_{T2}} \left| nq' \frac{\partial D_R/\partial \omega}{\partial D_R/\partial \theta_k} \right| dr \quad .$$

- $\tau_L$: Characteristic linear growth time

$$\tau_L = \bar{\gamma}_L^{-1} = \frac{\tau_A}{2} \left( \int_{r_{T1}}^{r_{T2}} \left| nq' \frac{\partial D_R/\partial \omega}{\partial D_R/\partial \theta_k} \right| \gamma_L dr \right)^{-1}$$

□ $\tau_R = \gamma_R^{-1}$: Characteristic lifetime of the bound state

$$\gamma_R \tau_A = -\ln(R_1 - T_1) - \ln(R_2 - T_2)$$




Determining $R, T$ requires the solution of the global problem: nonlocal behavior of the wave at the turning points. $R + T = 1$

oscillations/librations
rotations

# How does an eigenmode form in 2D

- WKB methods for wave propagation/transmission/conversion in 2D are highly non-trivial (A.N. Kaufman, E.R. Tracy, A. Jaun, Phys. Lett. A **279**, 309, 2001).
- WKB can more easily fails for parallel than for radial wave propagation. Present analysis provides an intermediate approach between full wave and higher order WKB (Pereverzev PoP98; Varenna04)
- Envelope Tracing Equations for $A(r,t) = \tilde{A} \exp \mathrm{i}\Phi$: (Cardinali, Zonca PoP03)

$$\dot{r} = -\frac{1}{nq'} \frac{\partial D_R / \partial \theta_k}{\partial D_R / \partial \omega} \quad ; \quad \dot{\theta}_k = \frac{1}{nq'} \frac{\partial D_R / \partial r}{\partial D_R / \partial \omega} \quad ; \quad \dot{\Phi} = -\theta_k \frac{\partial D_R / \partial \theta_k}{\partial D_R / \partial \omega}$$

$$\frac{d}{dt} \tilde{A} = \left[ \gamma + \frac{\partial^2 D_R / \partial \theta_k^2}{\partial D_R / \partial \omega} \left( \frac{\theta_k'}{2nq'} - \frac{\mathrm{i}/2}{(nq')^2} \partial_r^2 \right) \right] \tilde{A}$$

- After a complete oscillation/rotation:

$$\tilde{A}\exp\mathrm{i}\Phi \Rightarrow \tilde{A}\exp\mathrm{i}\Phi\exp\left(\mathrm{i}\Phi_0 + (\bar{\gamma}-\gamma_R)\tau_A\right)$$

$$\bar{\gamma} = \frac{2}{\tau_A}\int_{r_{T1}}^{r_{T2}} \gamma \left| nq' \frac{\partial D_R/\partial\omega}{(\partial D_R/\partial\theta_k)} \right| dr$$

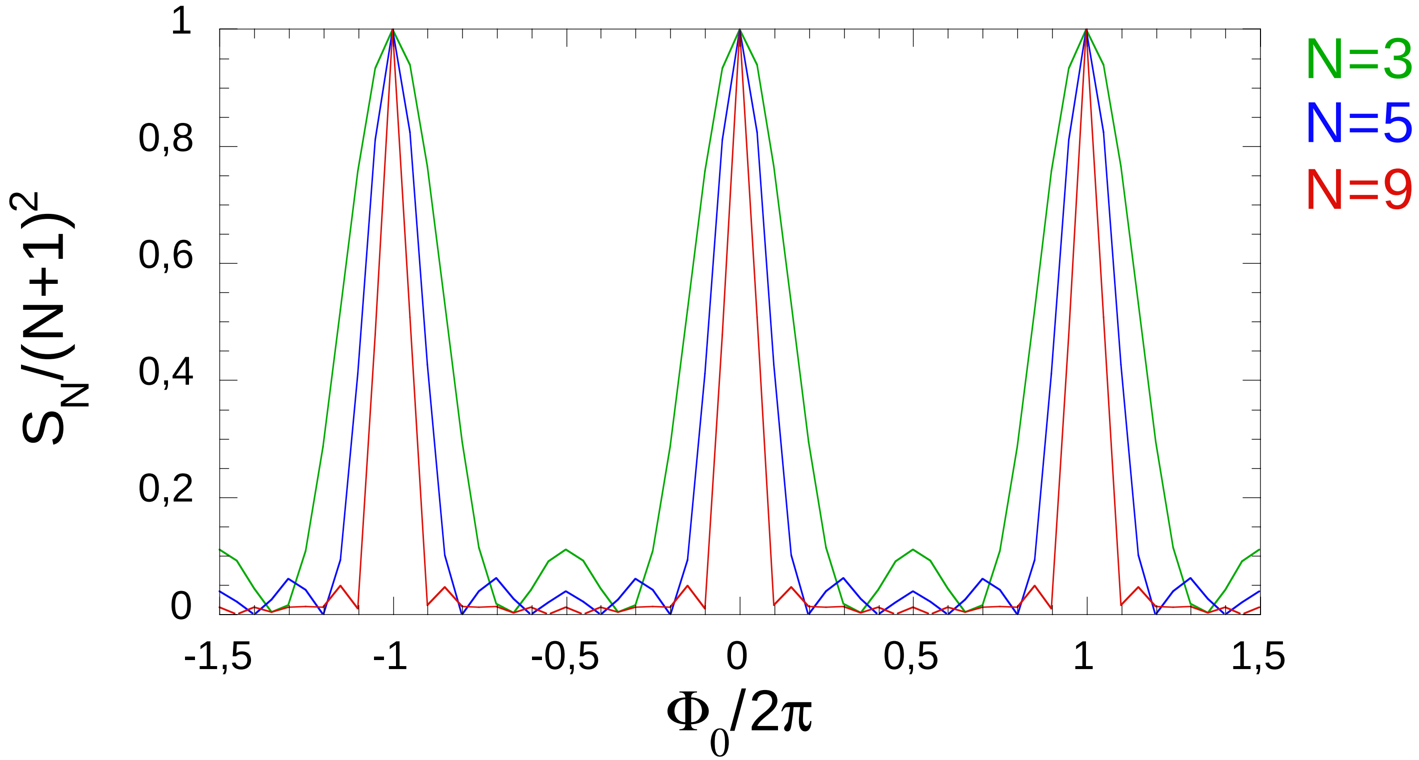
$$\Phi_0 = \oint nq d\theta_k - \overbrace{\mathrm{i}(\pi/2)(\sigma_1+\sigma_2)\mathrm{sgn}(\omega)}^{\text{Maslov Index}}$$

- After $N$ complete oscillations/rotations, $|\tilde{A}|^2 \Rightarrow |\tilde{A}|^2 S_N$:

$$S_N = \left| \frac{1-\mathcal{E}^{N+1}}{1-\mathcal{E}} \right|^2 \quad ; \quad \mathcal{E} \equiv \exp\left(\mathrm{i}\Phi_0 + (\bar{\gamma}-\gamma_R)\tau_A\right)$$

- For $(\bar{\gamma}-\gamma_R)\tau_A \gtrsim 1$ the mode grows at the local growth rate.

- For $(\bar{\gamma} - \gamma_R)\tau_A \ll 1$ the mode sets up the linear eigenmode structure.




- Broadening of spectral lines on short time scales. $\ell$ = radial wave number.

$$\Phi_0 = 2\ell\pi \pm \Delta \;\; ; \;\; \Delta = 2\pi\tau_A/(t+\tau_A) \;\; ; \;\; \Delta \Rightarrow \text{Broadening}$$

# Extension to weakly nonlinear problems

- The concept of MSD is of particular interest when the parallel group velocity is much larger than that in the radial direction, $|v_{\mathrm{gr},\parallel}| \gg |v_{\mathrm{gr},r}|$.
- Multiple time scales enter the problem: $\delta\Phi(\eta; r)$ forms on a $L/|v_{\mathrm{gr},\parallel}| \approx \omega^{-1}$ time scale; while the envelope slowly propagates radially on $\tau_A \approx L_A/|v_{\mathrm{gr},r}|$.
- Sufficiently close to marginal stability, such that $|\gamma_L/\omega| \ll 1$, parallel mode structure forms without significant nonlinear distortions: characteristic nonlinear time scale is $\tau_{\mathrm{NL}} \approx \gamma_L^{-1}$.
- Only linear wave dispersive properties need to be taken into account for determining $\delta\Phi(\eta; r)$ and $D(r, \omega, \theta_k)$, with $\theta_k \equiv (-i/nq')\partial_r$ (Dewar NF81).
- NL interactions reflect on the radial envelope only, for which one can systematically derive nonlinear equations, assuming a hierarchy among NL wave-wave interactions, where the $\tau_{\mathrm{NL}} \approx \gamma_L^{-1}$ is set by fast ion source modulations (or ITG-ZF interactions). (L. Chen *et al.* PoP00, PRL04, PoP04)

- Within this approach, it is possible to systematically generate standard NL equations in the form:

$$\Bigg\{\underbrace{\omega^{-1}\partial_t - \frac{\gamma}{\omega}}_{\text{drive/damping}} - \overbrace{\frac{\xi}{nq'\theta_k}\partial_r}^{\text{group vel.}} + \underbrace{i(\lambda+\xi)}_{\text{potential well}} + \overbrace{\mathrm{i}\frac{\lambda}{(nq'\theta_k)^2}\partial_r^2}^{\text{(de)focusing}}\Bigg\} A(r,t) \quad = \quad \text{NL TERMS}$$

- $\theta_k$ solution of $D_R(r,\omega,\theta_k) = 0$ and

$$\lambda = \left(\frac{\theta_k^2}{2}\right)\frac{\partial^2 D_R/\partial\theta_k^2}{\omega\partial D_R/\partial\omega}\,; \quad \xi = \frac{\theta_k(\partial D_R/\partial\theta_k) - \theta_k^2(\partial^2 D_R/\partial\theta_k^2)}{\omega\partial D_R/\partial\omega}\,; \quad \gamma = \frac{-D_I}{\partial D_R/\partial\omega}$$

- But why are we doing all this? ...

# Nonlinear Dynamics: local vs. global processes

- Mode saturation via wave-particle trapping (H.L. Berk *et al.* PFB 90, PPR 97) has been successfully applied to explain pitchfork splitting of TAE spectral lines (A. Fasoli *et al.* PRL 98): local distortion of the fast ion distribution function because of quasi-linear wave-particle interactions.

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- Compton scattering off the thermal ions (T.S. Hahm and L. Chen PRL 95): locally enhance the mode damping via nonlinear wave-particle interactions
- Mode-mode couplings generating a nonlinear frequency shift which may enhance the interaction with the Alfvén continuous spectrum (Zonca *et al.* PRL 95 and Chen *et al.* PPCF 98): locally enhance the mode damping via nonlinear wave-wave interactions.

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- EPM is a resonant mode (L. Chen PoP 94), which is localized where the drive is strongest: global readjustments in the energetic particle drive is expected to be important as well.

# Nonlinear Dynamics of a single-$n$ coherent EPM

- NL dynamics of a single-$n$ coherent EPM: neglect local phenomena and consider only global NL EPM dynamics. Consistency check *a posteriori* .
- Treat hot particle distribution consisting of a background plus a perturbation on meso time and space scales: the background is frozen in time.

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- The modification in the energetic particle distribution function in the presence of finite amplitude fluctuations is derived in the framework of nonlinear Gyrokinetics
- The non-adiabatic fast ion response – $\overline{\delta H}_k$ – is obtained from the NL gyrokinetic equation (Frieman & Chen PF 82). For details see Zonca et al, NF **45**, 477, (2005).

# NL Dynamics of a single-$n$ coherent EPM (cont'ed)

- NL dynamics of a single-$n$ coherent EPM: neglect local phenomena and consider only global NL EPM dynamics. Consistency check *a posteriori* .
- Treat hot particle distribution consisting of a background plus a perturbation on meso time and space scales: the background is frozen in time.
- Decompose fluctuating particle responses into adiabatic and non-adiabatic

$$\delta F_k = \frac{e}{m}\delta\phi_k \frac{\partial}{\partial v^2/2} F_0 + \sum_{\mathbf{k}_\perp} \exp\left(-\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{v} \times \mathbf{b}/\omega_c\right) \overline{\delta H}_k \quad ,$$

- $\overline{\delta H}_k$ from the NL gyrokinetic equation (Frieman & Chen PF 82):

$$\left(\partial_t + v_\parallel \partial_\ell + \mathrm{i}\omega_d\right)_k \overline{\delta H}_k = \mathrm{i}\frac{e}{m} QF_0 J_0(\gamma)\delta L_k - \frac{c}{B}\mathbf{b}\cdot\left(\mathbf{k}''_\perp \times \mathbf{k}'_\perp\right) J_0(\gamma')\delta L_{k'} \overline{\delta H}_{k''} \quad ,$$

$$QF_0 = \omega_k \frac{\partial F_0}{\partial v^2/2} + \mathbf{k}\cdot\frac{\hat{\mathbf{b}}\times\nabla}{\omega_c} F_0 \quad , \quad \delta L_k = \delta\phi_k - \frac{v_\parallel}{c}\delta A_{\parallel k} \quad ,$$

# Initial value radial envelope problem

- Using both time scale separation, $\omega = \omega_0 + i\partial_t$ as well as spacial scale separation, $\theta_k \Rightarrow (-\mathrm{i}/nq')\partial_r$ with $\partial_r$ acting on $A(r,t)$ only, the initial value radial envelope problem meso time and space scales becomes:

$$\underbrace{\left[D_R(\omega,\theta_k;s,\alpha) + iD_I(\omega,\theta_k;s,\alpha)\right]A_0}_{\text{LINEAR DISPERSION}} \quad = \quad \underbrace{\delta W_{KT} A_0}_{\text{LIN.}\,\oplus\,\text{NL EN. PART. RESP.}} \; ,$$

- $e_H\delta\phi/T_H = A(r,t) = A_0(r,t)\exp(-i\omega_0 t)$, with $|\omega_0^{-1}\partial_t \ln A_0(r,t)| \ll 1$

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- $e_H\delta\phi/T_H = \delta A(r,t) = A_0(r,t)\exp(-i\omega_0 t)$, with $|\omega_0^{-1}\partial_t \ln A_0(r,t)| \ll 1$
- Nonlinear $(n=0, m=0)$ distortion to the hot particle distribution on meso time and space scales: for details see Zonca et al, NF **45**, 477, (2005).

$$\frac{\partial}{\partial t} H_z = 2k_\theta^2 \rho_H^2 \frac{\omega_{cH}}{k_\theta} \frac{T_H}{m_H} \frac{\partial}{\partial r} \left[ \mathbb{I}\mathrm{m} \left( \frac{QF_0}{\omega} \frac{\bar{\omega}_d}{\bar{\omega}_d - \omega} \right) \Gamma^2 |A|^2 \right]_H .$$

- Assume isotropic slowing-down and EPM NL dynamics dominated by precession resonance.

$$
\left[D_R(\omega,\theta_k;s,\alpha)+iD_I(\omega,\theta_k;s,\alpha)\right]\partial_t A_0 = \frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}\alpha_H\left[1+\frac{\omega}{\bar{\omega}_{dF}}\ln\left(\frac{\bar{\omega}_{dF}}{\omega}-1\right)\right.
$$

$$
\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A_0 + i\pi\frac{\omega}{\bar{\omega}_{dF}}A_0\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H\left|A_0\right|^2\right) \quad .
$$

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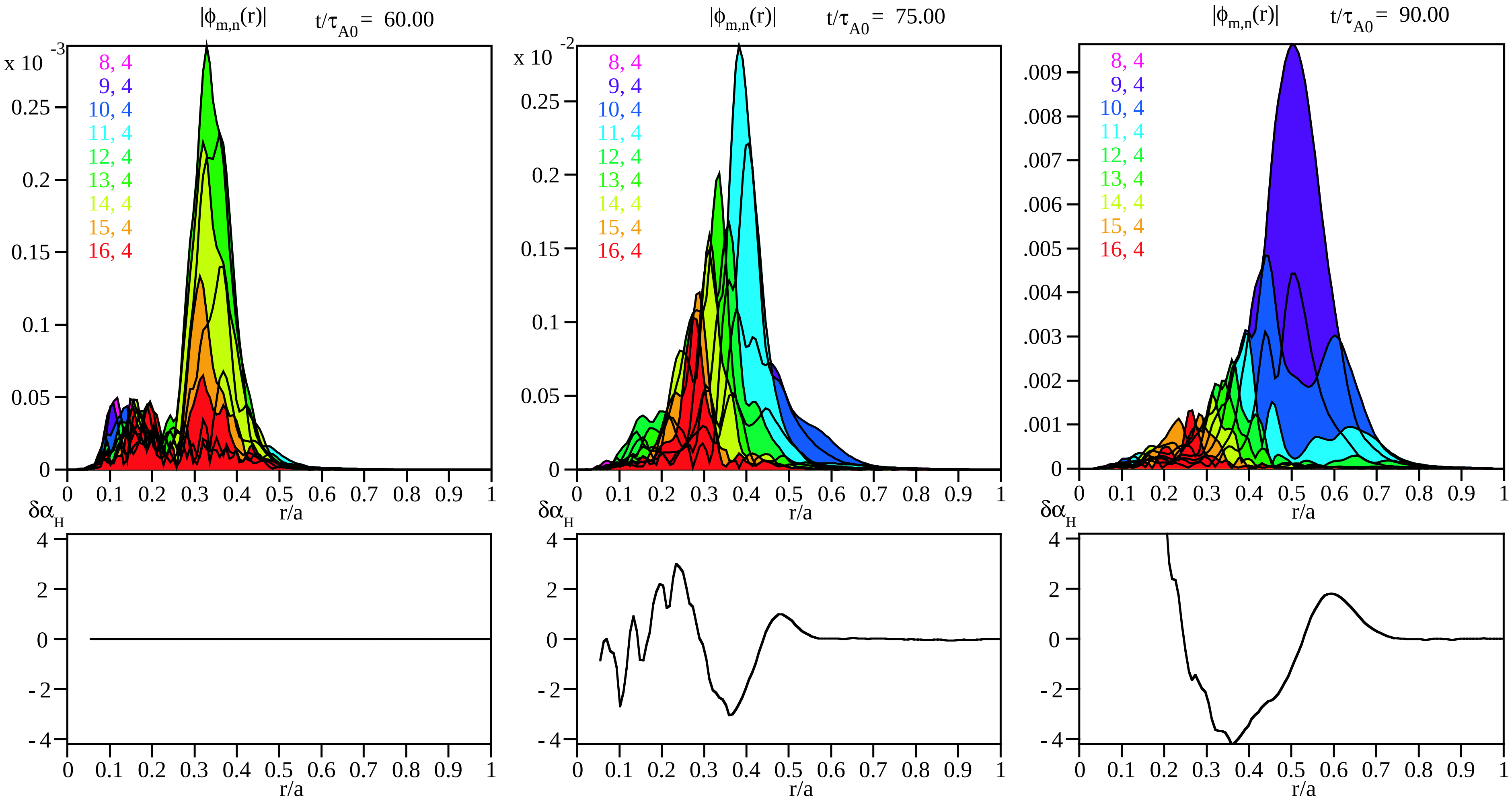
$$[D_R(\omega,\theta_k;s,\alpha)+iD_I(\omega,\theta_k;s,\alpha)]\,\partial_t A_0 = \frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}\alpha_H\left[1+\frac{\omega}{\bar{\omega}_{dF}}\ln\left(\frac{\bar{\omega}_{dF}}{\omega}-1\right)\right.$$
$$\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A_0+\underbrace{i\pi\frac{\omega}{\bar{\omega}_{dF}}A_0\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H\left|A_0\right|^2\right)} \; .$$

DECREASES DRIVE@ MAX $|A_0|$

INCREASES DRIVE NEARBY

# Avalanches and NL EPM dynamics (IAEA 02)



- Assume isotropic slowing-down and EPM NL dynamics dominated by precession resonance.

$$[D_R(\omega,\theta_k;s,\alpha)+iD_I(\omega,\theta_k;s,\alpha)]\,\partial_t A_0 = \frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}\alpha_H\left[1+\frac{\omega}{\bar{\omega}_{dF}}\ln\left(\frac{\bar{\omega}_{dF}}{\omega}-1\right)\right.$$

$$\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A_0+\underbrace{i\pi\frac{\omega}{\bar{\omega}_{dF}}A_0\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H\,|A_0|^2\right)}\;.$$

DECREASES DRIVE@ MAX $|A_0|$

INCREASES DRIVE NEARBY

- Assume localized fast ion drive, $\alpha_H = -R_0 q^2\beta'_H = \alpha_{H0}\exp(-x^2/L_p^2) \simeq \alpha_{H0}(1-x^2/L_p^2)$, with $x=(r-r_0)$.

- In order to maximize the drive the EPM radial structure is nonlinearly displaced by

$$(x_0/L_p)=\gamma_L^{-1}k_\theta\rho_H\,(T_H/M_H)^{1/2}\,(|A_0|/W_0)\;\;,$$

- $x_0$ is the radial position of the max EPM amplitude and $W_0$ indicating the typical EPM radial width in the NL regime

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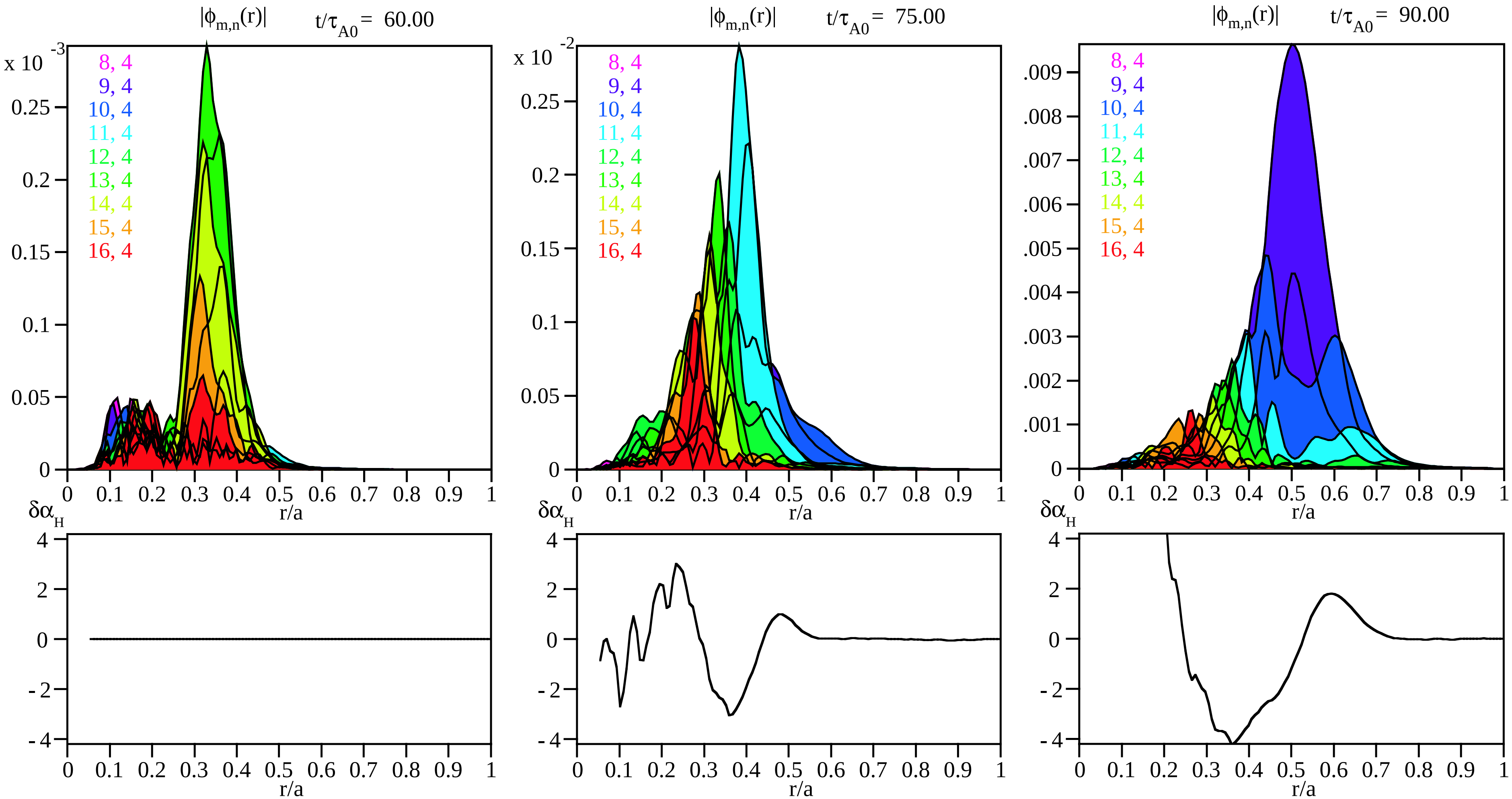
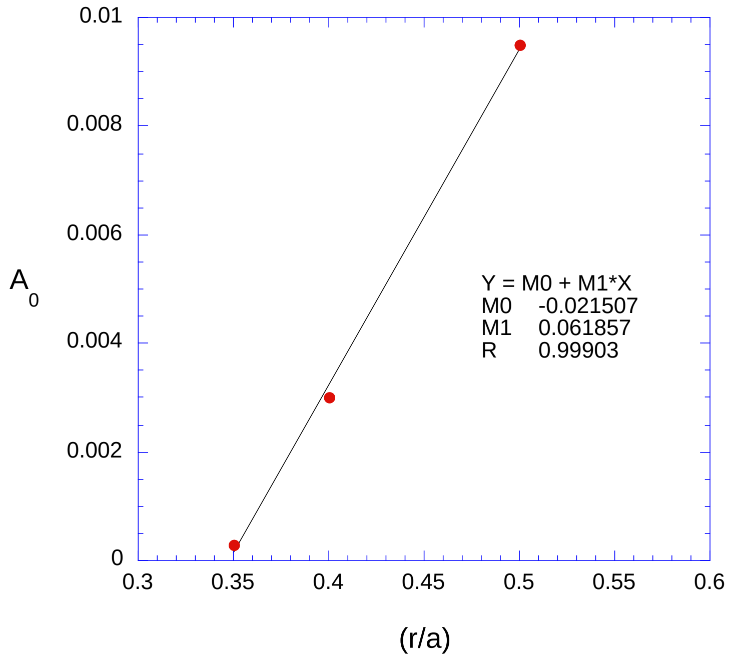


- During convective amplification, radial position of unstable front scales linearly with EPM amplitude.




- Real frequency chirping accompanies convective EPM amplification in order to keep $\omega \propto \bar{\omega}_d$

$$\Delta\omega = (s-1)\,\bar{\omega}_{dF}\big|_{x_0}\,(x_0/r)\,\left(\omega_0/\,\bar{\omega}_{dF}\big|_{x=0}\right) \quad .$$

# Conclusions: ... some homework!?!

• Nonlinear Gyrokinetics: derive the meso space-time scale nonlinear fast ion response to a given Energetic Particle Mode, characterized by an envelope with finite radial extent

• Resonant Particle Transport: verify that the avalanche process satisfies the condition for nonadiabatic frequency sweeping $\dot{\omega} \gtrsim \omega_B^2$, consistently with saturation different than wave particle trapping. See July 11th tutorial for details

• Numerical Techniques Wizards: solve the nonlinear initial value problem of slide 27 ... and win a bottle of wine

Use Zonca et al, NF **45**, 477, (2005) as reference

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