Accretion Disks

A somewhat eccentric and biased introduction for the plasma physicists in Aix en Provence

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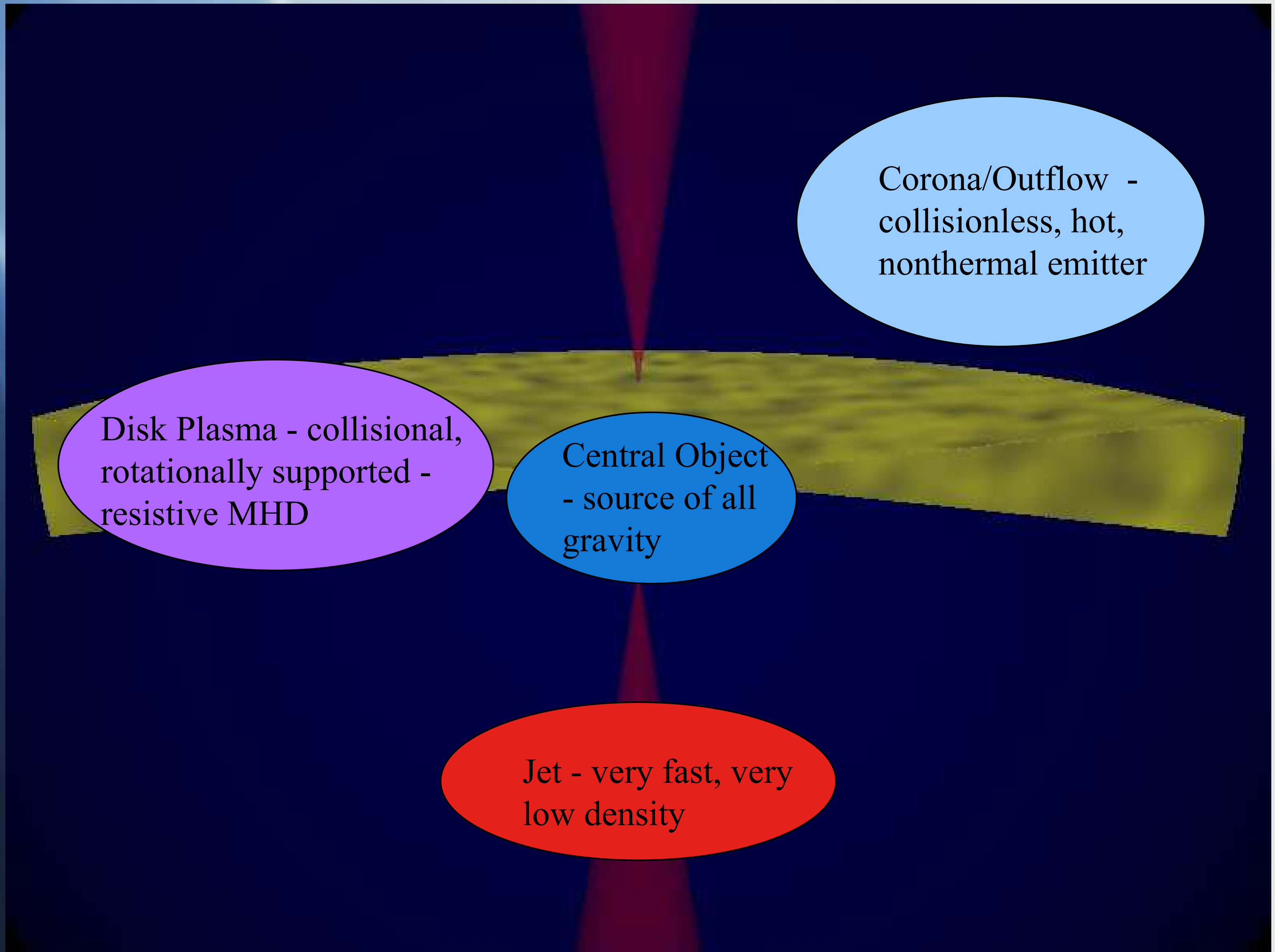
What is an Accretion Disk?

- Flattened, rotionally supported, gas accreting onto a central gravitating object over the course of many orbital periods.
- A good fluid, i.e. a high collision rate and a short mean free path (not true in a disk corona or jet).
- Negligible gravity due to disk (not true in a galactic disk).
- Not necessarily ionized, but here we will emphasize good conductors.

Accretion Disks are Ubiquitous

- Protostars - accretion from environment- winds & jets.
- Cataclysmic variables (white dwarf accretors) - accretion from binary companion - winds.
- Active Galactic Nuclei (supermassive black holes) - accretion from environment - winds & jets.
- Galactic black holes/neutron stars - accretion from companion - winds & jets.

A Theoretical Cartoon:



An Observational Perspective:

- Broad spectral energy distribution.
- Power law emission at high frequencies (X-rays).
- Radio emission seen in some cases, associated with jets.
- Conspicuous emission lines, often in conjunction with optically thick continuum.

Accretion Disks are Important

- Active Galactic Nuclei (AGN) visible over cosmological distances.
- Protostellar disks define the environment in which planets form (as well as being an important stage in stellar evolution).
- Some CVs are the precursors of Type Ia supernovae.
- Astrophysical example par excellence of dynamically important magnetic fields.

The Flow of Mass, Energy and Angular Momentum

- The specific angular momentum of a fluid element determines its radial position in the disk.
- Fluid elements transfer angular momentum outward, which leads the gas to move inward.
- Half the gravitational binding energy gained by a fluid element goes into its orbital motion. The other half is either dissipated as heat, either locally or at a larger radius. (The outward flux of angular momentum has an associated energy flux.)

The outward flow of angular momentum drives the release of gravitational energy.

Viscosity and Other Fictions

- Accretion disks rotate differentially, with the angular frequency decreasing outwards.
- Consequently, friction between annuli will automatically transfer angular momentum outward.
- Real viscosity is orders of magnitude too small in real accretion disks. (We can estimate ν from cataclysmic variable systems.)
- Shakura and Sunyaev (1973) proposed a useful parameterization, inspired by expectations of turbulence.

$$\alpha_{SS} \equiv \frac{\nu\Omega(r)}{c_s^{\ 2}}$$

Physical Equilibrium for Shakura-Sunyaev Disks:

- Vertical hydrostatic equilibrium implies

  $h\Omega^2 \approx c_s^2/h$ or $h\Omega \approx c_s$

- For thin disks, $h << r,\ c_s << r\Omega$, gravity dominates the radial structure so that $\Omega(r) \propto r^{-3/2}$

- Energy dissipated per unit area is $\approx \dot{M}\Omega^2$ which implies that $T_{surface} \propto \left(\dot{M}\Omega^2\right)^{1/4} \propto r^{-3/4}$

- The mass accretion rate is roughly

$$\dot{M} \approx \alpha_{ss} \Sigma h c_s \approx \alpha_{ss} P h \Omega^{-1}$$

- So for a constant flux of mass (and ignoring the difference between the surface temperature and the midplane temperature

$$\Sigma \propto r^{-3/4}$$

$$P \propto r^{-21/8}$$

$$h \propto r^{9/8}$$

All these rules are based on the notion that angular momentum is passed outward within the disk, and that energy dissipated locally is efficiently radiated away, that is, in much less time than it takes for the gas to move inward.

Time Scales

Dynamical rates (for vertical pressure balance etc.) $\tau_{dyn.}{}^{-1} \approx \Omega(r)$

Thermal rates (for vertical heat transfer, also magnetic flux loss) $\tau_{thermal}{}^{-1} \approx \alpha_{ss}\Omega(r)$

Infall rate $\tau_{\inf all}{}^{-1} \approx \alpha_{ss}\Omega(r)\left(h/r\right)^2$

Inefficient radiators have h/r of order unity and become inefficient accretors as well (ADIOS models - Begelman and Blandford).

Diagnostics and Constraints

- The surface temperature does not depend on α, and diagnostics of a stationary disk reveal only products of α and other unknown quantities.
- However, disks whose average temperatures lie in the partial ionization range for hydrogen are thermally unstable, cycling between high temperature, largely ionized states and low temperature, largely neutral states.
- Such systems are seen as `dwarf novae' and the times they spend in their high and low states suggest

$$\alpha_{ss}(high) \approx 0.1$$

$$\alpha_{ss}(low) \approx 0.01$$

The Physics of α

- Originally (1973) it was assumed that accretion disks would be violently, hydrodynamically unstable, giving rise to an α of order unity. After 30 years it has become apparent that hydrodynamical instabilities are extremely weak (Barranco and Marcus 2005).
- The evidence for a substantially larger α in ionized disks led to a widespread belief that in such disks the transport of angular momentum is driven by magnetic fields, which are created in the disk by some dynamo process.
- In the last 15 years this belief has become the community consensus, based on the existence of magnetic field instability which acts to transport angular momentum outward (Balbus and Hawley 1991 and many papers thereafter -- and two papers much earlier, see Chandrasekhar (1961) and Velikhov (1959).

The Magneto-Rotational Instability (MRI)

- Radial ripples in an azimuthal or vertical magnetic field embedded in an accretion disk will be stretched by the differential rotation of the disk. This stretching will have the effect of adding angular momentum to segments that are displaced outward and subtracting it from line segments that are displaced inward. (Like the tethered satellite experiment, except that it works.)
- This torque will reinforce the outward (or inward moti... field line segments, leading to an instability with a growth rate

$$\gamma \approx \sqrt{3}\, k_{\parallel} V_A \ \text{ for } \gamma << \Omega$$

Growth saturates below (3/4)Ω and is suppressed at higher wavenumbers.

Simulations of the MRI

- This MRI has been simulated in `shearing box' and `global' 3D simulations by a variety of groups (Hawley, Stone, Matsumoto, Brandenberg)
- In three dimensions $k_\theta \approx {k_r}/{6} \approx \frac{\Omega}{6V_A}$ s saturat k_z s in a turbulent state wit k_r and a

  few times larger than

- The angul rt is mostly mediated $\langle -b_r b_\theta \rangle \approx \frac{B^2}{2} > 4\pi\rho \langle v_r v_\theta \rangle$

- The local value of a is typically 0.01 or less near the disk mid-plane (or everywhere in simulations with no vertical gravity). However, the average magnetic stress is almost uniform over a few density scale heights, so that the vertically averaged a is probably close to 0.1. The mechanism for saturation is not well understood (boundary conditions, diffusion, buoyancy).
- The magnetization of the corona has not been properly simulated.
- The simulations all produce "large scale" magnetic fields in addition to turbulent components, which are more or less axisymmetric. $B^2 \approx b^2$
- The simulations are all "toy models" with very simple models for the disk plasma. The hope is that if can understand the dynamo process and the MRI instability in detail, we won't need to run some very large number of incredibly detailed numerical simulations. (This isn't a realistic goal anyhow.)

The Disk Dynamo: Generating *B* for the MRI

An accretion disk dynamo is a standard example of an "α-Ω" dynamo, in which the azimuthal field component is generated by shearing the radial field, and the radial field is generated by eddy-scale motions from the azimuthal field.

$$\partial_t \vec{B} = \nabla \times \left(\vec{V} \times \vec{B} + \langle \vec{v} \times \vec{b} \rangle \right)$$

$$\Rightarrow \partial_t B_\theta \approx -\frac{3}{2}\Omega B_r + \left(\nabla \cdot D_T \cdot \nabla \right) B_\theta$$

$$\partial_t B_r = \nabla \times \left(\overleftrightarrow{\alpha} \cdot \vec{B} \right) + \left(\nabla \cdot D_T \cdot \nabla \right) B_r$$

The usual treatment involves keeping only vertical gradients. $\vec{B} \cdot \langle \vec{v} \times \vec{b} \rangle$ is critical for the dynamo effect.

How do we estimate <v x b> in MRI driven turbulence (or anywhere else)?

Set it equal to zero. Take its time derivative, and multiply by the turbulent correlation time. In an environment without bulk flows this gives

$$\left\langle \vec{v} \times \vec{b} \right\rangle_i \approx \varepsilon_{ijk} \left\langle v_j \partial_l v_k - b_j \partial_l b_k \right\rangle B_l \tau_{corr}, \quad 4\pi\rho \equiv 1$$

According to the usual rules of this game, we can set the velocity term equal to some value set by the environment, and it will determine the electromotive force until the magnetic back reaction (from the second term) becomes large. Unfortunately, the second term does not stay small for long (Gruzinov and Diamond, Hughes) and the “kinematic dynamo” is throttled in its cradle

Magnetic Helicity

The trace of the magnetic term is $\vec{j}\cdot\vec{b}$. In the coulomb gauge this is closely related to the magnetic helicity $\vec{j}\cdot\vec{b} = k^2 \vec{a}\cdot\vec{b}$

This is a conserved quantity.

$$\partial_t H = -\nabla \bullet \left(H\vec{\mathrm{v}} + \vec{B}\left(\Phi - \vec{A}\bullet\vec{\mathrm{v}}\right)\right)$$

$$\nabla^2\Phi = \nabla \bullet \left(\vec{\mathrm{v}} \times \vec{B}\right)$$

$$\nabla \bullet \vec{A} = 0$$

These last two conditions are equivalent.

The Conservation of Magnetic Helicity

- If we average over eddy scales, then the magnetic helicity becomes

$$\left\langle \vec{A}\bullet\vec{B}\right\rangle = \left\langle \vec{A}\right\rangle\bullet\left\langle \vec{B}\right\rangle + \left\langle \vec{a}\bullet\vec{b}\right\rangle \equiv H + h$$

- The evolution of H is given by

$$\partial_t H = 2\vec{B}\bullet\left\langle \vec{v}\times\vec{b}\right\rangle - \nabla\bullet\left(\left\langle(\vec{b}\bullet\vec{A})\vec{v}\right\rangle + \vec{B}\Phi - \left\langle \vec{b}\,\vec{v}\bullet\vec{A}\right\rangle\right)$$

$$\nabla^2\Phi = \nabla\bullet\left\langle \vec{v}\times\vec{b}\right\rangle$$

$$\nabla\bullet\vec{A} = 0$$

$$\partial_t h = -2\vec{B}\cdot\left\langle \vec{v}\times\vec{b}\right\rangle - \nabla\cdot\vec{J}_h$$

- In the absence of any magnetic helicity current, the dynamo can only work by creating equal and opposite amounts of magnetic helicity on large and small scales, limiting the large scale magnetic energy to the ratio of the eddy scale to the large scale, times the small scale magnetic field energy.
- Even this is unrealistic. The magnetic helicity h will interact coherently with the large scale field, inducing motions (through the c$\left\langle \vec{b} \bullet \vec{j} \right\rangle$t helicity) which will transform h into H. The rate for this is

$$\tau^{-1}{}_{cascade} \approx k^2 B^2 \tau_{corr}$$

A successful dynamo requires a systematic magnetic helicity current, driving local accumulations of "h", which then drives the dynamo through the nonlinear

The Eddy-Scale Magnetic Helicity Current

Unless the large scale field is very weak, the inverse cascade is faster than anything else, so that

$$\langle \vec{v} \times \vec{b} \rangle = -\frac{\vec{B}}{2B^2} \nabla \cdot \vec{J}_h$$

The eddy scale magnetic helicity current can be calculated explicitly. It is

or

$$\vec{J}_h = h\vec{V} + \int \frac{d^3k}{(2\pi)^3 k^2} \left(\left\langle \vec{v}(\vec{B} \cdot \vec{j}) - \vec{b}\left(\vec{B} \cdot \vec{\omega}\right) \right\rangle \right) - 2r\Omega' \int \frac{d^3k}{(2\pi)^3 k^4} \left\langle j_r \partial_\theta \vec{b} + j_\theta \partial_r \vec{b} \right\rangle$$

This will be zero in perfectly symmetric turbulence. What do we need to get a non-zero effect?

Symmetry Properties of the Magnetic Helicity Flux

This flux is invariant under a reversal of the magnetic field. It will depend on even powers of the large scale magnetic field.

This flux is invariant under a parity transformation. It can be nonzero in mirror-symmetric turbulence.

A non-zero flux requires symmetry breaking in 2 directions. Differential rotation will provide this.

A constant flux is uninteresting, but if the large scale magnetic field varies with position, then div J will not vanish.

A Simple Model of the Accretion Disk Dynamo

- In order to see what this does to the accretion disk dynamo, we need to plug in the correlations expected from the MRI. We can gain some insight by simply writing the magnetic helicity current in the vertical direction as AB_θ^2 Plugging this into the dynamo, and neglecting the dissipative terms, we get

$$\partial_t B_r \approx A\partial_z^2 B_\theta \qquad \partial_t B_\theta = -\frac{3}{2}\Omega B_r$$

- This will give an exponentially growing magnetic field if the magnetic helicity current

The Magnetic Helicity Flux for the MRI

We can evaluate the magnetic helicity flux to quasilinear order by taking the time derivative of correlations between the fluctuating magnetic and velocity fields and multiplying by the correlation time. In the presence of differential rotation we need to invert the effects of shear and the Coriolis and centrifugal forces. The final expression is quite complicated. However, the MRI produces eddies with $k_\theta < k_r < k_z$, and $\Omega\tau_{corr} \sim 3$

We can use this to get

$$J_{hz} \approx 2 B_\theta^2 \tau_{corr} \int \frac{d^3 \vec{k}}{(2\pi)^3 k^2} k_\theta^2 \left[\langle v_r v_\theta \rangle + \frac{3}{2} \langle v_r^2 \rangle + (\Omega \tau_{corr})^2 \left(\langle b_r^2 \rangle - 2 \langle b_r b_\theta \rangle \right) \right]$$

The MRI Dynamo

- We conclude that the MRI should drive a positive vertical magnetic helicity current, and a dynamo.
- By the same line of reasoning, whatever drives turbulence in the Sun drives a dynamo, and the predicted magnetic helicity flux should be negative.
- If we treat accretion disk as a periodic shearing b then this gives us a dynamo growth rate of $\sim \frac{V_A}{L_B}$

- The existence of a dynamo depends on the sign of the magnetic helicity current. If it ran the other way the dynamo would be suppressed (as it is in magnetic Kelvin-Helmholtz simulations).

- The implication is that the dynamo field always grows on a time scale of a few eddy turn over times, and is always dominated by scales which are a few eddy scales in size (vertically). This is what is seen in the MRI simulations.

Towards More Realistic Models

- Real disks are vertically stratified, which will suppress the MRI driven dynamo when the eddy shapes are squeezed vertically.
- The dynamo field will probably vary in time, and the effects of this on disk structure are largely unknown. The time scale for variations, the dynamo time, is ~ the disk thermal time (based on balancing buoyant losses with dynamo growth).
- We expect, based on the simulations done to date, that the time averaged Shakura-Sunyaev "α" will not be a constant but will vary with the distance from the midplane.
- Computer simulations need to conserve magnetic helicity and vertical boundary conditions need to

The Effect of Accretion Disks on Their Environment: Coronae

- In real disks, as opposed to periodic simulations, the magnetic helicity current will emerge from the disk photosphere.
- Since the scale of the helicity current will change from <h at the photosphere to ~r at the edge of the corona/wind, it will shed most of its energy, amounting to about α_{ss} of the disk energy budget, in the corona. This is similar to a popular explanation for the solar corona, although the disk can also inject additional material from runaway heating. It may also run an additional dynamo in the corona.

The Effect of the Environment on Accretion Disks - The Accretion of Poloidal Flux

- *The usual assumption is that entrained flux moves with the accreting plasma. This process is balanced by the outward turbulent diffusion of bent magnetic field lines. This gives (van Ballegooijen)*

$$\alpha c_s \left(H/r\right) \sim \left(B_r/B_z\right) D_T/H \sim \alpha c_s \left(B_r/B_z\right)$$

In other words, the field lines bend very slightly as they pass through a narrow disk, which limits the accretion of external flux to a negligible level.

Neglecting this effect, Blandford and Payne proposed magneto-centrifugal jets, launched

Magnetic Helicity and Turbulent Diffusion

The electromotive force in a turbulent medium, has a piece of the form:

$$-D_T \vec{J}, \qquad D_T \approx \left(\langle v^2 \rangle - \langle b^2 \rangle\right)\tau_{corr}/3$$

This is zero in Alfvenic, isotropic turbulence. When the turbulence is anisotropic due to shearing, the diffusion coefficients differ and attach to the appropriate gradient. When the back reaction is large, the diffusion is suppressed by the factor $I - \hat{b}\hat{b}$*. The radial diffusion of entrained flux is suppressed by the ratio of the perpendicular components to the total field.* $B_\perp^2 / B^2$

Outward diffusion = inward drag when….

Balancing the two, as before, we get a critical poloidal field strength

$$B_{crit} \sim B_\theta \left(h/r\right)^{1/2} \sim c_s \left(\alpha h/r\right)^{1/2}$$

This is just big enough that the radial and vertical field components will be similar (Lubow, Papaloizou and Pringle 1994) and the stress (and angular momentum transport) through the magnetosphere will be comparable to the internal transport. When the poloidal field is weaker than this, it will be dragged inward. Due to this, and the scaling of disk properties, the poloidal field will typically affect the innermost part of the accretion disk, or

When does the environment supply enough magnetic flux?

Since the accreted poloidal field has nowhere to go, whether or not one gets an inner annulus dominated by a large scale poloidal field will depend on the environment. In general, it's not obvious what this means for systems that accrete from the ISM. We can look at binary accreting systems, and assume that the accretion disk can accrete some fraction of the donor star's poloidal field, determined by the size of the disk. This will be important if

$$B_p \left(r_{disk} / r_{inner} \right)^2 \geq B_{crit} \sim \sqrt{\dot{M} \left(\Omega / r \right)}$$

Plugging in Numbers….

Obviously, for otherwise identical accreting systems, this is more likely to give us a strong poloidal field when the accreting object is a compact object, the smaller the better.

A white dwarf near the Chandrasekhar limit, accreting at 10^18 gm/sec, with an externally imposed field of about a gauss, and an inner disk radius of ~3x10^10 cm, will fail to acquire a strong poloidal field by a bit more than an order of magnitude. In the same kind of system a black hole will easily acquire a strong poloidal field.

Conclusions

- The magnetorotational instability can drive an internal disk dynamo. This makes angular momentum transport within an ionized accretion disk self-consistent.
- As a consequence, we can expect accretion disks to pollute their environment with a magnetic helicity. This does not depend on any particular level of organization in magnetic field.
- The environment may be a controlling influence for black holes in binary systems. Jets? AGN?