Relaxation by forced magnetic reconnection

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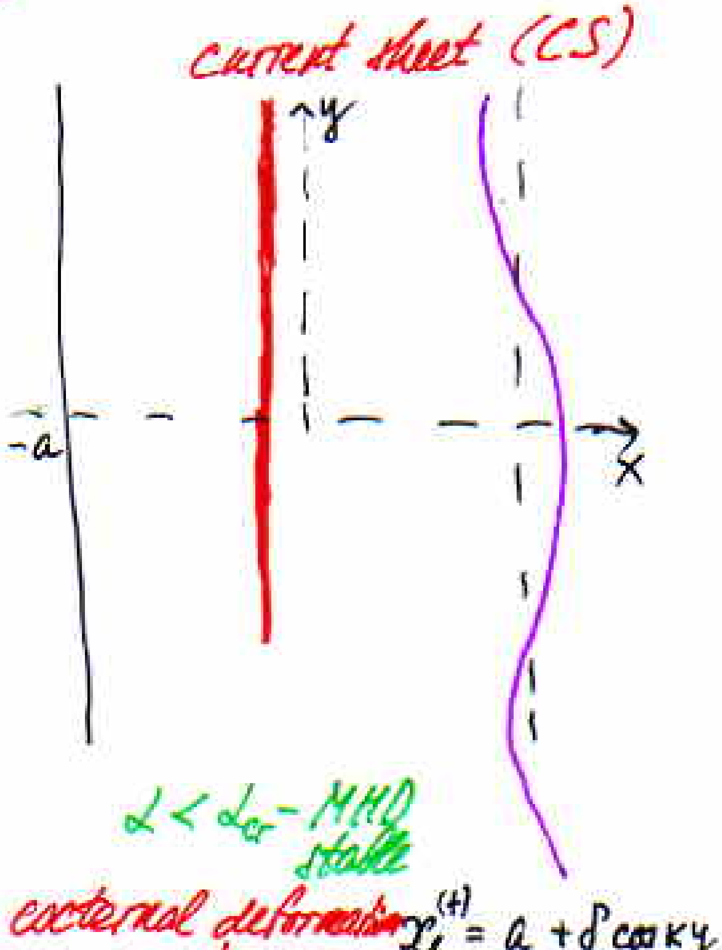
Two types of magnetic reconnection

1) spontaneous reconnection (via resistive MHD instabilities)
2) forced reconnection (triggered in an MHD stable field by external perturbation)

Simple example: sheared planar force-free field

$\underline{B} = \{0, B_0 \sin \alpha x, B_0 \cos \alpha x\}$



tearing unstable if the shear parameter

$$\alpha > \alpha_{cr} = \frac{\pi}{2a}$$

(Hahm and Kulsrud, 1985)

$\alpha < \alpha_{cr}$ – MHD stable

external deformation $x_b^{(+)} = a + \delta \cos ky$

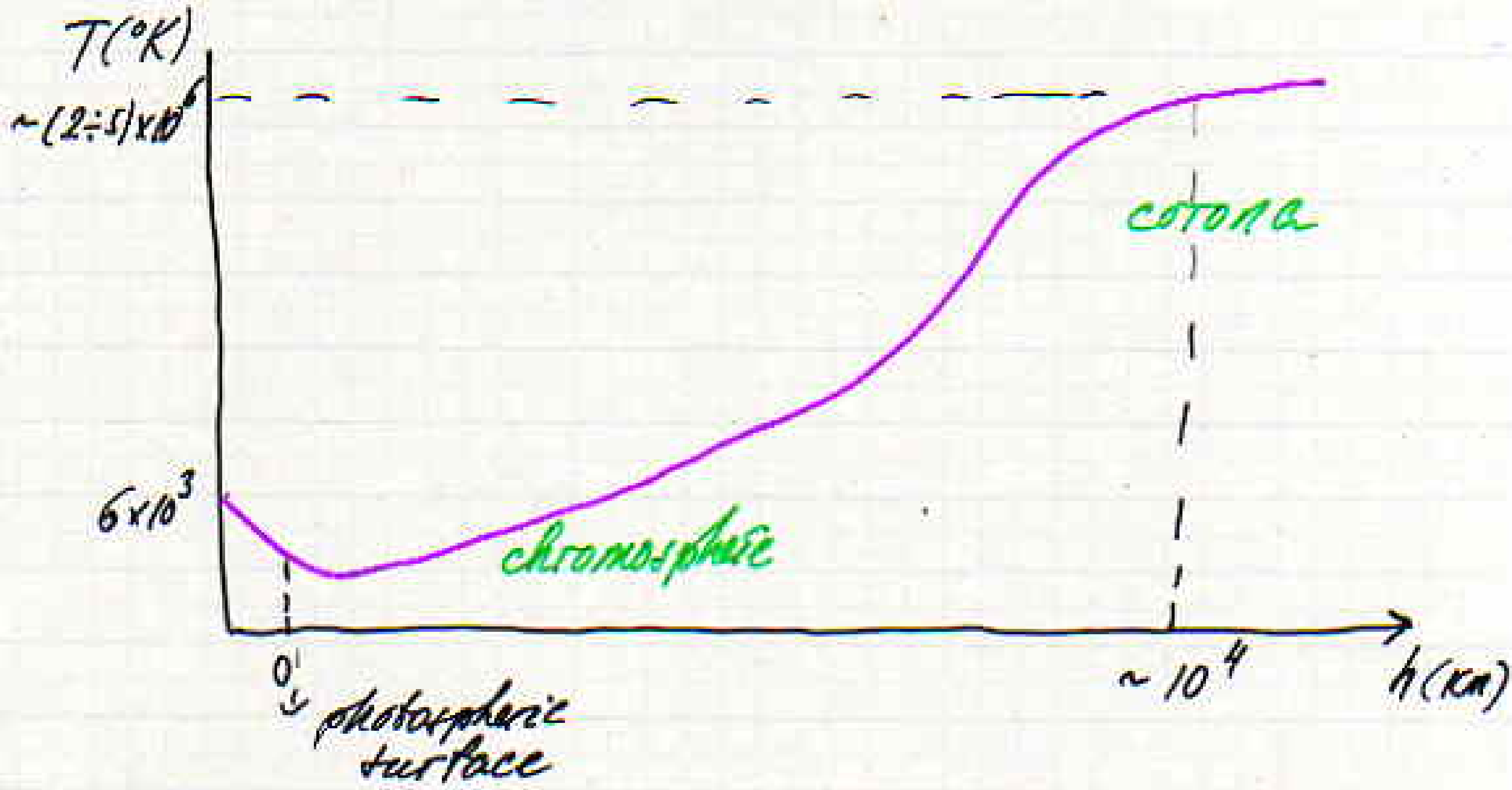
↓

current sheet formation

↓

forced reconnection

Motivation: mechanism of the solar (and stellar) coronal heating



Coronal plasma: $T \sim 10^2 eV$, $n \sim (10^9 \div 10^{10}) cm^{-3}$

cooling by radiation and thermal conduction } $\Rightarrow$ $q \simeq (3\times10^5 \div 10^7)$ erg/cm²·sec

is required to maintain the high-temperature corona

$\Downarrow$

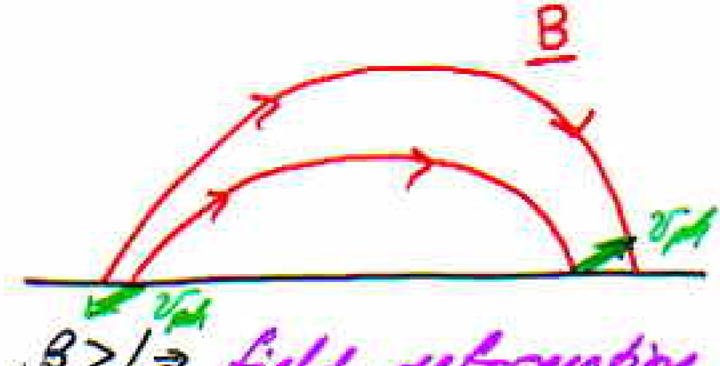
$L_\odot \simeq 10^{11} \frac{erg}{cm^2 \cdot sec}$ $\Rightarrow$ a tiny fraction of the whole energy budget of the Sun

Magnetic coronal heating

$\beta \ll 1 \Rightarrow$ magnetically dominated

$B_c \sim 10^2$ G (active region) $\Rightarrow$

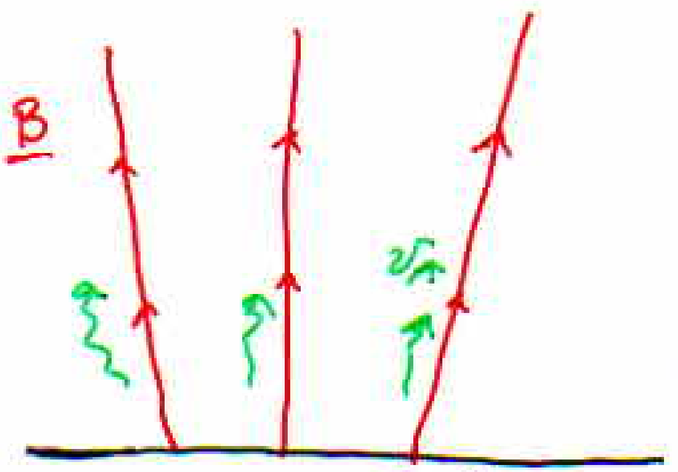
$V_A \sim 10^3$ km/sec



-- photospheric surface

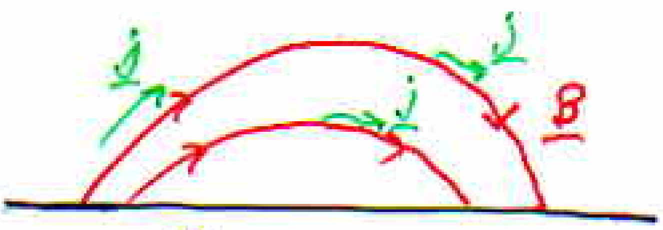
$\beta > 1 \Rightarrow$ field deformation by convective motions

$$\left.\begin{array}{l} v_{ph} \sim 1 \text{ km/sec} \\ l_{ph} \sim 10^3 \text{ km} \end{array}\right\} \Rightarrow \tau_{ph} \sim \frac{l_{ph}}{v_{ph}} \sim 10^3 \text{ sec}$$



coronal hole $\rightarrow$

generation of Alfven waves



active region magnetic loop

$L \sim 10^4$ km $\Rightarrow \tau_A \sim \frac{L}{V_A} \sim (1 \div 10)$ sec

$\underbrace{\tau_{ph} \gg \tau_A}$

quasistatic field deformation

$\Rightarrow$ force-free magnetic field $\Rightarrow$

excess magnetic energy potentially available for coronal heating

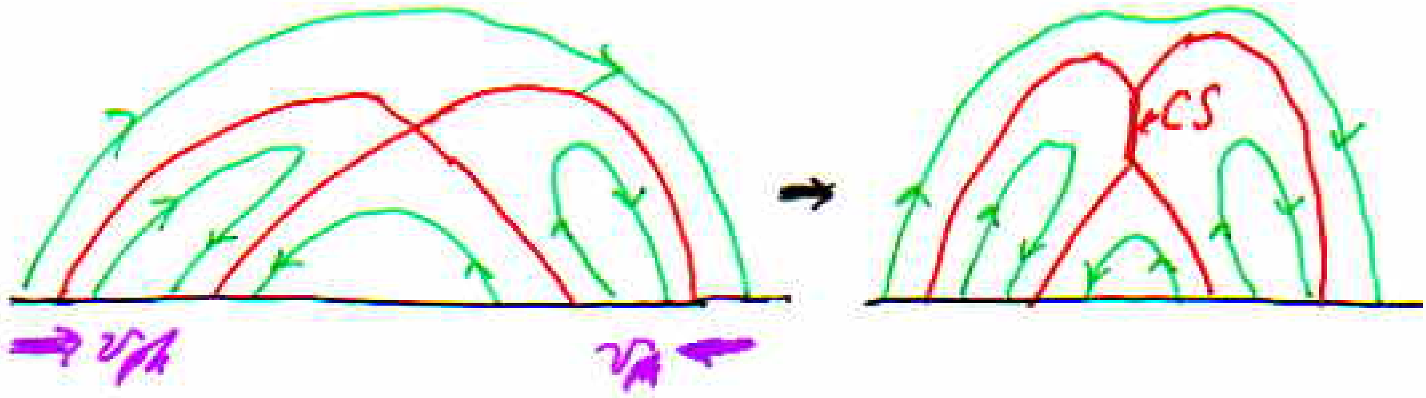
Major difficulty: high electric conductivity of the coronal plasma

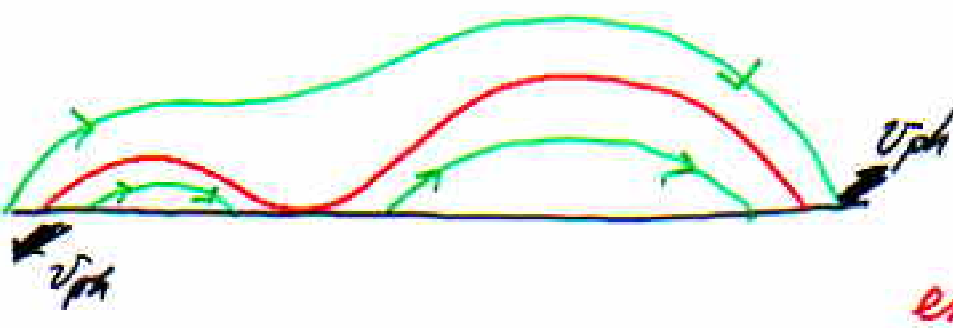
$\tau_\eta = L^2/\eta$ is extremely large

$S \equiv \tau_\eta/\tau_A \sim (10^{11} \div 10^{12}) \Rightarrow$

simple Ohmic dissipation is irrelevant

Way out $\Rightarrow$ current sheets (Parker, 1972)





photospheric shearing $\Rightarrow$

CS on the entire separatrix surface

Current sheets + finite resistivity $\Rightarrow$

forced magnetic reconnection

Slow perturbation: $\Delta t \gg \tau_A \Rightarrow$

force-free equilibrium

$$\underline{B} = (\underline{\nabla}\psi(x,y) \times \hat{z}) + B_z(x,y)\,\hat{z}$$

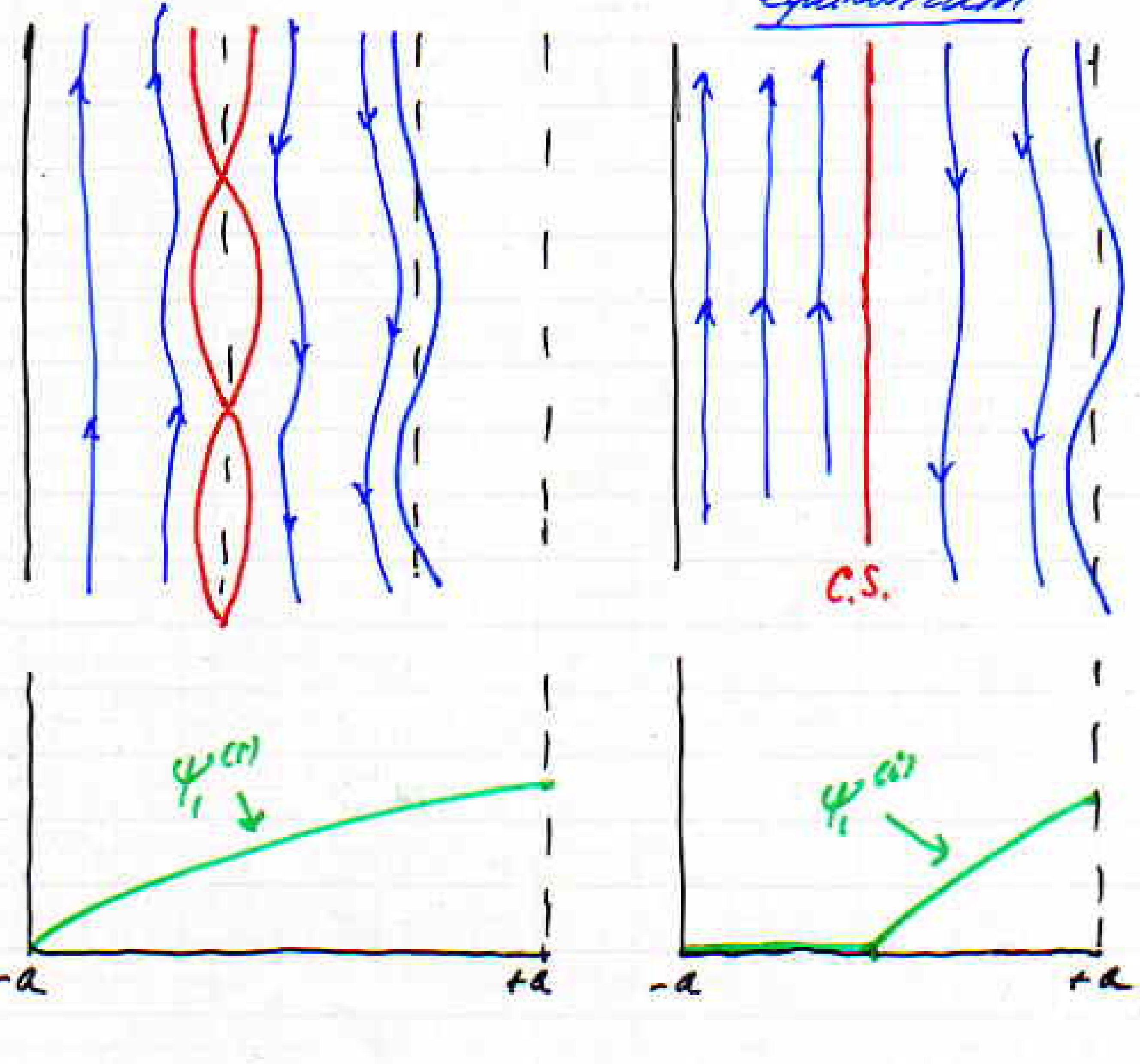
$$\nabla^2\psi + B_z \frac{dB_z}{d\psi} = 0\,, \quad B_z(x,y) \equiv B_z(\psi)$$

Linear approximation: $\delta \ll a \Rightarrow$

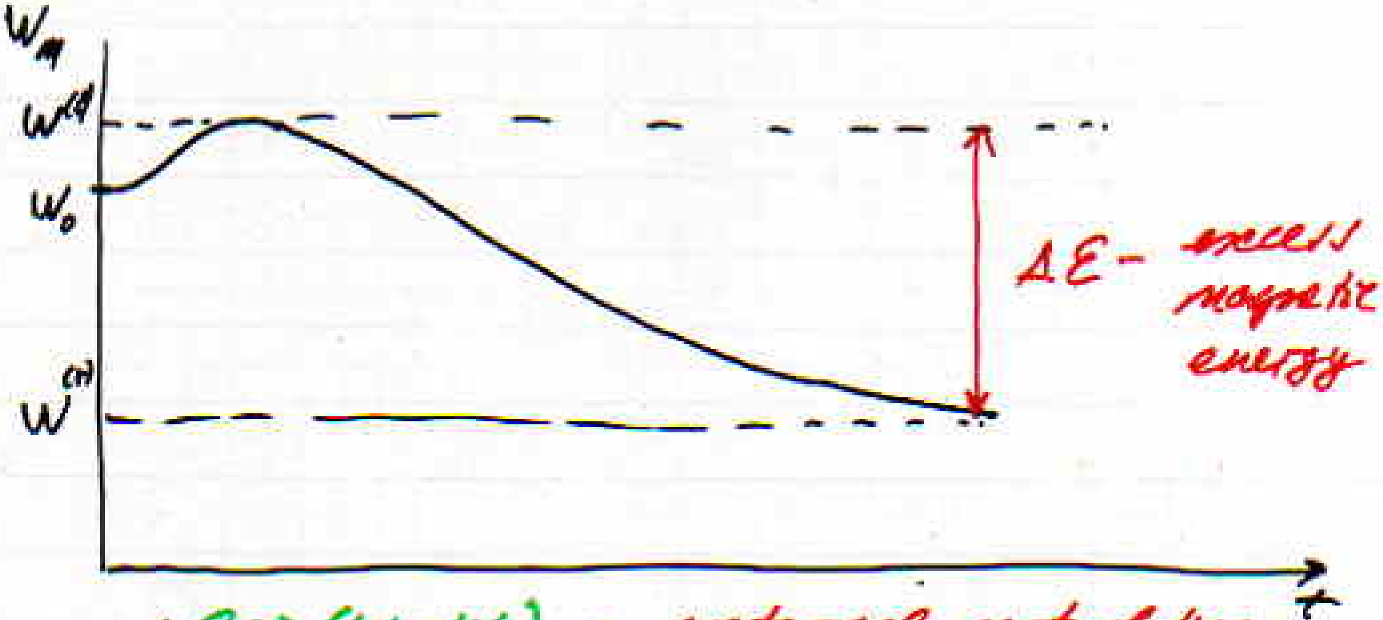
$$\psi = \frac{B_0}{\alpha}\cos\alpha x + \psi_1(x)\cos ky\,; \quad B_z(\psi) = \alpha\psi$$

Two possible equilibria

Regular solution | Singular ideal MHD equilibrium



Evolution of the magnetic energy

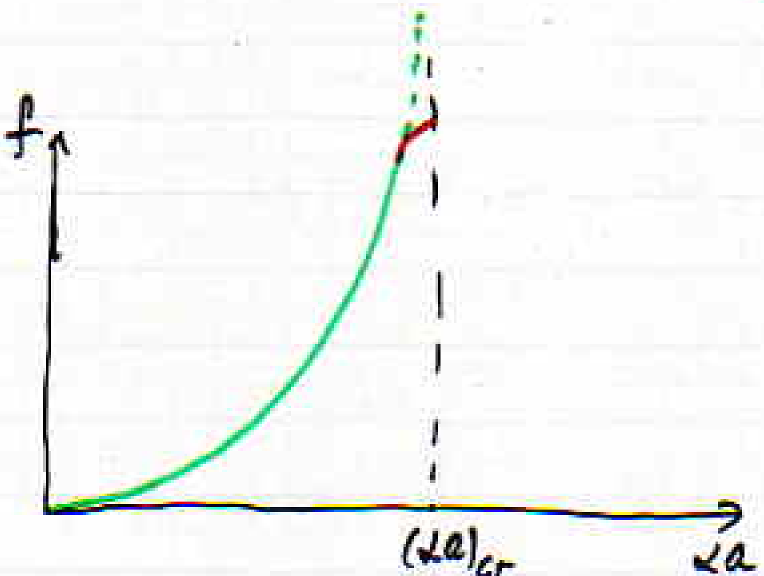


$\Delta E \gg (W_i - W_0) \to$ external perturbation acts as a trigger for magnetic relaxation ⇒

ΔE is tapped from the energy stored in the initial magnetic field

$$\Delta E = \frac{B_0^2}{8\pi}\cdot\frac{\delta^2}{a}\underbrace{f(\lambda a)}$$

→ dependence on the shear of the initial field



f becomes very large as $\lambda \to \lambda_{cr}$ ⇒ the link between the forced and spontaneous reconnection

Marginally stable state:

$$f \sim \left(\frac{\delta}{a}\right)^{-1/3} \gg 1$$

Energy dissipation under ongoing reconnection

$$\underbrace{\delta = \delta_0 e^{-i\omega t}}_{\text{continuous external driving}}; \quad \omega \tau_A \ll 1 \text{ – quasistatic perturbation}$$

$$\underbrace{\psi_1 = A\psi_1^{(i)} + (1-A)\psi_1^{(e)}}_{\text{external solution}}$$

$A = A_1 + iA_2$ – amplitude of the ideal state

The dissipation power

$$Q = \left\langle\left\langle \frac{B^2(x_s^{(H)})}{8\pi}\frac{d\delta}{dt}\cos ky \right\rangle\right\rangle = -\omega A_2 \langle \Delta \mathcal{E} \rangle$$

A is determined by the internal structure of the current sheet $\Rightarrow$ internal solution

$$\boxed{\begin{aligned} \rho\frac{d\underline{v}}{dt} &= (\underline{j}\times\underline{B}) + \rho\nu\nabla^2\underline{v} \\ \frac{\partial \underline{B}}{\partial t} &= \underline{\nabla}\times(\underline{v}\times\underline{B}) + \eta\nabla^2\underline{B} \end{aligned}}$$

$\tau_\eta = a^2/\eta$ – global resistive time-scale

$\tau_\nu = a^2/\nu$ – global viscous time-scale

$\tau_\eta = S\,\tau_A$; $S \gg 1$ – Lundquist number

$P_m = \nu/\eta$ – magnetic Prandtl number

$$\tau_\nu = P_m^{-1} S\,\tau_A \gg \tau_A \rightarrow \underline{P_m < S}$$

Linear approximation ($\delta_0 \ll a$) for an incompressible plasma flow: $\underline{v} = \nabla\varphi(x,y,t)\times\hat{z}$ ⑤

$\varphi(x,y,t) = \varphi(x)\sin ky\, e^{-i\omega t}$ — streamfunction

$$\boxed{\begin{gathered}\omega^2\varphi'' - i\omega\tau_v^{-1}\varphi^{(iv)} = kx\,\tau_A^{-2}\psi_1'' \\ -i\psi_1 = -ikx\varphi + (\omega\tau_R)^{-1}\psi_1''\end{gathered}}$$

Constant-ψ-approximation

External solution: $\Delta' = \frac{2x}{\alpha}\cot(xa)\frac{A}{1-A}$

Internal solution: $\Delta' = \int \psi_1''\,dx$ ⇒

$$\boxed{\begin{gathered}\frac{A}{1-A} = \frac{\tan(xa)}{k(1-k^2/x^2)}\int\frac{dx}{x}\left[(\omega\tau_A)^2\varphi'' - i(\omega\tau_A)\frac{P_m}{S}\varphi^{(iv)}\right] \\ \varphi^{(iv)} + iSP_m^{-1}(\omega\tau_A)\varphi'' + S^2P_m^{-1}(kx)^2\varphi = S^2P_m^{-1}kx\end{gathered}}$$

Two regimes of forced reconnection

1) Low driving frequency: $\underline{\omega < \omega_1 \sim \tau_A^{-1}S^{-1/3}P_m^{1/3}}$

plasma inertia isn't important ⇒

viscous reconnection

2). high driving frequency: $\omega > \omega_1$ ⇒

viscosity plays no role ⇒

inertial reconnection

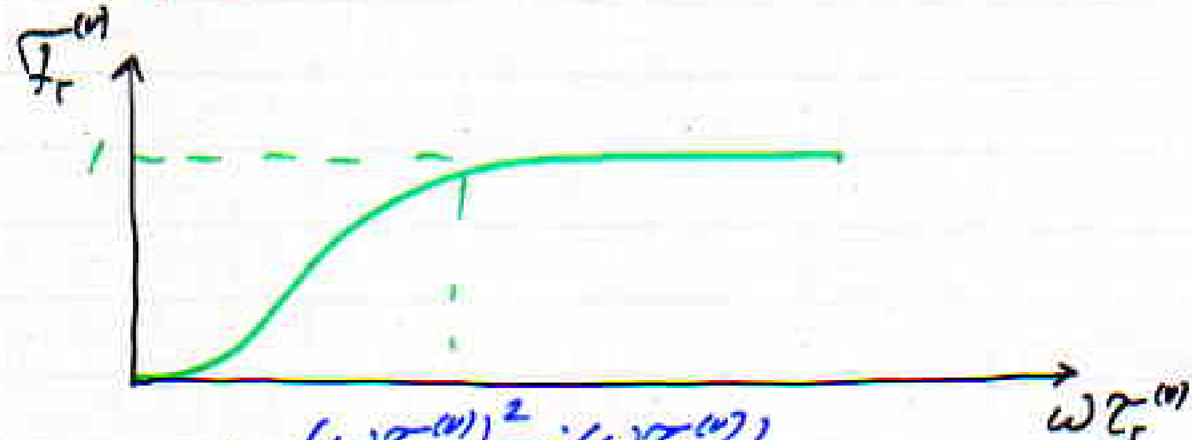
Viscous reconnection ($\omega < \omega_1 \sim \tau_A^{-1} S^{-1/2} P_m^{3/4}$)

$$\tau_r^{(\nu)} \sim \tau_A S^{2/3} P_m^{1/6}$$

$$(\Delta x)^{(\nu)} \sim a S^{-1/3} P_m^{1/6}$$

$$Q^{(\nu)} = \frac{\langle \Delta E \rangle}{\tau_r^{(\nu)}} \mathcal{F}_r^{(\nu)}(\omega \tau_r^{(\nu)})$$

$$\mathcal{F}_r^{(\nu)}(\omega \tau_r^{(\nu)}) = (\omega \tau_r^{(\nu)})^2 / 1 + (\omega \tau_r^{(\nu)})^2$$



$$A = \frac{(\omega \tau_r^{(\nu)})^2 - i(\omega \tau_r^{(\nu)})}{1 + (\omega \tau_r^{(\nu)})^2}$$

Two dissipation channels: resistive and viscous

$$Q^{(\nu)} = Q_\eta + Q_\nu$$

$$\int \eta j^2 dx \qquad \int \rho \nu (\varphi'')^2 dx$$

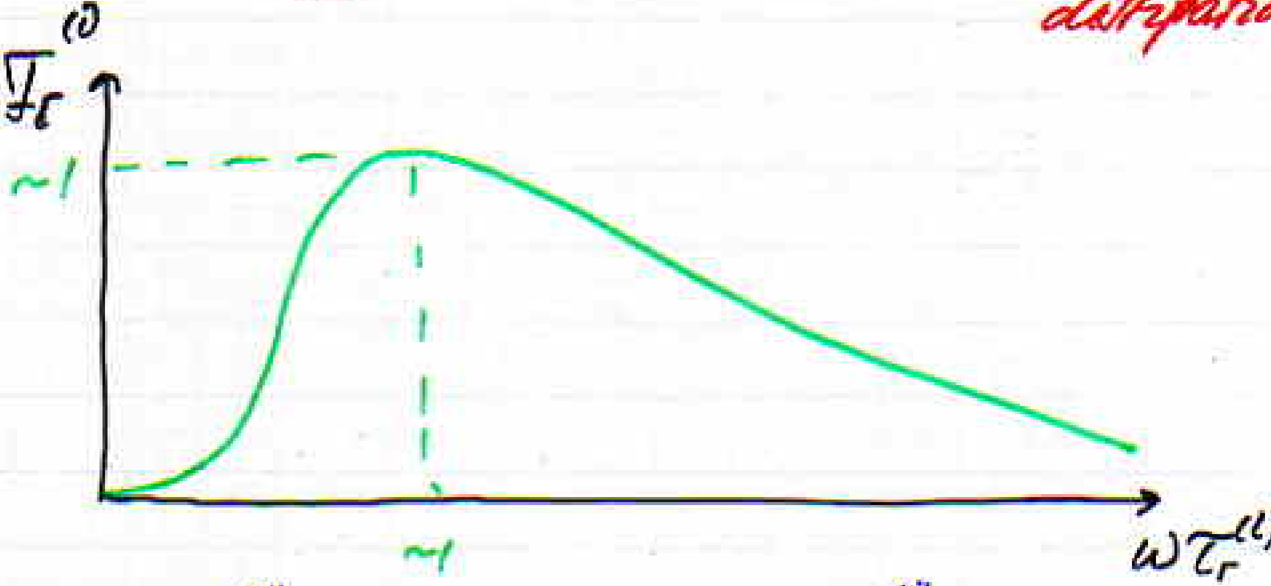
$Q_\eta \sim Q_\nu$ irrespective of the magnitude of the magnetic Prandtl number

Inertial reconnection ($\omega_1 < \omega < \omega_2 \sim \tau_A^{-1} S^{-1/3}$)

$$\tau_r^{(i)} \sim \tau_A S^{3/5}$$

$$\Delta x^{(i)} \sim a S^{-1/4} (\omega \tau_A)^{1/4}$$

$$Q^{(i)} = \frac{\langle \Delta E \rangle}{\tau_r^{(i)}} \mathcal{F}_r^{(i)}(\omega \tau_r^{(i)}) \equiv Q_2 \text{ - ohmic dissipation}$$



$\omega \tau_r^{(i)} \ll 1$
$\Downarrow$
$\mathcal{F}_r^{(i)} \sim (\omega \tau_r^{(i)})^{3/4}$

$\omega \tau_r^{(i)} \gg 1$
$\Downarrow$
$\mathcal{F}_r^{(i)} \sim (\omega \tau_r^{(i)})^{-1/4}$

Upper frequency bound ω_2 for the inertial reconnection $\Rightarrow$

violation of the quasistatic approximation for the external solution $\Rightarrow$

$$\omega < \omega_2 \sim \tau_A^{-1} S^{-1/3}$$

As $\omega_1 \sim \tau_A^{-1} S^{-1/3} P_m^{-2/3}$ $\Rightarrow$ no inertial regime of forced reconnection if $P_m > 1$

Regimes of plasma heating

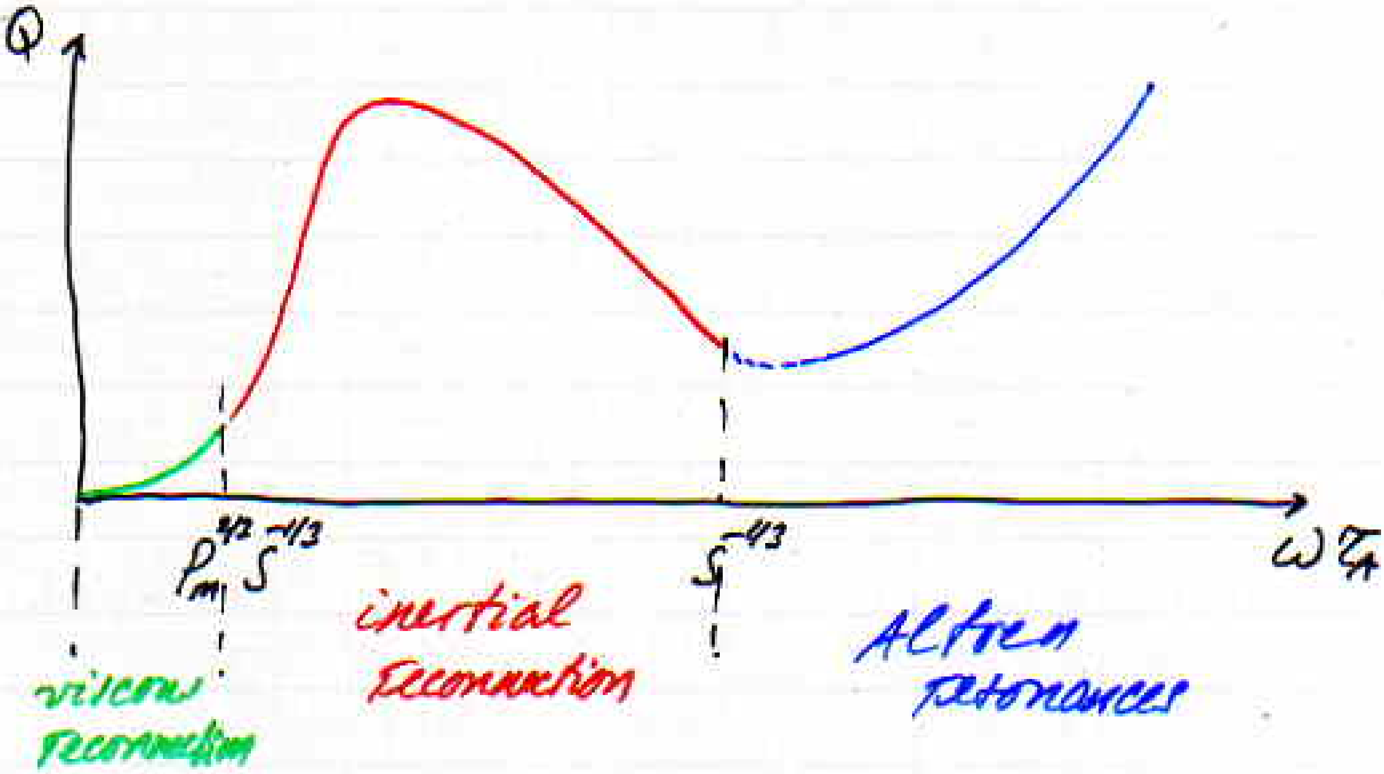
3 non-dimensional control parameters:

$$S \gg 1, \quad P_m, \quad \omega\tau_A \ll 1$$

$\swarrow$

3 different dissipation regimes

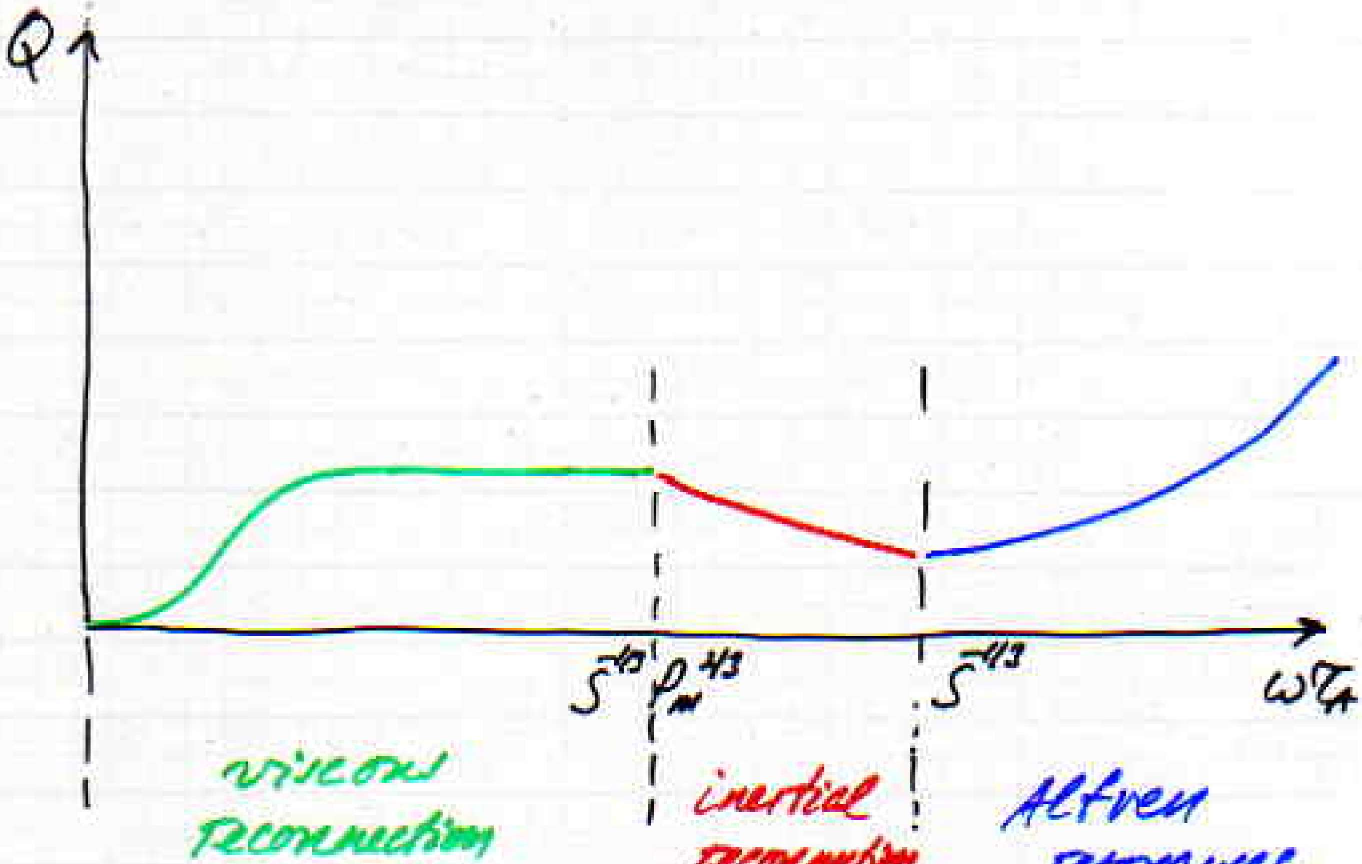
i. $P_m < S^{-2/5} \rightarrow \omega_1 \sim \tau_A^{-1} S^{-1/3} P_m^{1/3} \ll \tau_1^{(i)-1} \sim \tau_A^{-1} S^{-3/5}$



Viscosity plays only a minor role

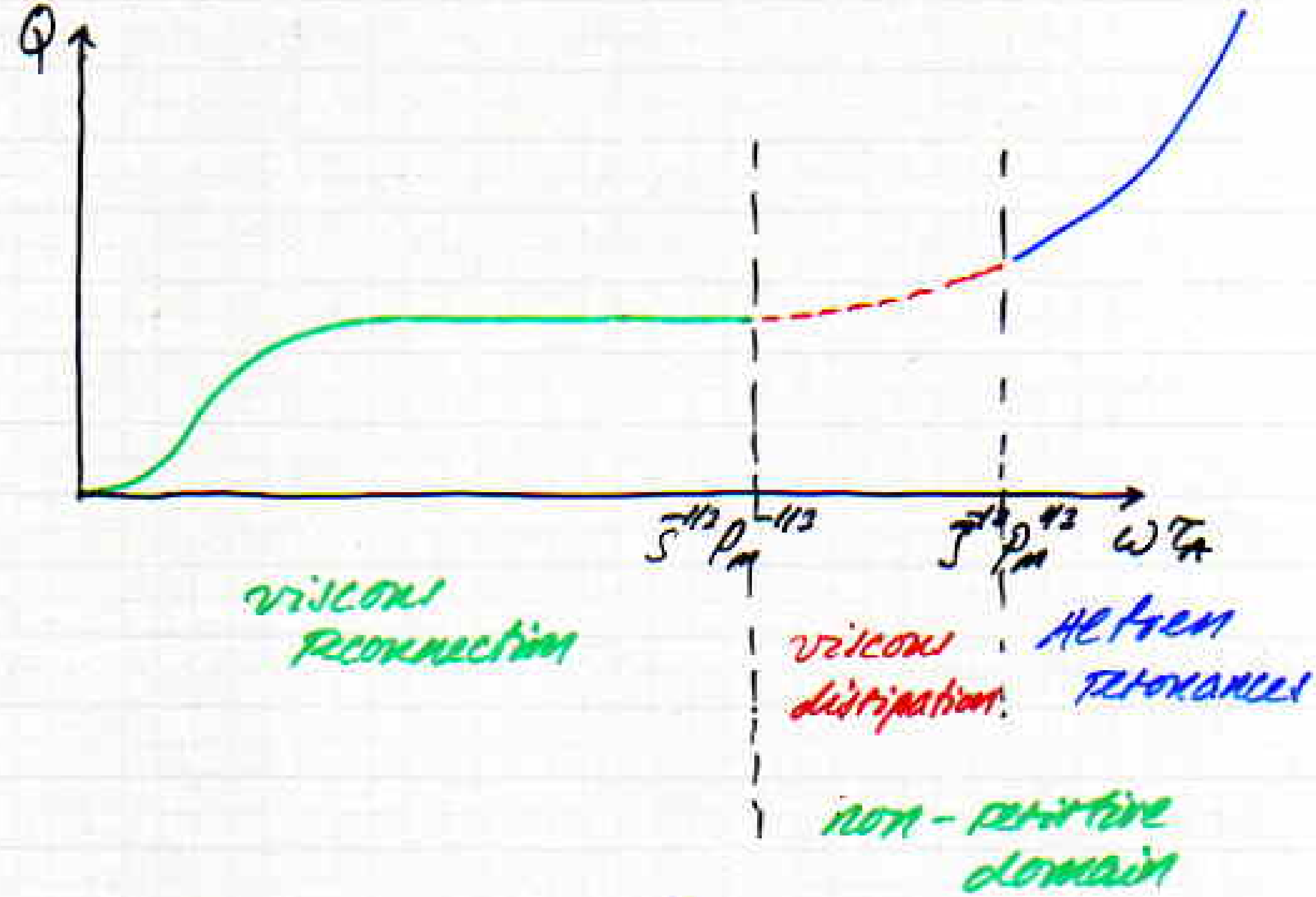
2) $S^{-2/5} < P_m < 1 \Rightarrow$

$$\omega_1 \sim \tau_A^{-1} S^{-1/3} P_m^{2/3} \gg \tau_r^{(m)-1} \sim \tau_A^{-1} S^{-3/5} P_m^{-1/6}$$



<u>Viscosity plays a dominant role even when $P_m \ll 1$</u>

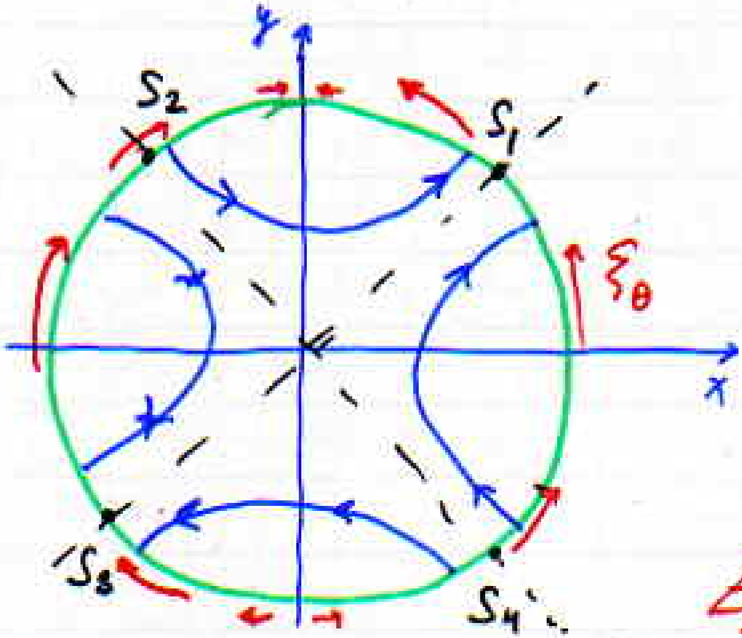
3). $P_m > 1$ → <u>no inertial reconnection</u>



$\omega > \omega_3 \sim \tau_A^{-1} S^{-1/3} P_m^{-1/3}$ → viscous stresses violate the quasistatic approximation for the external solution

$\omega > \omega_4 \sim \underbrace{\tau_A^{-1} S^{-1/3} P_m^{1/3}}_{\omega_4 \text{ doesn't depend on resistivity}}$ → viscosity doesn't destroy Alfven resonances

Forced magnetic reconnection at the neutral X-point



Initial potential quadrupole field

$$\Psi_0(r,\theta) = \frac{B_0}{2R} r^2 \cos 2\theta$$

$$B_{0r} = -\frac{1}{r}\frac{\partial \Psi_0}{\partial \theta} = B_0 \frac{r}{R} \sin 2\theta$$

$$B_{0\theta} = \frac{\partial \Psi_0}{\partial r} = B_0 \frac{r}{R} \cos 2\theta$$

External perturbation:

displacements at the boundary surface $r = R \Rightarrow$

redistribution of the flux function $\Psi(R,\theta)$

Example:

$\xi_\theta = \delta_0 \cos\theta$

$\Downarrow$

$$\Psi(r=R,\theta) = \Psi_0(\theta - \delta\theta) \simeq \Psi_0(\theta) - \frac{\partial \Psi_0}{\partial \theta}\delta\theta \Rightarrow$$

$$\Psi(r=R,\theta) = \underbrace{\frac{B_0}{2} R \cos 2\theta}_{\text{initial}} + \underbrace{\frac{B_0}{2}\delta_0 \sin\theta + \frac{B_0}{2}\delta_0 \sin 3\theta}_{\text{perturbation}}$$

When the perturbation results in the current sheet formation inside the boundary surface? $\Rightarrow$

If the new <u>regular</u> solution is not consistent with the initial field in <u>the</u> framework of ideal MHD

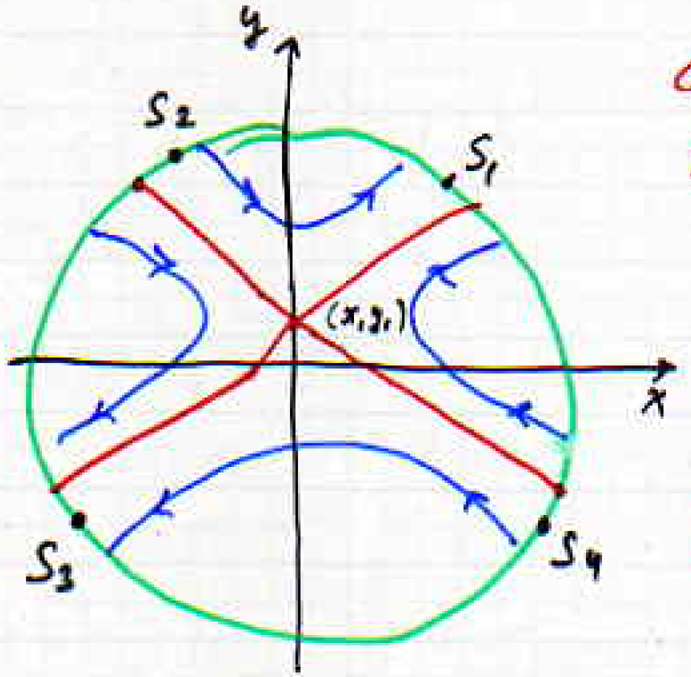
*) $\Delta\Psi_3 = \frac{B_0}{2}\delta_0 \sin 3\theta \Rightarrow \Delta\Psi_3(r,\theta) = \frac{B_0}{2}\frac{r^3}{R^3}\delta_0 \sin 3\theta \Rightarrow$

X-point remains at $r=0$
and $\Psi(0)=0 \Rightarrow$ compatible with
the initial field $\Rightarrow$
no current sheet $\Rightarrow$
<u>no forced reconnection</u>

The same is true for all perturbations
with $m \geq 2$, since $\Delta\Psi_m \propto r^m$

**) <u>a special role of the dipole perturbation</u>

$\Delta\Psi_1 = \frac{B_0}{2}\delta_0 \sin\theta \Rightarrow \Delta\Psi_1(r,\theta) = \underbrace{\frac{B_0\delta_0}{2}\frac{r}{R}\sin\theta}$



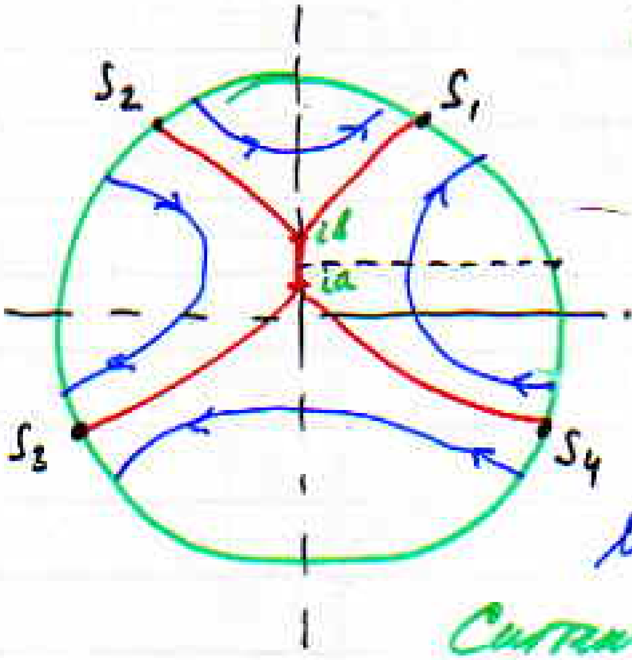
corresponds to a uniform
field $B_{1x} = -B_0\delta_0/2R \Rightarrow$

neutral X-point is
shifted to (x_1, y_1)
$x_1 = 0, \ y_1 = \delta_0/2$

$\Psi_S = \Psi(x_1, y_1) = \Psi_0(x_1, y_1) + \Psi_1(x_1, y_1) =$
$= \frac{B_0\delta_0^2}{8R} \neq 0 \Rightarrow$

not compatible with the
initial field $\Rightarrow$ flux
redistribution between the
lobes $\Rightarrow$ analogue of the
$\Psi_1^{(2)}$ solution $\Rightarrow$
reconnected state

what is the singular solution $\psi_1^{(c)}$ that preserves the initial field connectivity?



Current sheet in the place of a neutral X-point

Planar potential magnetic field

$B_x(x,y) - iB_y(x,y) = f(z)$; $z = x + iy$

$f(z)$ – analytic function

Neutral X-point: $f(z) = -\frac{iB_0}{R} z$

Current sheet extended from ia to ib

$$B_x - iB_y = -\frac{iB_0}{R}(z - ia)^{1/2}(z - ib)^{1/2}$$

What are a and b?

(2) ⇒ a, b ⇒ $B_y = \frac{B_0}{R}x$; $B_x = \frac{B_0}{R}y - \frac{B_0}{2R}(a+b)$ ⇒ $a + b = \delta_0$

C.S. is centred at $y_1 = \delta_0/2$.

$a = \frac{\delta_0}{2} - \varepsilon$; $b = \frac{\delta_0}{2} + \varepsilon$

C.S. corresponds to $\psi_S = 0$ ⇒

$$\Delta\psi = \int_{c.s.}^{R} B_y\,dx = \psi^{(c)}(0) \Rightarrow \varepsilon = \frac{\delta_0}{2} L^{-1/2};\ L = \ln\frac{R}{\delta_0} \gg 1.$$

Magnetic reconnection: transition from $\psi_1^{(c)}$ to $\psi_1^{(r)}$

How much energy is released?

$\Delta W_m = \frac{1}{2c} I \cdot \Delta\psi$

I – total current $= \frac{cB_0}{4R}\varepsilon^2 = \frac{cB_0}{4R}\frac{\delta_0^2}{4}L^{-1}$; $\Delta\psi = \psi_S^{(c)} = \frac{B_0\delta_0^2}{8R}$ ⇒

$$\boxed{\Delta W_m = \frac{B_0^2}{8\pi}\cdot\frac{\pi R^2}{32}\left(\frac{\delta_0}{R}\right)^4 L^{-1}}$$