Magnetic Islands in a Tokamak: Introduction and current status…

A. Smolyakov[*]

University of Saskatchewan, Saskatoon, Canada

***CEA Cadarache, France**

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Outline

- Basic island evolution -- extended Rutherford equation
- Finite pressure drive: Bootstrap current-NTM
- Stabilization mechanisms
- Critical plasma parameters for NTM onset and scalings, NTM control
- NTM theory issues: island rotation frequency
- Summary

Current status

- **Relatively large scale magnetic perturbations are often observed in tokamaks, m/n=2/1,3/,4/3,…; δr=3-10cm**
- **Critical for operation in advanced regimes**

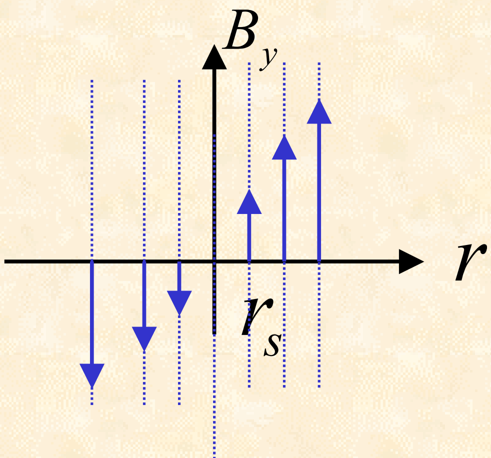
  deteriorate plasma confinement by 10-50 %

  lead to loss of catastrophic loss of discharge (disruptions)
- **Driven by pressure gradient (bootstrap) current and external perturbations (helical error field from coil imperfections)**

  - firmly established experimentally with a reasonable support from theory
- **"Theory based empirical scaling" are <u>absolutely</u> not reliable for future devices**
- **Number of "singular" effects have been identified theoretically (small but very important for e.g. the threshold of the excitation); practical importance is not clear**
- **Several critically important (both experimentally and in theory) effects remain poorly studied: finite banana width, rotation frequency, …**
  - insufficient data/hard to measure
  - analytical theory is diffucult/insufficient efforts (in modeling, in particular)

- toroidal particle code which resolves the structure of the magnetic island (3D), with ion-ion and ion-electron collisions, trapped and passing particles

Basics of Nonlinear Magnetic Islands

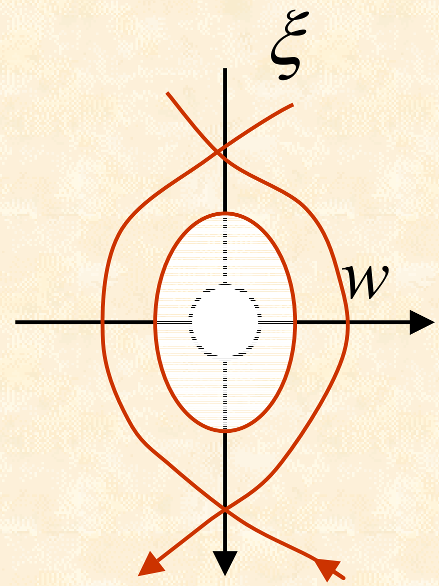


Unpeturbed magnetic shear layer

around the rational surface

$B \cdot \nabla = 0$

Perturbed (reconnected) magnetic surfaces



- Effective helical flux function for the rotating island

$$\psi(x,t) = -\frac{x^2}{2L_s}B_0 + \widetilde{\psi}(t)\cos\xi.$$

$x = r - r_s$ is the distance from the rational surface, $L_s = qR/S$ is the shear length, $S = q' r_s/q$, and the helical coordinate $\xi = m\widetilde{\theta} - \int^t \omega(t')dt'$, $\widetilde{\theta} = \theta - \zeta/q_s$. Magnetic island with half-width $w^2 = 4L_s\widetilde{\psi}/B_0$.

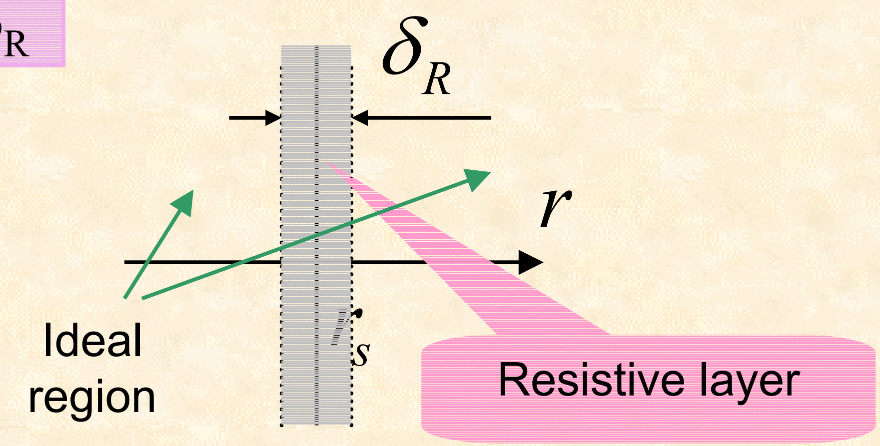
Nonlinear Magnetic Islands

- Helical flux function for the rotating island

$$\psi(x,t) = -\frac{x^2}{2L_s}B_0 + \tilde{\psi}(t)\cos\xi.$$

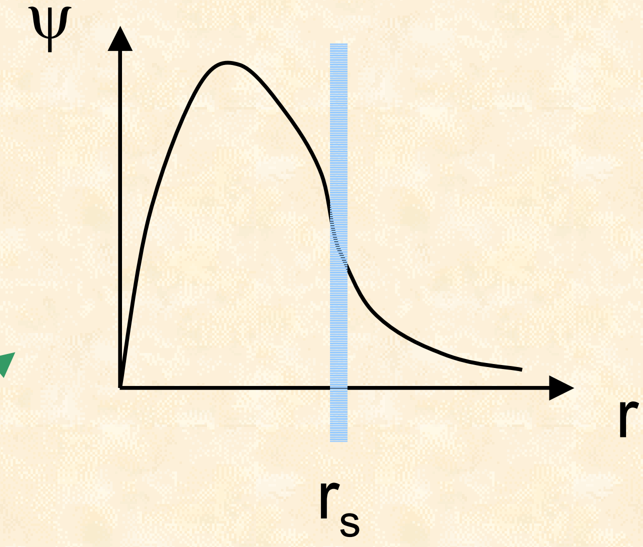
- Rutherford regime: $w > \delta_R$. Typical values: $\delta_R < 0.3$ cm for $S = 10^5 - 10^8$, $w \simeq 1$ cm
- Constant ψ approximation. Single helicity

Magnetic islands are nonlinear for $w > \delta_R$



Resistivity is important only in a narrow layer around the rational surface, δ_R

Current driven vs pressure gradient driven tearing modes



Ideal region: $B \bullet \nabla (J/B) = 0$

Solved with proper boundary conditions to determine

$$\Delta' \equiv \frac{1}{\psi}\frac{d\psi}{dx}\Big|_{-\varepsilon}^{+\varepsilon}$$

Current drive

Nonlinear/resistive layer:

Full MHD equations (including

neoclassical terms/bootstrap current)

are solved

$$\rho \frac{dV}{dt} = \frac{1}{c} J \times B - \nabla p - \nabla \cdot \Pi$$

$$E + \frac{1}{c} V \times B - \eta (J - J_b) = 0$$

Bootstrap current drive

Rutherford Equation–Basic Evolution Equation

- The nonlinear equations for the evolution of the magnetic island follow from the matching conditions obtained by integration of the Ampere's law, $4\pi J_{\|}/c = \nabla_{\perp}^2 \psi$, across the nonlinear region

$$\int_{-\pi}^{\pi} d(m\tilde{\theta}) \int_{-\infty}^{\infty} dx J_{\|} \cos\xi = \frac{c}{4} \Delta_c' \tilde{\psi}$$

$$\frac{\partial w}{\partial t} = D_R \Delta'$$

Rutherford equation

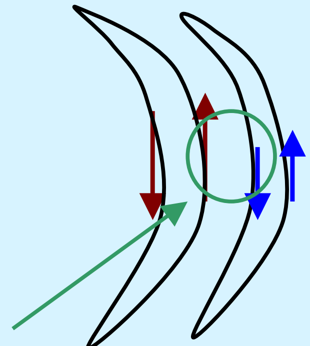
Negative energy mode driven by dissipation, unstable for $\Delta' > 0$

$$\int \delta B^2 dx d\xi = -\Delta' {\psi_s}^2$$

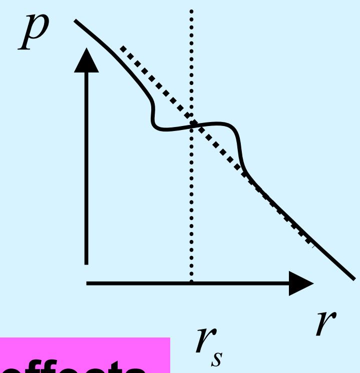
Bootstrap Current Drive

- Pressure driven current due to friction between trapped and untrapped particles

$$J_{\parallel} = \sqrt{\epsilon}\frac{c}{B_\theta}\frac{dp}{dr}$$



Diamagnetic banana current +friction effects



Loss of the bootstrap current around the island

Driving mechanism

- Generalized Ohm law

$$0 = -en\left(-\nabla\phi - \frac{1}{c}\frac{\partial\psi}{\partial t}\right) - \mathrm{b}\cdot\nabla p_e - \mathrm{b}\cdot\nabla\cdot\Pi_e + enJ_{\parallel}/\sigma$$

Bootstrap current

$$\mathrm{b}\cdot\nabla\cdot\Pi_e = n_e n\mu_e V_{\theta e}$$

Constant on magnetic surface

$$V_{\theta e} = -\frac{c}{enB_0}\frac{\partial}{\partial r}(p_e + p_i) + \frac{B_\theta}{B_0}(V_{ze} - V_{zi}]$$

$$J_b = \langle J_b \rangle$$

$$e_{i,e}nE_r - \nabla p_{i,e} + e_{i,e}n\left(V_\theta^{\ i,e}B_z - V_z^{\ i,e}B_\theta\right) = 0$$

Radial force balance, but $V_\theta^{\ i} = 0$

Extended Rutherford Equation–Basic Evolution Equation

- The nonlinear equations for the evolution of the magnetic island follow from the matching conditions obtained by integration of the Ampere's law, $4\pi J_{\|}/c = \nabla_{\perp}^2 \psi$, across the nonlinear region

$$\int_{-\pi}^{\pi} d(m\widehat{\theta}) \int_{-\infty}^{\infty} dx J_{\|} \cos\xi = \frac{c}{4} \triangle_c' \widetilde{\psi}$$

- Rutherford equation

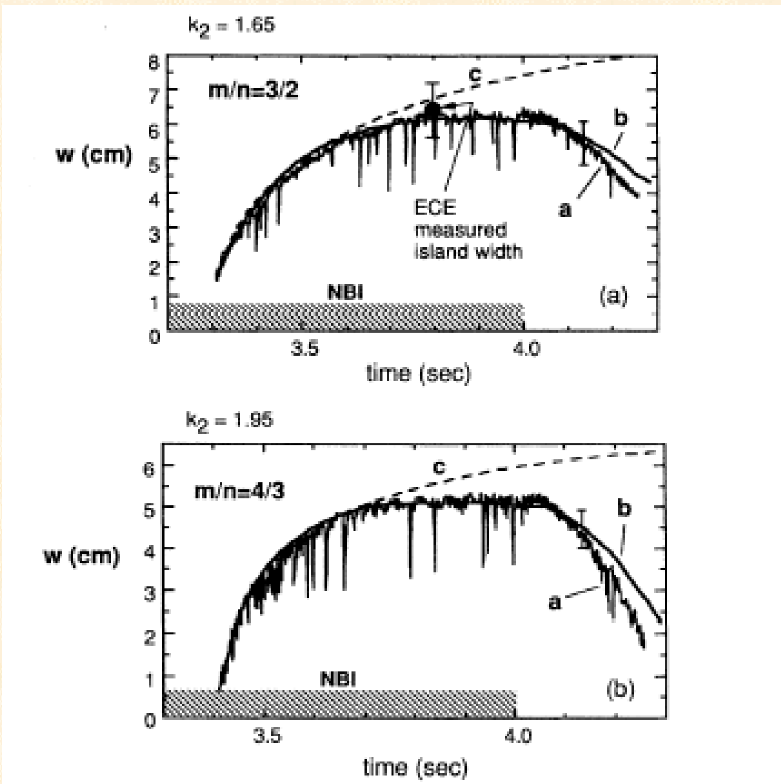
Qu, Callen 1985

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle_c'}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{S w} + \ldots$$

$$J_{\|} = \frac{\sigma}{c}\frac{\partial \psi}{\partial t} + \sqrt{\epsilon}\frac{c}{B_\theta}\frac{dp}{dr}$$

Neoclassical Magnetic Islands

- Neoclassical modes in TFTR, Z. Chang et al. PRL 74, 4663 (1995)



$$w_{sat} \sim \beta / \Delta'$$

$$w \sim (\beta t / \tau_R)^{1/2}$$

Saturation for $\Delta < 0$

$$\tau_R \frac{\partial w}{\partial t} = \Delta' + \frac{\beta}{w}$$

Rutherford growth

$$w \sim \Delta' t / \tau_R$$

Bootstrap growth

Beta dependence signatures are critical for NTM identification

Some problems in a simplest version of the extended Rutherford equation:

- Rutherford equation

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle'_c}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{Sw} + ...$$

- Theoretical problems: Transition to the linear limit $w \longrightarrow 0$? All m mode numbers are unstable? Does not happens in the experiments: most often m/n=3/2, 2/1, 4/3, 5/4, ...

  Experimentally: NTM do not appear in all discharges; must be triggered by an external perturbation. Hysteresis: critical β for mode excitation and mode suppression are different. The threshold mechanism?

Neoclassical Tearing Modes are metastable – Thresholds

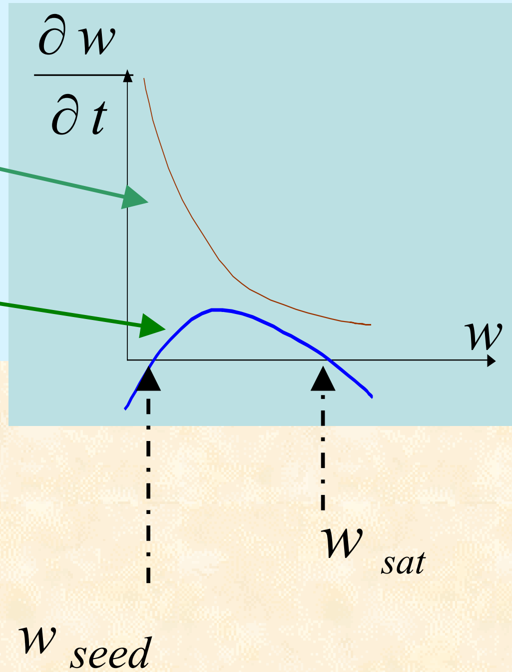
- Modification of the bootstrap current for small island width (finite parallel heat conductivity)

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle_c'}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{S w} \qquad \textit{No threshold}$$

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle_c'}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{S}\frac{w}{w_c^2 + w^2} \qquad \textit{Threshold}$$

Fitzpatrick, 1995

Gorelenkov, Zakharov, 1996



Threshold mechanisms

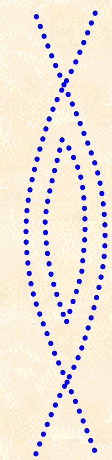
I. Finite $\chi_{II} / \chi_{\perp}$ threshold- transport threshold

$$\chi_{//}\nabla_{//}T >> \chi_{\perp}\nabla_{\perp}T$$

Temperature is constant along the field lines -> flat

Inside the closed surfaces

However for narrow island $\chi_{//} / L_{//}^2 \approx \chi_{\perp} / L_{\perp}^2$ $L_{\perp} \approx w$



Competition between the parallel (pressure flattening) and transverse (restoring the gradient) heat conductivity ->

restores finite pressure gradient

II. Polarization current threshold

III. Neoclassical: enhanced polarization current and other effects (e.g. ion sound)

Other stabilizing mechanisms? Polarization threshold!

Drift/Inertial, Neoclassical, Curvature, etc Effects

- Quasineutrality equation

$$\nabla_{\|} J_{\|} + \nabla_{\perp} \cdot \mathrm{J}_{\perp} = 0$$

$$J_{\|} = \nabla^{-1} \nabla_{\perp} \cdot \mathrm{J}_{\perp}$$

Bootstrap current is divergent free:

$$J_b = \langle J_b \rangle \qquad \nabla_{//} J_b = 0$$

- Perpendicular current

$$\mathrm{J}_{\perp} = \frac{c}{B} \mathrm{b} \times \nabla p + \frac{c m_i n_0}{B} \mathrm{b} \times \frac{d_0}{dt} \mathrm{V} + \frac{c}{B} \mathrm{b} \times \nabla \cdot \Pi$$

Diamagnetic current

Glasser-Green Johson

Inertia, polarization current

Neoclassical viscosity, enhanced polarization

Polarization Current Effects

- Polarization current

$$J_{\|} = \nabla^{-1}\nabla_{\perp} \cdot \mathrm{J}_{\perp} \qquad \mathrm{J}_{\perp} = \frac{c m_i n_0}{B} \mathrm{b} \times \frac{d_0}{dt} \mathrm{V}$$

- Rutherford equation

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle_c'}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{S w} + g\frac{\beta_\theta}{w}\left(\frac{\rho_s}{w}\right)^2 \frac{\omega(\omega - \omega_{*i})}{\omega_{*e}^2}$$

Bootstrap current drive

Slab polarization current, Smolyakov 1989

In toroidal geometry: Smolyakov, Lazzaro, Callen, PoP 1995

Note the dependence on the frequency of island rotation!

Mechanisms for NTM Thresholds:

- Modification of the bootstrap current for small island width due to finite parallel heat conductivity, part of the lost bootstrap current is restored → less drive

- Polarization current threshold

Smolyakov, 1989; Zabiego, Callen 1995; Wilson et al, 1996

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle_c'}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{S}\frac{w}{w_c^2 + w^2} - \alpha\beta_\theta\frac{1}{w^3}$$

Fitzpatrick, 1995; Gorelenkov, Zakharov, 1996

- Magnetic field curvature (Glasser-Green-Johnson) effect is also stabilizing. Especially important for small aspect ratio (MAST, *R.J. Buttery et al., PRL 88, 125005-1 (2202), H.Lutjens, J-F Luciani, and X. Garbet, POP 8, 4267 (2002)*).

$$\triangle_{GGJ}' = g_{GGJ}\frac{D_R}{\sqrt{w_c^2 + w^2}}$$

Also finite banana width, *Poli et al., 2002*

- Neoclassical effects of the poloidal flow damping; enhanced polarization: $v_A^2 \to v_A^2 B_0^2/B_\theta^2$?

Neoclassically Enhanced Polarization Current

Coupling of the transverse and longitudinal flows/Neoclassical flow damping

- Current closure equation

$$\nabla_{\parallel} J_{\parallel} + \nabla_{\perp} \cdot \left(\frac{c m_i n_0}{B} \mathrm{b} \times \frac{d_0}{dt} \mathrm{V} \right) + \nabla \cdot \left(\frac{c}{B} \mathrm{b} \times \nabla \cdot \Pi \right) = 0$$

Neoclassical viscous current

$$\Pi_{\parallel} = \frac{3}{2} \pi_{\parallel} \left(\mathrm{bb} - \frac{1}{3} \mathrm{I} \right) \qquad \frac{3}{2} \pi_{\parallel} = p_{\perp} - p_{\parallel}$$

- Neoclassical current

$$\nabla \cdot \mathrm{J}_{nc} \equiv \nabla \cdot \left(\frac{c}{B^2} \mathrm{B} \times \nabla \cdot \Pi \right) = \frac{c}{B_\theta} \frac{\partial}{\partial x} \langle \mathrm{b} \cdot \nabla \cdot \Pi \rangle$$

- Divergence of the transverse current is related to the component of the parallel force

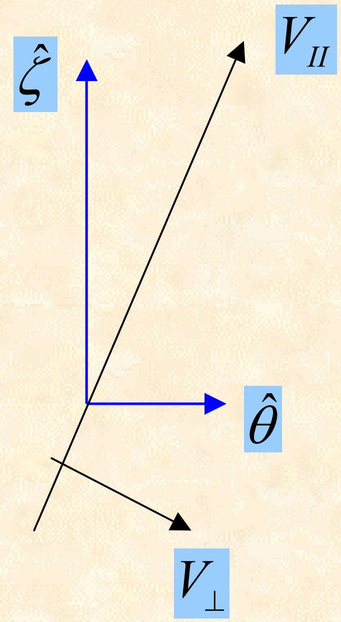
$$m_i n_0 \frac{d}{dt} V_{\parallel} = -\nabla_{\parallel} p - \langle \mathrm{b} \cdot \nabla \cdot \Pi \rangle_\theta$$

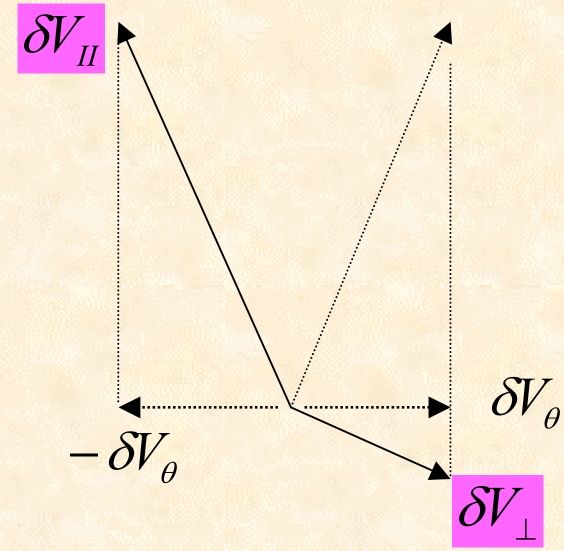
$$\nabla \cdot \mathrm{J}_{nc} = \frac{c}{B_\theta} m_i n_0 \frac{\partial}{\partial x} \frac{d_0}{dt} V_{\parallel} + \frac{c}{B_\theta} \frac{\partial}{\partial x} \nabla_{\parallel} p$$

Parallel ion dynamics effects

Enhanced inertia, replaces the standard polarization current

Neoclassical inertia enhancement





Transverse inertia was replaced with parallel. How to determine $V_{\|}$?

Neoclassical Flow Damping

- Neoclassical force

$$\langle \mathrm{b}\cdot\nabla\cdot\Pi\rangle = -\frac{3\,\varepsilon}{2\,q}\left\langle \pi_{\|}\frac{1}{r_s}\frac{\partial}{\partial\theta}\nabla_{\perp}\ln B\right\rangle = m_i n_0 \chi_\theta V_\theta$$

$$\chi_\theta = \frac{q^2}{\varepsilon^{1/2}}\left(\frac{d_0}{dt}+\frac{\nu_i}{\varepsilon}\right) \qquad V_\theta = V_y + \frac{\varepsilon}{q}V_{\|} \qquad V_y = \frac{c}{B_0}\frac{\partial\phi}{\partial x}$$

Resulting equation for the parallel flow velocity is

$$\frac{d_0}{dt}V_{\|} = -q\varepsilon^{1/2}\left(\frac{d_0}{dt}+\frac{\nu_i}{\varepsilon}\right)\left(V_y+\frac{\varepsilon}{q}V_{\|}\right) - \frac{1}{m_i n_0}\nabla_{\|}p$$

Neoclassically Enhanced Polarization Current II

- Neoclassical current

$$\nabla \cdot \mathrm{J}_{nc} = \frac{c}{B_\theta} m_i n_0 \frac{\partial}{\partial x}\frac{d_0}{dt} V_\parallel + \frac{c}{B_\theta}\frac{\partial}{\partial x}\nabla_\parallel p$$

- From the radial momentum balance

$$V_\parallel \simeq V_\zeta = V_\theta \frac{B_\zeta}{B_\theta} + \frac{c}{e n_0 B_\theta} E_r - \frac{c}{B\theta}\frac{\partial p}{\partial r} \qquad V_\theta = k\frac{cT'}{eB}$$

- Extended Rutherford equation

$$\tau_R \frac{\partial w}{\partial t} = \frac{\triangle'_c}{4} + \sqrt{\epsilon}\frac{\beta_\theta}{Sw} + g\frac{\beta_\theta}{w}\left(\frac{\rho_s}{w}\right)^2 \frac{\omega(\omega - \omega_{*i})}{\omega_{*e}^2} + g_{neo}\frac{\beta_\theta}{w}\left(\frac{\rho_s}{w}\right)^2 \frac{\omega(\omega - k\omega_{*i})}{\omega_{*e}^2} + ?$$

standard inertia

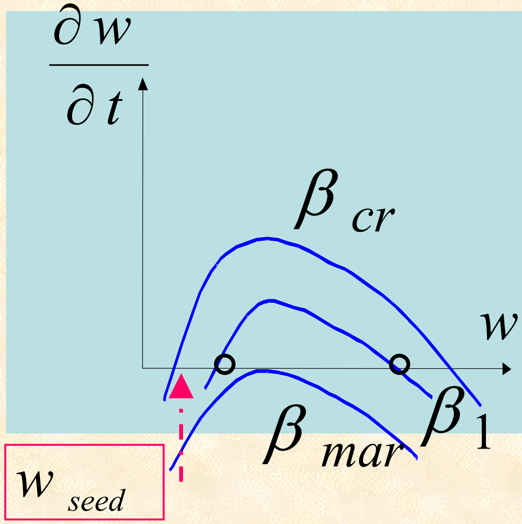
Neoclassically enhanced inertia

$$g_{neo} = \left\{ \begin{array}{lll} q^2/\epsilon^2 & \nu_i \gg \epsilon\omega & Smolyakov\ \ et\ \ al., PoP\ 2,\ 1581\ \ (1995) \\ q^2/\sqrt{\epsilon} & \nu_i \ll \epsilon\omega & Wilson\ \ et\ \ al.\ PoP\ 3,\ \ 248\ (1996) \end{array} \right\}$$

g_{neo} depends on collisionality regime and may have further dependence on frequency, Mikhailovskii et al PPCF 2001

Uniformly valid fluid theory,
Smolyakov, Lazzaro, PoP, 2004

Metastable modes: threshold and marginal beta



β_{cr} – NTM excitation

β_{mar} – suppression

No mode at β_1

$$\beta_{cr} > \beta_1 > \beta_{mar}$$

MHD activity, sawteeth, ELM, … Hysteresis → $\beta_{cr} > \beta_{mar}$

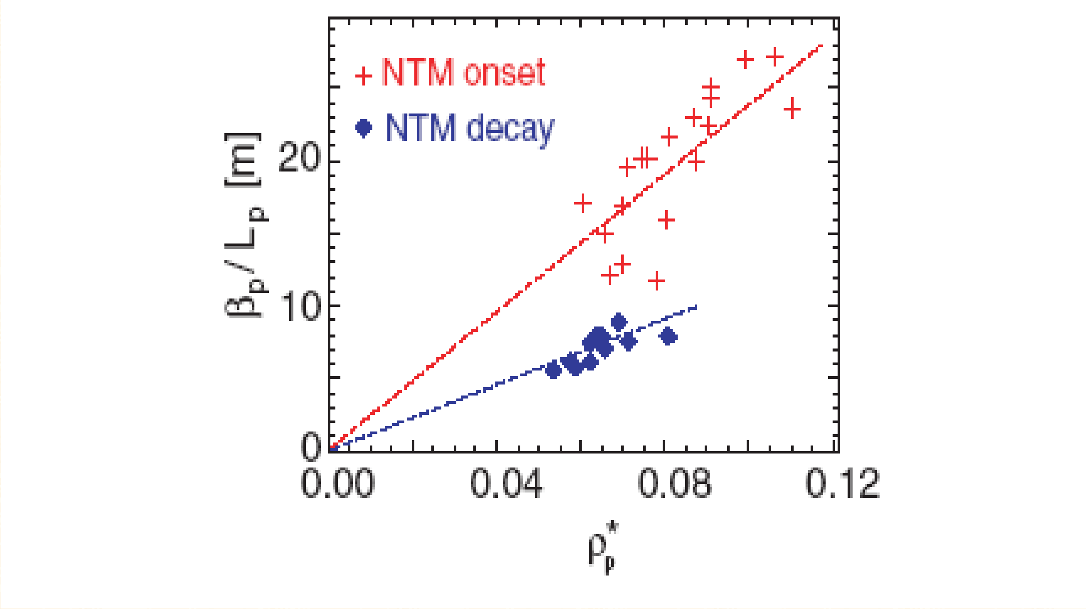
$$\frac{\tau_R}{r^2}\frac{\partial w}{\partial t} = \Delta' + a_{bs}\varepsilon^{1/2}\frac{L_q}{L_p}\frac{\beta_p}{w}\left(\frac{1}{1 + w_d^{\,2}/w^2} - \frac{w_{pol}^{\,2}}{w^2}\right)$$

$$w_{pol}^{\,2} = \varepsilon\frac{L_q}{L_p}g(\nu_{ii},\varepsilon)\rho_{\theta i}^{\,2}\frac{T_e}{T_i}\frac{\omega(\omega - \omega_{*pi} - k\omega_{*Ti})}{\omega_{*e}^{\,2}}$$

Collisionality

NTM critical parameters?

- Critical beta for NTM onset β_{cr} ; determined by the size of a seed island, w_d and w_{pol}
- Marginal beta for complete NTM stabilization (NTM are unconditionally stable); β_{mar} depends on w_d and w_{pol}, no dependence on the seed island size
- Linear scalings with $\rho_{*\theta}$, weak dependence on ν_{*ii}, $\sim \nu_{*ii}^{-(0.1 \div 0.2)}$



Asdex U, S. Gunter et al., PPCF 43 (2003) 161

Seed MHD activity is crucial for NTM onset!

NTM seeding by ELM

NTM destabilization by ELMs, DIII-D, R J La Haye et al, Nucl Fus, v 40, (53) 2000

$q(0) \geq 1$ removes sawteeth, fishbones remain– modest increase in the critical β

$q(0) > 1$ sawteeth and fishbones are removed -> β increase almost to the ideal limit

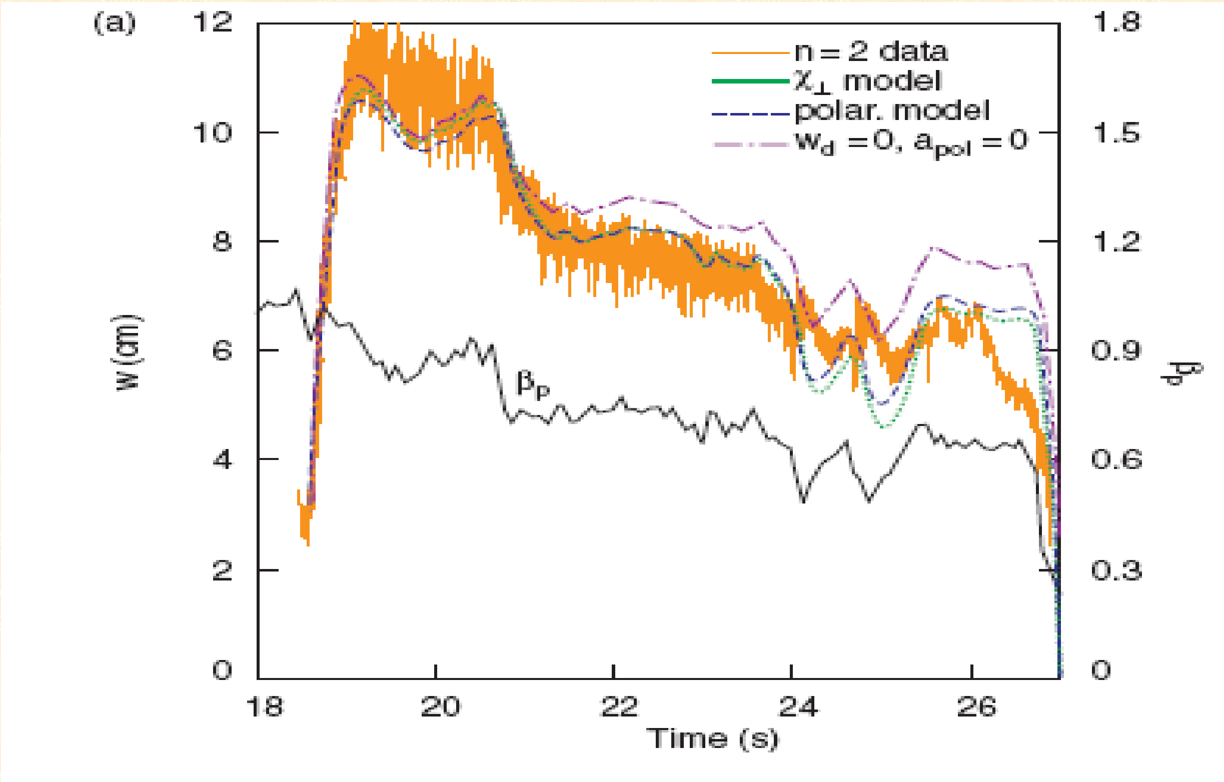
Seed islands are small (due to ELM). Gentler frequent ELM would help,

q(0)>1 not very well reproducible

Transport $(\chi_{II} / \chi_{\perp})$ vs polarization threshold models?

- No definite conclusions: smaller tokamaks data seem to suggest polarization mechanism
- JET data – transport mechanism or both (not conclusive)

R J Buttery, et al, JET
Nucl Fusion 43 (2003), 69



Prognosis to future devices

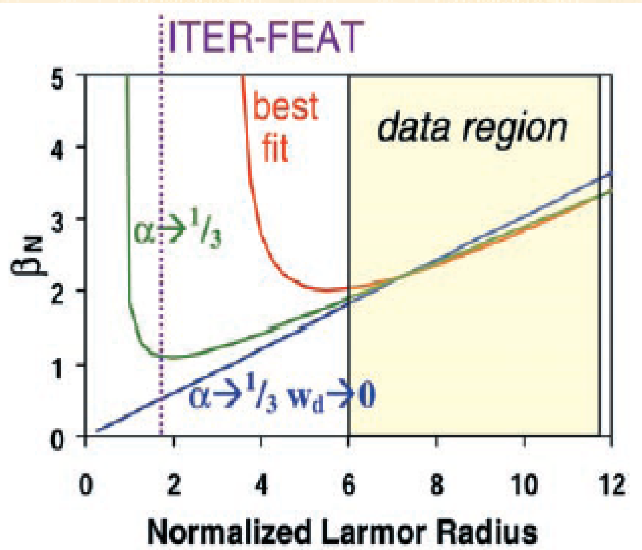
Include extrapolation over several different directions:

extrapolation of the critical and marginal plasma pressure in the NTM model (s)

extrapolation of the size of a seed island and screening/shielding factors

profiles effects, local gradients, etc are important

Small variations in fit parameters weakly affect the data region with huge differences for extrapolated values



ρ_{*i} scaling predicts lower values of β_N for ITER

However: $\rho_{*i} - \nu$ scalings may not be predictive,

R.J. Buttery, Nucl Fusion 44 (2004), p 678:

Different devices show similar β_N. Neural network analysis shows the sawtooth period as a key parameter. Correlation with seed amplitude?

R.J. Buttery et al., PPCF 42 (2000), B61

α stabilization of sawteeth in ITER?

NTM control

-Replace the missing bootstrap current with external CD;
ECCD applied to O-point: Asdex-U, JT-60U, DIII-D, FTU
NTM is suppressed, plasma beta is raised again with further heating
~10 % of the total heating power is required into ECCD; ~25 MW in ITER

FTU, Berrini et al, IEEE NPSS, 2005

-NTM mode stabilization via magnetic coupling, *Yu et al, PRL 2000*,
separatrix stochastization -> enhanced radial transport ->
radial plasma pressure gradient is restored -> bootstrap current is restored -> island destabilization is reduced

DIII-D, La Haye et al, PoP 9, 2002. m=1,n=3 $B_r/B_t=1.6\times10^{-3}$ field is applied before 3/2 NTM onset: 3/2 NTM is suppressed.
However, no confinement improvement! Reduced rotation due to n=3 ripple?

Magnetic islands theory issues:

Finite banana width effects?

Provides the threshold, depends on rotation (Poli, 2003,2005)

Island rotation frequency? Sign of the polarization term depends on the rotation frequency

Nonlinear trigger/excitation mechanism? Magnetic coupling: not every sawtooth crash results in the NTM, resonant conditions for m/n=1/1 and m/n=3/2?

"Cooperative effects" of the error field and neoclassical/bootstrap drive in a finite pressure toroidal plasma? NTM and resistive wall modes?

Rotation of magnetic islands

in collaboration with X. Garbet, M. Ottaviani, E Lazzaro

What defines the rotation frequency? – **Dissipation!**

We consider stationary states: $w = const$, $\omega = const$

Two components of the Ampere law

$$\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} d\xi J_{\|}(x,\xi) \cos\xi = \frac{c}{4}\Delta'_c \widetilde{\psi}.$$

$$\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} d\xi J_{\|}(x,\xi) \cos\xi = \frac{c}{4}\Delta'_s \widetilde{\psi}.$$

$\Delta'_s \neq 0$ due to the interaction with the wall/error field/shear external flow. Consider $\Delta'_s = 0$ for simplicity (localized island)

The rotation frequency is determined by the $\sin\xi$ part of the non-ambipolar current which can be written as

$$\int\limits_{-\infty}^{\infty} d\psi \int\limits_{-\pi}^{\pi} d\xi \nabla_{\|} J_{\|}(x,\xi) = 0$$

The longitudinal current is driven by the non-ambipolar current

$$\frac{1}{e}\nabla_{\|} J_{\|} = \frac{\partial}{\partial x}\left(\Gamma_e - \Gamma_i\right).$$

- $\sin\xi$ component defines the rotation frequency

- $\cos\xi$ component enters the island evolution equation (e.g., polarization current)

Sources of the non-ambipolar fluxes:

- "coherent"– single helicity case (polarization current)
- "incoherent" –small scale perturbations, $l << w$

- – small scale electrostatic fluctuations
- – small scale magnetic fluctuations (drift waves + symmetry breaking/stochastization)
- – neoclassical (toroidicity + trapped paricles)

Flux-forces relationships:

$$mn\frac{d\mathbf{V}}{dt} = en\left(\mathbf{E}+\frac{1}{c}\mathbf{V}\times\mathbf{B}\right) - \nabla p - \nabla\cdot\Pi_{gv} - \nabla\cdot\Pi_{\parallel} - \mathbf{R}+\mu nm\nabla^2 V$$

- Neoclassical viscosity

$$\Pi_{\parallel} = \frac{3}{2}\pi_{\parallel}\left(\mathbf{bb}-\frac{1}{3}\mathbf{I}\right)$$

- Π_{gv} -gyroviscosity, contributes to the $\cos\xi$ part, island evolution equation

- $\mathbf{R}$ - friction force

- $\nabla\cdot\Pi_{\parallel}$ - neoclassical viscous force

- $\mu nm\nabla^2 V$ - viscosity force (anomalous?)

Fluxes: $\Gamma = \Gamma_R + \Gamma_I + \Gamma_t + \Gamma_{neo} + \Gamma_\mu$

- Γ_R–friction force flux, ambipolar (classical)

- Γ_I–inertial (polarizaton) flux, affects the island evolution equation $\Gamma_I = n\frac{1}{\omega_{ci}}\mathbf{b}\times\frac{d}{dt}(\mathbf{V}_E + \mathbf{V}_p) + \frac{c}{eB}\mathbf{b}\times\nabla\cdot\Pi_{gv}$

- Γ_t - turbulent flux

$$\Gamma_t = \overline{\tilde{n}\tilde{V}_{Er}} + \overline{nV_\parallel \frac{B_r}{B_0}}$$

  – We assume that there is a sufficient scale separation between the characteristic size of the magnetic island and the scale of microscopic fluctuations that define the anomalous transport across the magnetic surfaces in the island

- $\overline{\tilde{n}\widetilde{V}_{Er}}$ - the electrostatic component, locally ambipolar due to $n_e = n_i$; but could be non-ambipolar for for sub-Larmor size fluctuations

- $\overline{nV_{\parallel}\frac{B_r}{B_0}}$ - magnetic flatter, also stochastisation near the separatrix, mainly in the electron component- locally Non ambipolar. Ambipolar on average over the magnetic surfaces (globally)

- Neoclassical flux (toroidicity is important) $\Gamma_{neo} = \overline{nV_{pr} + nV_{\pi r}}$ where $V_p = \frac{c}{eB}\mathbf{b} \times \nabla p, \quad V_\pi = \frac{c}{eB}\mathbf{b} \times \nabla \cdot \pi$

- Γ_μ - transverse viscosity flux $\Gamma_\mu = \frac{n}{\omega_{ci}}\mu\mathbf{b}\times\nabla^2\mathbf{V}$

Non-ambipolar turbulent/stochastic flux

Assume non-ambipolar electron and ion fluxes in the form

$$\Gamma_e = -nD_e\left(\frac{\partial n}{n\partial r} + \alpha\frac{\partial T}{T\partial r} - e\frac{\partial\phi}{T\partial r}\right)$$

$$\Gamma_i = -nD_i\left(\frac{\partial n}{n\partial r} + e\frac{\partial\phi}{T\partial r}\right)$$

Plasma profiles around the magnetic island

$$\phi = \frac{\omega B_0}{k_\theta c}\left[x - \lambda(\psi)\right]$$

$$n = -\frac{en_0}{T_e}\frac{B_0\omega_*}{k_\theta c}\lambda(\psi)$$

$$T = -\frac{eB_0\omega_*\eta_e}{k_\theta c}\lambda(\psi)$$

$$\int_{-\infty}^{\infty} d\psi \int_{-\pi}^{\pi} d\xi \; \frac{\partial}{\partial x} (\Gamma_e - \Gamma_i) = 0$$

$$\omega = \frac{D_e (\omega_* + \alpha \omega_* \eta_e) - D_i \omega_*}{D_e + D_i}$$

Samain, PPCF, 1988; (also Fitzpatrick, Waelbroeck, 2005)

Requires non-ambipolar flux due to small scale fluctuations, $k_\perp^{-1} \ll w$;

Rotation is in the electron direction if $D_e \gg D_i$ (non-ambipolar flux is mainly in electron component, due to the magnetic fluctuations), but for fluctuations with $k_\perp \rho_i \gg 1$, the electrostatic transport is not ambipolar, $D_e/D_i =$?

Trapped particles contribution?

Non-ambipolar flux due to the viscosity (Fitzpatrick, Waelbroeck, PoP, 2005)

$$\Gamma_i = \frac{n}{\omega_{ci}}\mu_i \mathbf{b}\times\nabla^2\mathbf{V}_i \quad \Gamma_e = \frac{n}{\omega_{ce}}\mu_e \mathbf{b}\times\nabla^2\mathbf{V}_i$$

$$V_i = \frac{c}{B}\mathbf{b}\times\nabla\phi + \frac{c}{enB}\mathbf{b}\times\nabla p_i \quad V_e = \frac{c}{B}\mathbf{b}\times\nabla\phi - \frac{c}{enB}\mathbf{b}\times\nabla p_e$$

$$\omega = \frac{m_e\mu_e\omega_* - m_i\mu_i\omega_*}{m_e\mu_e + m_i\mu_i}$$

Fitzpatrick, Waelbroeck, 2005, ($T_i = T_e = const$)
Rotation is mainly in the ion direction if $\mu_i \simeq \mu_e$
Assuming $\mu \simeq D$

$$\Gamma_\mu = \frac{n}{\omega_{ci}}\mu_i \mathbf{b}\times\nabla^2\mathbf{V}_i \propto \frac{n}{\omega_{ci}}\mu_i \frac{cT}{enB_0}\frac{1}{w^2}\frac{\partial n}{\partial r} \propto \frac{\rho^2}{w^2}\Gamma_t$$

anomalous viscosity driven flux is small?

Non-ambipolar neoclassical flux (due to poloidal flow damping)

Neoclassical transport is not automatically ambipolar!

$$\Gamma^i_{neo} = D^i_{neo}\ \left(\frac{p_i^{'}}{p_0} - \frac{e}{T_i}\left(E_r - B_\theta U^i_{\|}\right)\right) \sim D^i_{neo}\ (\ E_r - E_r^{neo})$$

$$\Gamma^e_{neo} = D^e_{neo}\ \left(\frac{p_e^{'}}{p_0} + \frac{e}{T_i}\left(E_r - B_\theta U^e_{\|}\right)\right) \sim D^i_{neo}\ (\ E_r - E_r^{e})$$

Ion flux is dominant : $D^i_{neo} = \mu_i \rho^2_{\theta i} \gg D^e_{neo} = \mu_e \rho^2_{\theta e}$

As a result of the quasineutrality constrain the ambipolar neoclassical flux becomes

$$\Gamma_{neo} = D^e_{neo}\ (\ E_r^{neo} - E_r^e) \sim D^e_{neo}\frac{n^{'}}{n},$$

and independent of the electric field

With magnetic island plasma profiles are modified

$$\phi = \frac{\omega B_0}{k_\theta c}\left[x - \lambda(\psi)\right]$$

$$n = -\frac{en_0}{T_e}\frac{B_0\omega_*}{k_\theta c}\lambda(\psi)$$

$$T = \frac{eB_0\omega_{*i}\eta_i}{k_\theta c}\lambda(\psi)$$

$$\nabla_\parallel J_\parallel = \left(\frac{c}{B_0}\right)^2\left(\frac{q}{\varepsilon}\right)^2 m_i n\left(D^i_{neo}\frac{\partial^2}{\partial r^2}\left(\phi - \frac{p_i^{'}}{en_0}\right) + D^e_{neo}\frac{\partial^2}{\partial r^2}\left(\phi + \frac{p_e^{'}}{en_0}\right)\right)$$

The D^i_{neo} transport is responsible for the fast poloidal momentum damping (non-ambipolar process). As a result of strong non-ambipolar flux, the electric field induced around the island rotation changes in a such way to annihilate the non ambipolar flux

$$\omega = \omega{*}i(1 + \eta_i(1 + k))$$

Conclusions on the island rotation

- The island rotation in a tokamak is determined by the dominant dissipative process

- The non-ambipolar neoclassical current/poloidal flow damping is dominant

- The island rotation is in the ion direction (lock into the ions poloidally, no toroidal rotation is assumed)