Effect of sheared flows on neoclassical tearing modes

Abhijit Sen

Institute for Plasma Research,
Bhat, Gandhinagar 382428, India

Collaboration: D. Chandra, P.K. Kaw (IPR)
M.P. Bora (Guwahati)
S. Kruger (Tech-X)

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Outline

- Introduction and mini-overview of NTM s
- Effect of sheared flows
    - Numerical simulation
    - Model calculation
- Open issues and future work

What are NTMs?

- NTMs are relatively large size **magnetic islands** that develop slowly at mode rational surfaces with low (m,n) mode numbers in **high temperature tokamak** plasmas.

- Like the classical TMs they are current driven but the current source is the **bootstrap current** - a neoclassical (toroidal geometry driven) source of free energy.

- They limit the attainable β in a tokamak to values well below the ideal MHD limit - hence they are a **major concern** for all reactor grade machines i.e. long pulse (steady state) devices.

Classical Tearing Modes

- Asymptotic theory- uses two regions of the plasma
  - Outer region - marginal ideal MHD - kink mode
  - Inner region - include effects of inertia, resistivity, nonlinearity, viscosity etc.
- Matching between inner and outer region

$$\frac{1}{2}\Delta'\psi_1 = \mu_0 R \int_{-\infty}^{\infty} d\rho \oint \frac{d\alpha}{2\pi}\cos(m\alpha) J_{\parallel},$$

- Linear theory : $\gamma \sim (\Delta')^{4/5} S^{-3/5}$

Magnetic island evolution in classical tearing modes

- Near mode rational surface **k·B = 0** ,

  $B_0 = B(r{=}r_s) - B_\theta(nq'/m)(r-r_s)\alpha$, $\alpha = \theta - (n/m)\varsigma$

  $\delta B = \delta B_r \sin(m\alpha)\,\mathbf{r}$

- Leads to the formation of a **magnetic island**

- Island width $w = 4(\delta B_r\, r_s / B_\theta\, nq')^{1/2}$

- when w > resonant layer thickness - nonlinear effects important

- Nonlinear evolution – Rutherford regime

$$\frac{dw}{dt} \approx \eta \Delta'$$

$$\Rightarrow \; w \propto t$$

- The form of the Rutherford equation can be traced to the form of Ohm's Law which governs the inner region solution, e.g.

$$E_{\|} = \eta J_{\|}$$

$$E_{\|} \sim -\frac{\partial A_{\|}}{\partial t}$$

$$J_{\|} \sim -\nabla^2 A_{\|}$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{d\delta B}{dt} = \eta \frac{\Delta'}{w} \delta B$$

$$\Rightarrow$$

$$\frac{dw}{dt} \approx \eta \Delta'$$

- In high temperature tokamaks neoclassical effects need to be retained

Modified Ohm's Law

$$< E_{\|} > \;=\; \eta J_{\|} + \frac{1}{neB} < B \cdot \nabla \cdot \pi_{\|e} >$$

$\Downarrow$

Bootstrap current

$\Uparrow$

$$\frac{1}{neB} < B \cdot \nabla \cdot \pi_{\|e} > \;\approx\; \frac{\mu_e}{\nu_e} \frac{1}{B_\theta} \frac{dp}{dr} + \eta \frac{\mu_e}{\nu_e} J_{\|}$$

Electron viscous stress which describes damping of poloidal electron flows - new free energy source. Has dependence on pressure gradient

Modified Rutherford Equation

$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + \frac{D_{nc}}{w})$$

where

$$D_{nc} = -\sqrt{\epsilon}\,\frac{2\mu_0}{B_\theta^2}p'\frac{q}{q'}k_0$$

$$p'q' < 0, \quad D_{nc} > 0$$

Unstable for normal tokamak operation

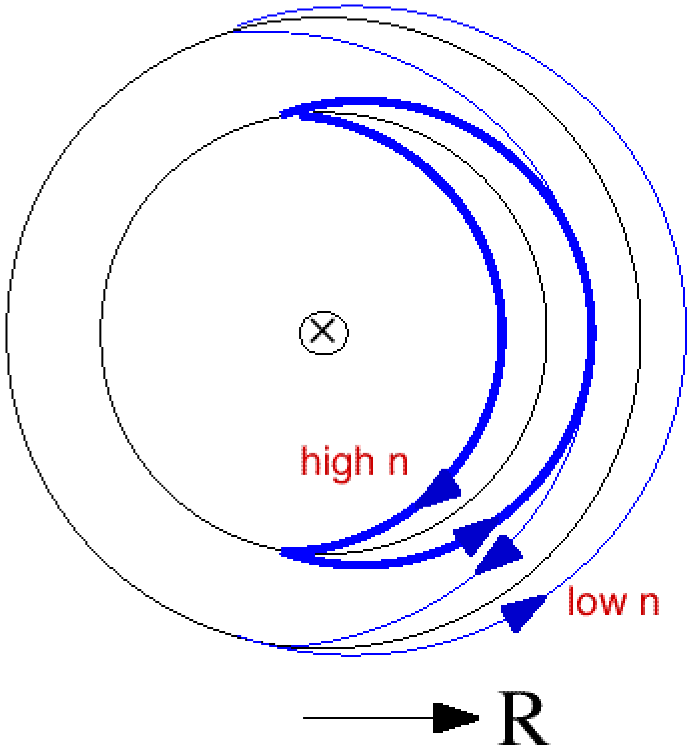
$$p'q' > 0, \quad D_{nc} < 0$$

Stable in reversed shear regions

- Can be unstable for $\Delta' < 0$ $\Rightarrow$ $$w_{sat} = \frac{D_{nc}}{-\Delta'} \approx \frac{r_s \beta_\theta}{m}$$
- for small islands $w \sim \sqrt{\eta t}$

Bootstrap Current

Projection into a poloidal plane



generated by trapped particles:

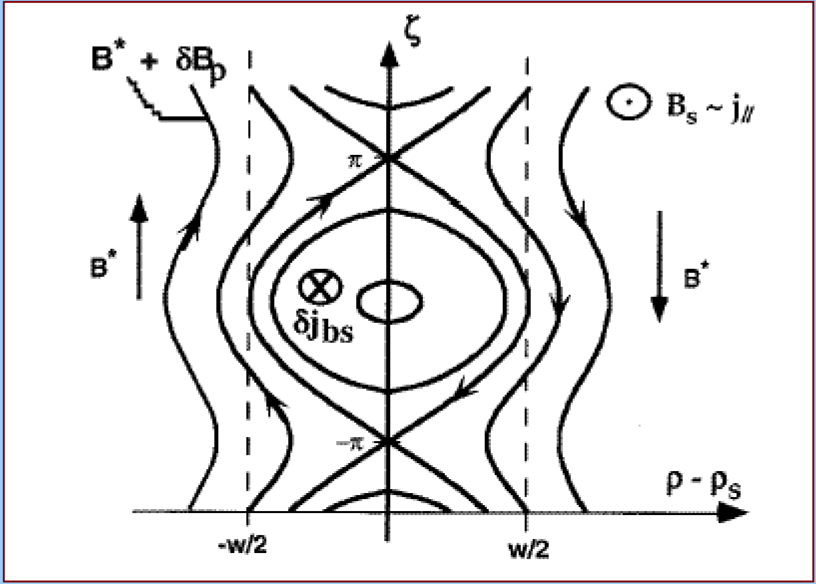
example: banana particles

- electrons drift from flux surfaces due to the ∇B-drift
- electrons with low parallel velocity are trapped in the toroidal mirror
  $\Rightarrow$ **banana orbits**
- at the intersection of 2 banana orbits a net current results due to the density gradient
- passing particles exchange momentum with trapped particles
  $\Rightarrow$ **bootstrap current**

similar: helically trapped particles

Physics of NTM

- Plasma pressure profile is flattened within the island - J_{bs} **is turned off**
- **This triggers a δJ_{bs} with the same helical pitch as the island**
- the corresponding induced δB has the same direction as the initial perturbation and **enhances it**



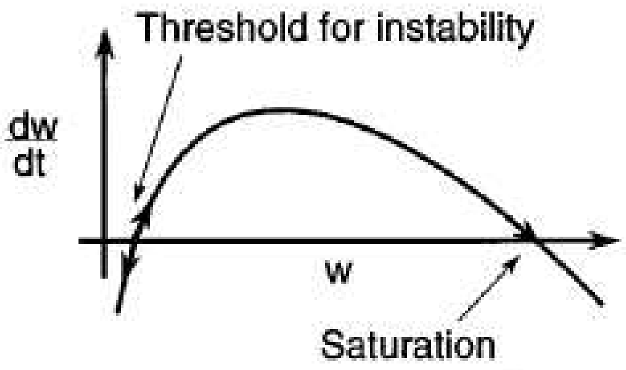
This picture neglects finite perpendicular thermal conductivity within the island - important for small island widths - leads to **threshold size**.

Generalized Rutherford Equation

$$I_1 \frac{dW}{dt} = \Delta' + 9.26\epsilon_s^{0.5} \frac{\beta_s'}{s_s} \frac{W}{W^2 + W_d^2}$$

$$W_d \simeq 1.8 W_c$$

$$W_c = 2.83 \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{0.25} \left(\frac{1}{\epsilon_s s_s n}\right)^{0.5},$$



"Phase diagram"

$$W_{threshold} \simeq \frac{2.80}{n} \frac{|\Delta'|}{\beta_s' \epsilon_s^{1.5}} \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{0.5}$$

$$W_{sat} \simeq 9.26\epsilon_s^{0.5} \frac{\beta_s'}{|\Delta'| s_s}.$$

NTM characteristics

- **low (m,n) islands that are driven by perturbations of the bootstrap current.**

- Can grow even if $\Delta^{/} < 0$ - distinct from classical TMs

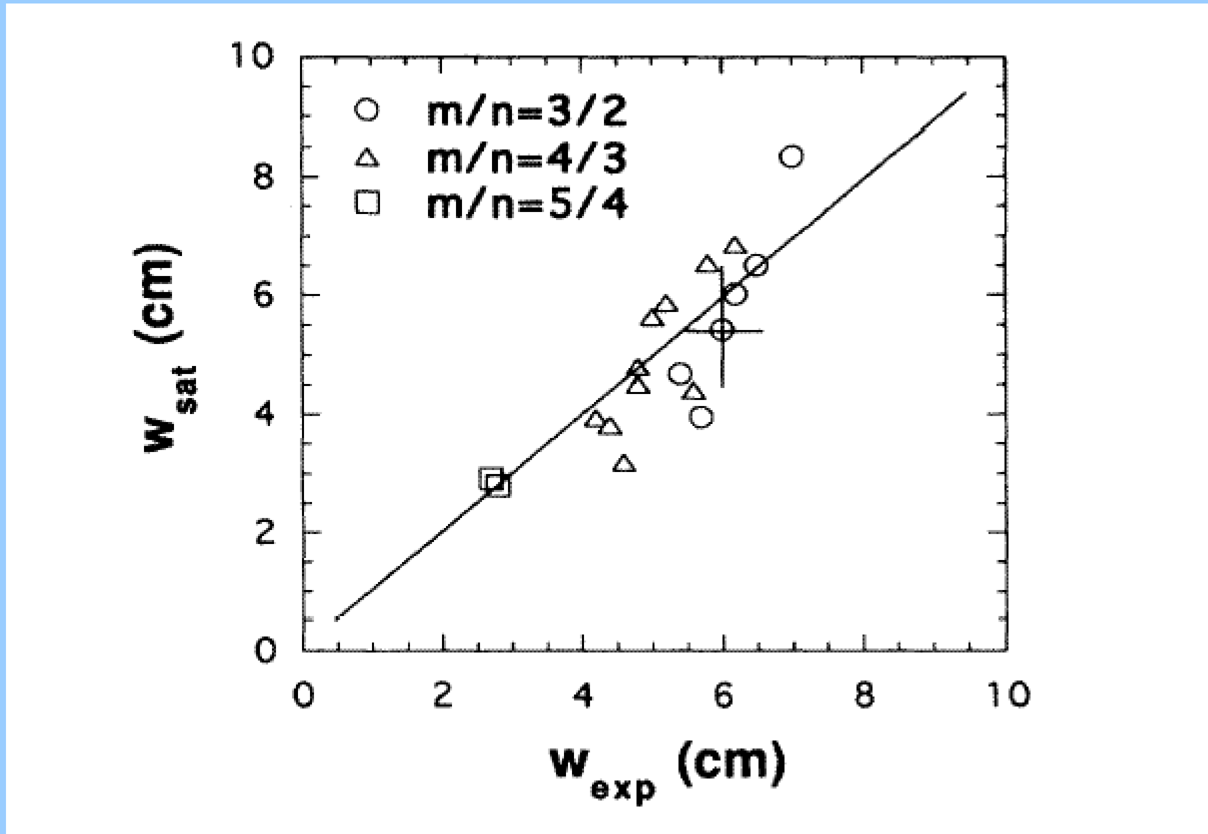
- A **“seed”** island is necessary for growth - so NTM is a nonlinear mode - no linear analog.

- Saturation width proportional to β_θ - hence limits plasma pressure

- NTMs have been observed experimentally on many tokamaks starting with TFTR in 1995 – associated with degradation in plasma confinement

- Broad agreement with scaling features given by the Rutherford model

TFTR



Theory - experiment comparison of saturated island widths

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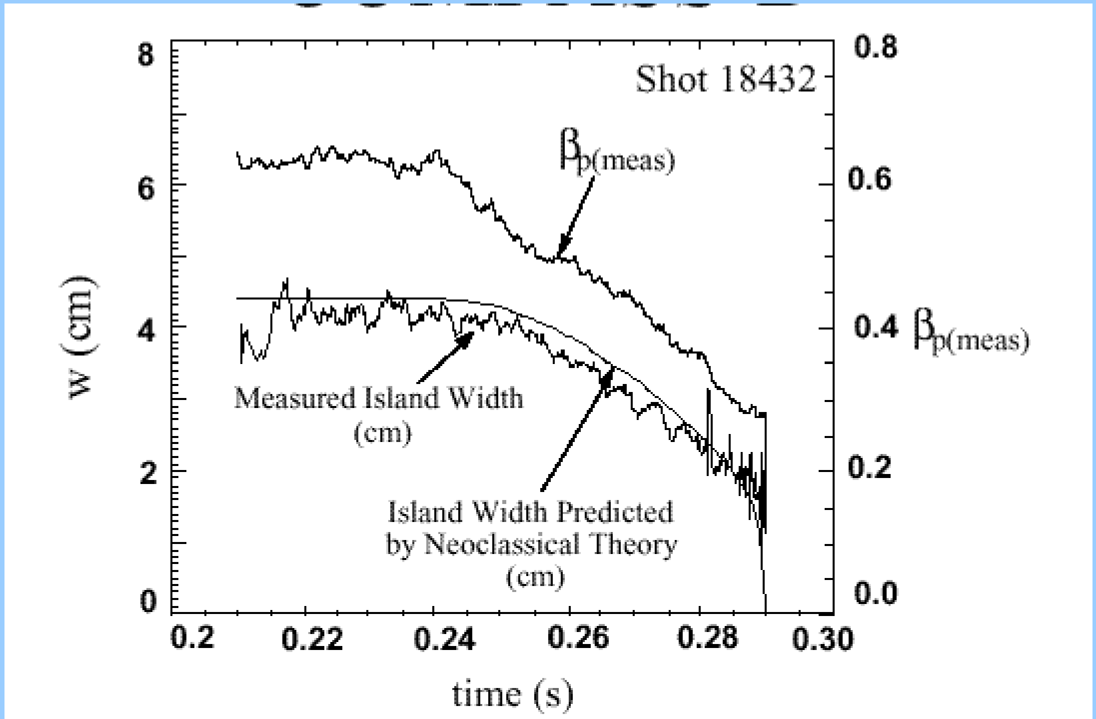


Saturated island width scales like β_p

$$w_{sat} = -a_1 \varepsilon^{1/2} \left(\frac{L_q}{L_p} \right) \frac{\beta_p}{\Delta'}$$

[D.A. Gates et al, Nuclear Fusion **37** (1997) 1593]

- Also experimental studies on suppression of NTMs using ECCD, LHCD, local heating etc. – basically restoring the bootstrap current in the island

- NTM trigger and nonlinear evolution characteristics still not well understood

- Large number of theoretical studies including some numerical simulations.

- Interaction of flows with NTMs not widely studied
  - an important issue since flows can arise from NB injection, RF heating or from turbulence drive

How can flows affect NTMs?

• Flows can influence both outer layer and inner layer dynamics for resistive modes.

• They can also bring about changes in linear coupling mechanisms such as toroidal coupling between harmonics.

• Past nonlinear studies – mainly numerical – and often limited to simple situations (e.g. poloidal flows, non-self consistent) reveal interesting effects like oscillating islands, distortion in eigenfunctions etc.

• Also some recent analytic work on the the effect of flow on the threshold and dynamical properties of magnetic islands which are relevant to NTMs

Refs: Chen &Morrison, '92, 94; Bondeson & Persson, '86,'88,'89; M.Chu,'98 Dewar & Persson, '93; Pletzer & Dewar, '90,'91,'94;Smolyakov '93,'95

Our present work:

- Investigates the nonlinear evolution of NTMs in the presence of **sheared equilibrium flows**

- **Primary approach is numerical** – we solve a set of model reduced MHD equations (3D and toroidal geometry) that contain viscous forces based on neoclassical closures and that permit inclusion of equilibrium flows in a consistent manner

- We also look at the nonlinear evolution of **classical tearing modes** for a comparative study and to obtain a better understanding of the role of flows

- Carry out a **generalized Rutherford model** calculation to seek qualitative understanding of the nonlinear numerical results

Model Equations

- Generalized reduced MHD equations

  *(Kruger, Hegna and Callen, Phys. Plasmas **5** (1998) 4169.)*

- *Applicable to any toroidal configuration – no constraint on aspect ratio – exploits smallness of* $(\lambda_\perp/\lambda_{||})$ *and* $(\lambda_\perp/\boldsymbol{a})$

- Clear separation of time scales – MHD equilibrium, perp. wave motion and parallel wave motion

- *Final equations evolve scalar quantities on shear Alfven time scales*

- Energy conservation, divergence free magnetic field to all orders

- neoclassical closures, sub-Alfvenic equilibrium flows included

Model Equations (GRMHD)

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1) \cdot \nabla \phi_1 - \boldsymbol{b}_1 \cdot \nabla \phi_0 = \eta \tilde{J}_{||} - \frac{1}{ne} \boldsymbol{b}_0 \cdot \nabla \cdot \mathbf{\Pi}_e$$

bootstrap current

$$\nabla \cdot \left(\frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (\boldsymbol{V}_1 \cdot \nabla) \left(\nabla \cdot \left(\frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) = (\boldsymbol{B}_0 \cdot \nabla) \frac{\tilde{J}_{||}}{B_0} + (\boldsymbol{B}_1 \cdot \nabla) \frac{J_{T||}}{B_0}$$
$$+ \nabla \cdot \frac{\boldsymbol{B}_0 \times \nabla p_1}{B_0^2} \qquad + \nabla \cdot \frac{\boldsymbol{B}_0}{B_0^2} \times \nabla \cdot \mathbf{\Pi}$$

GGJ

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot \boldsymbol{V}_1 = (\Gamma - 1) \left[\eta J_{T||}^2 - \mathbf{\Pi} : \nabla \boldsymbol{V} + \mathbf{\Pi}_{\boldsymbol{e}} : \nabla \frac{\boldsymbol{J}}{ne} - \nabla \cdot \boldsymbol{q} \right]$$

heat flow

$$\rho \frac{d\widetilde{V}_{||}}{dt} + (\boldsymbol{V}_1 \cdot \nabla) V_{||_0} = -\boldsymbol{b}_0 \cdot \nabla p_1 - \boldsymbol{b}_1 \cdot \nabla p_T - \boldsymbol{b}_0 \cdot \nabla \cdot \mathbf{\Pi}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla$$

$$\boldsymbol{V} = \Omega(\psi) R^2 \boldsymbol{\nabla}\zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0 \times \nabla\Phi_0}{B_0^2} + V_{0||}\boldsymbol{b}_0 + \frac{\boldsymbol{B}_0 \times \nabla\Phi_1}{B_0^2} + \tilde{V}_{||}\boldsymbol{b}_T$$

Equilibrium flow

- **Neoclassical closure**

$$\vec{\nabla} \cdot \Pi_s = \rho_s \mu_s \left\langle B^2 \right\rangle \frac{\vec{V}_s \cdot \vec{\nabla}\Theta}{\left(\vec{B} \cdot \vec{\nabla}\Theta\right)^2} \vec{\nabla}\Theta,$$

- appropriate for long mean free path limit
- reproduces poloidal flow damping
- gives appropriate perturbed bootstrap current

Numerical simulation

- **GRMHD** eqns solved using code *NEAR* – toroidal initial value code – Fourier decomposition in the poloidal and toroidal directions and central finite differencing in the flux coordinate direction.

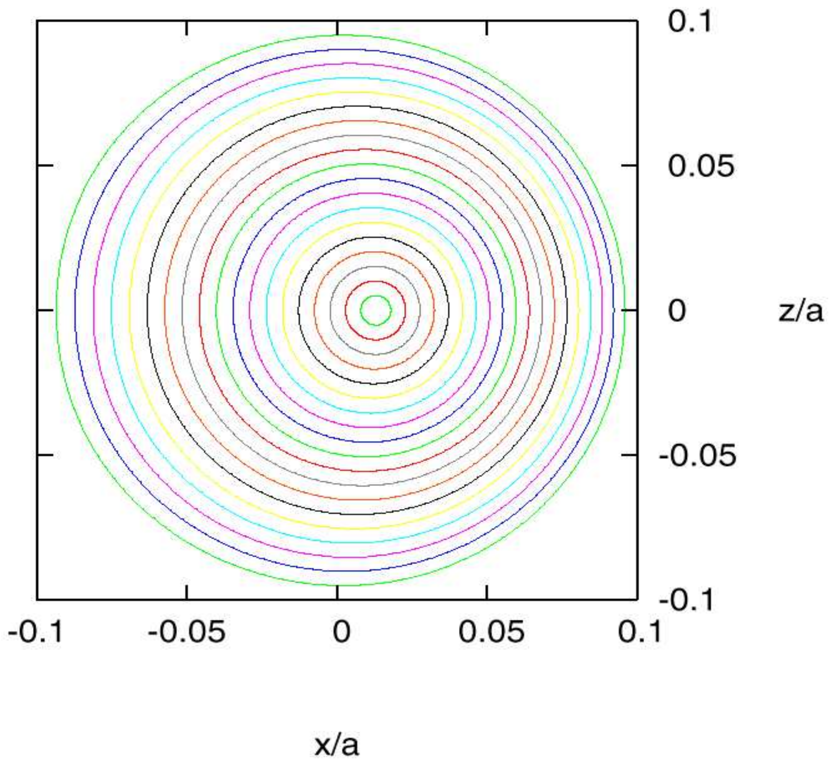
- Self-consistent equilibrium generated from another code **TOQ** **(http://fusion.gat.com/toq)**

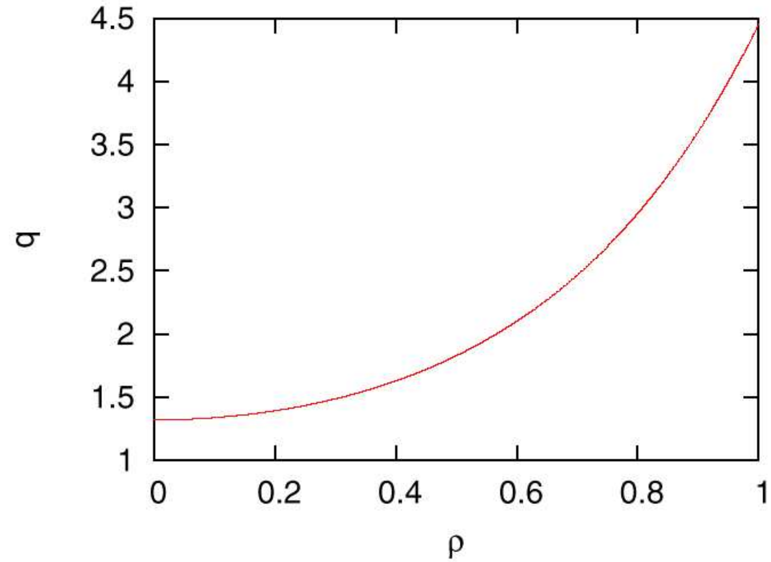
- Typical runs are made at **$S \sim 10^5$**

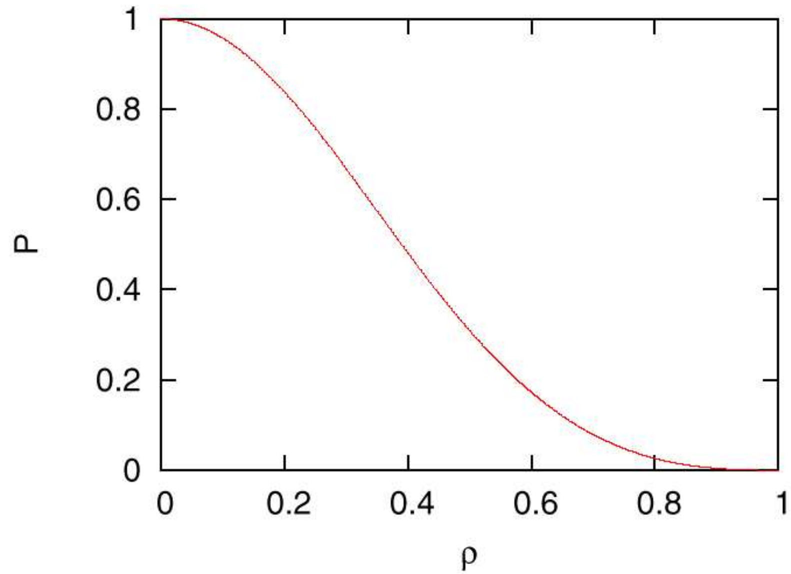
- modes investigated:(2,1) and (3,1)

- Present study restricted to **sheared toroidal flows**

Typical equilibrium without flow





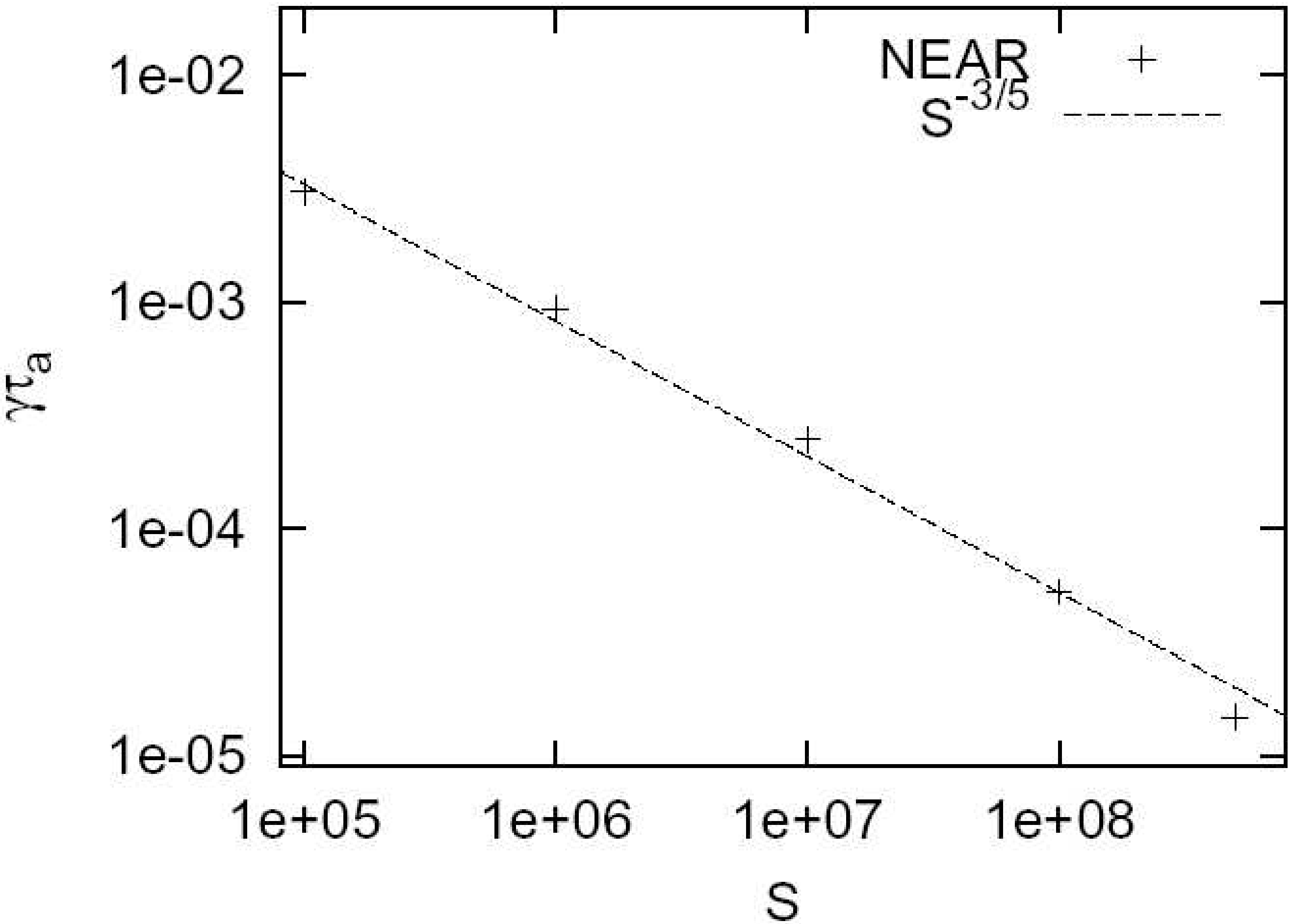


B_o= 4.8 T I=0.28 MA

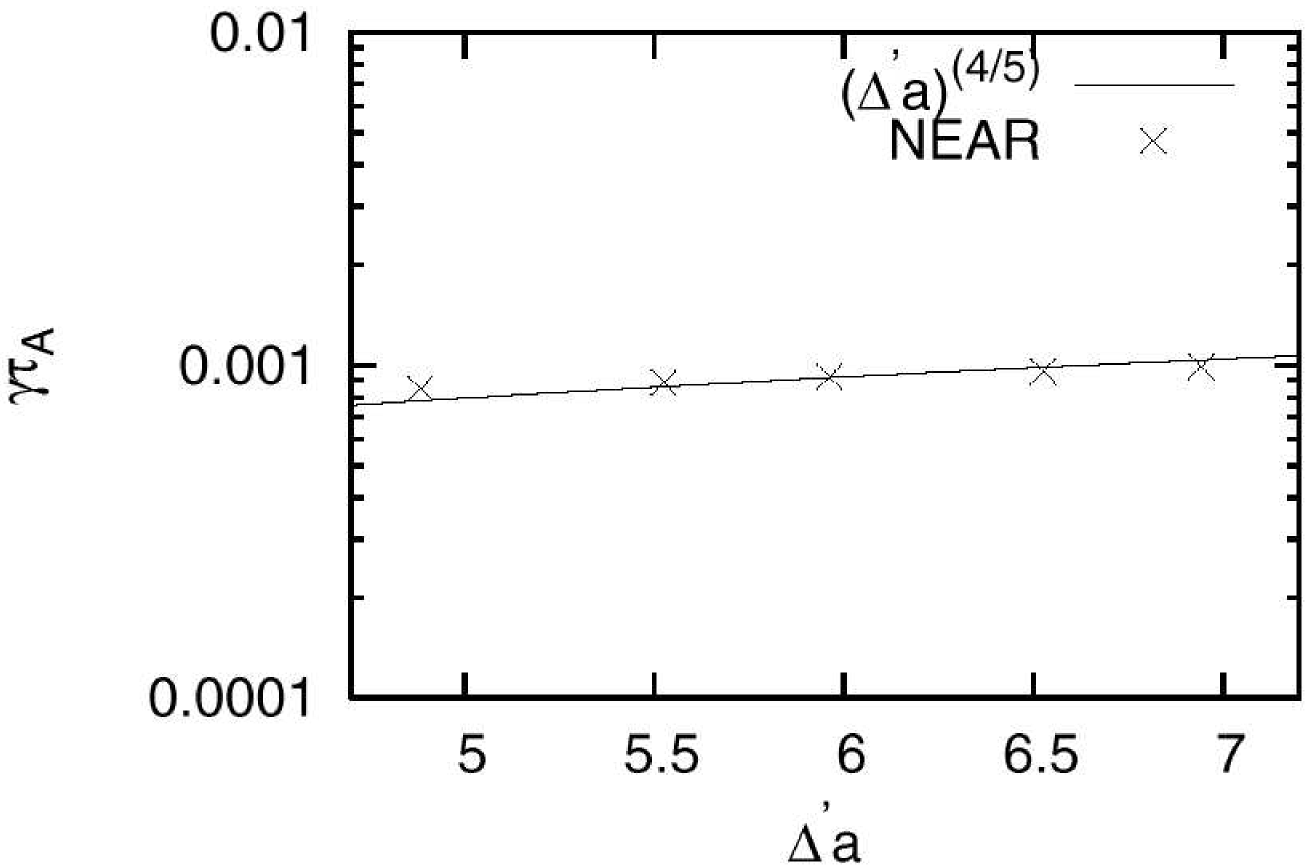
R=5.00 m a =0.50 m

β_0 = 0.6 % β_p =2.2

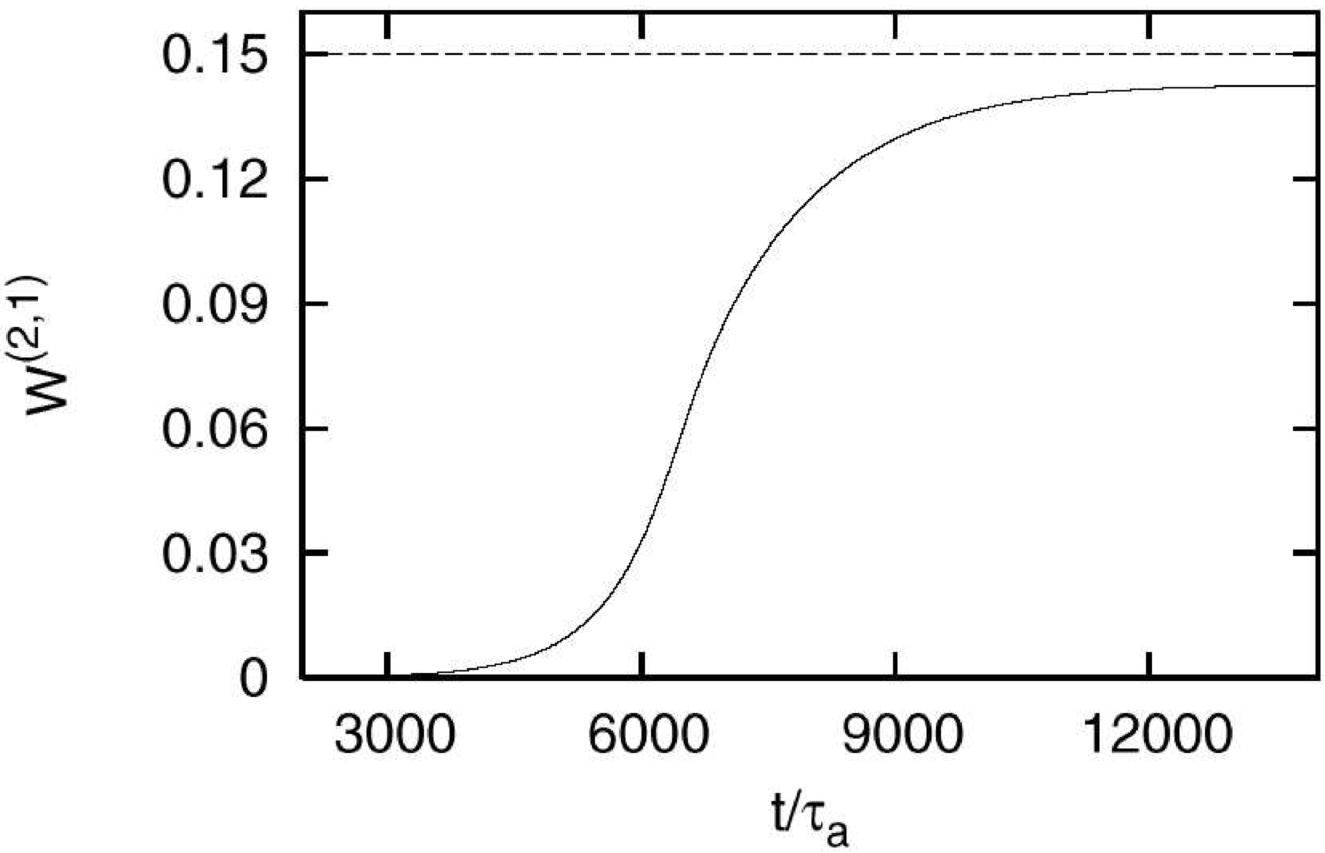
Linear Benchmarking of resistive TMs – S scaling



Linear Benchmarking of resistive TMs - Δ' scaling



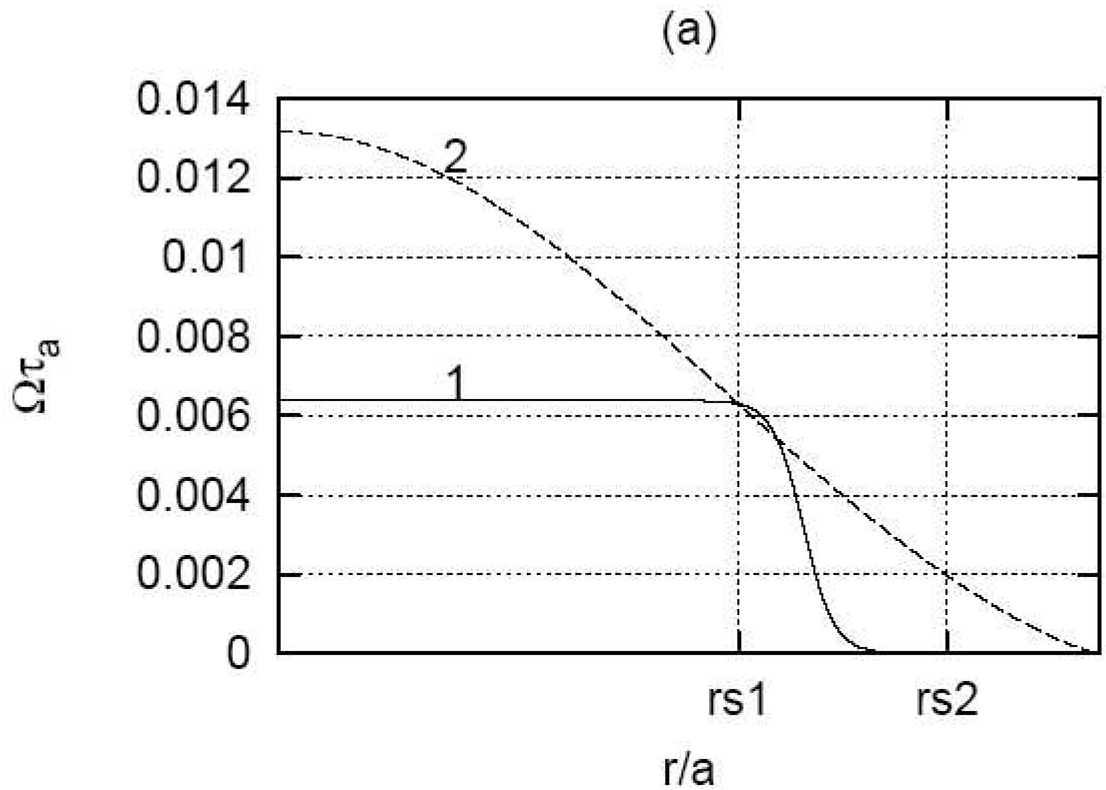
Nonlinear saturation of (2,1) resistive TMs



$$\Delta' \rightarrow \Delta'(1 - W/W_{sat})$$

Hegna, C.C., Callen, J.D., Phys. Plasmas **1** (1994) 2308.

Toroidal flow profiles



1- differential flow

2- negative sheared flow (usual tokamak profile)

Main points of investigation

- Effects arising from equilibrium modifications

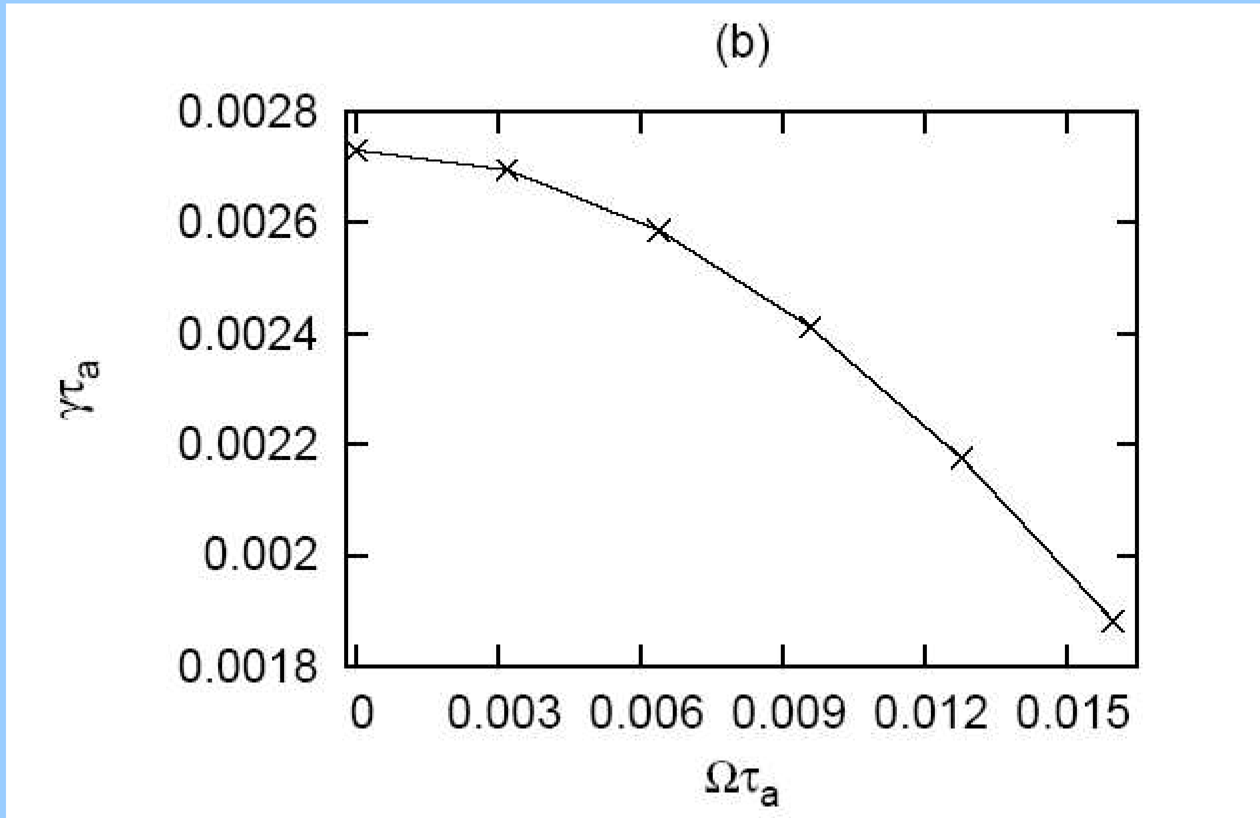
- Influence on toroidal coupling

- Influence on inner layer physics

- Changes in outer layer dynamics

- Nonlinear changes – saturation levels etc.

Reduction of (2,1) resistive TM growth with differential flow



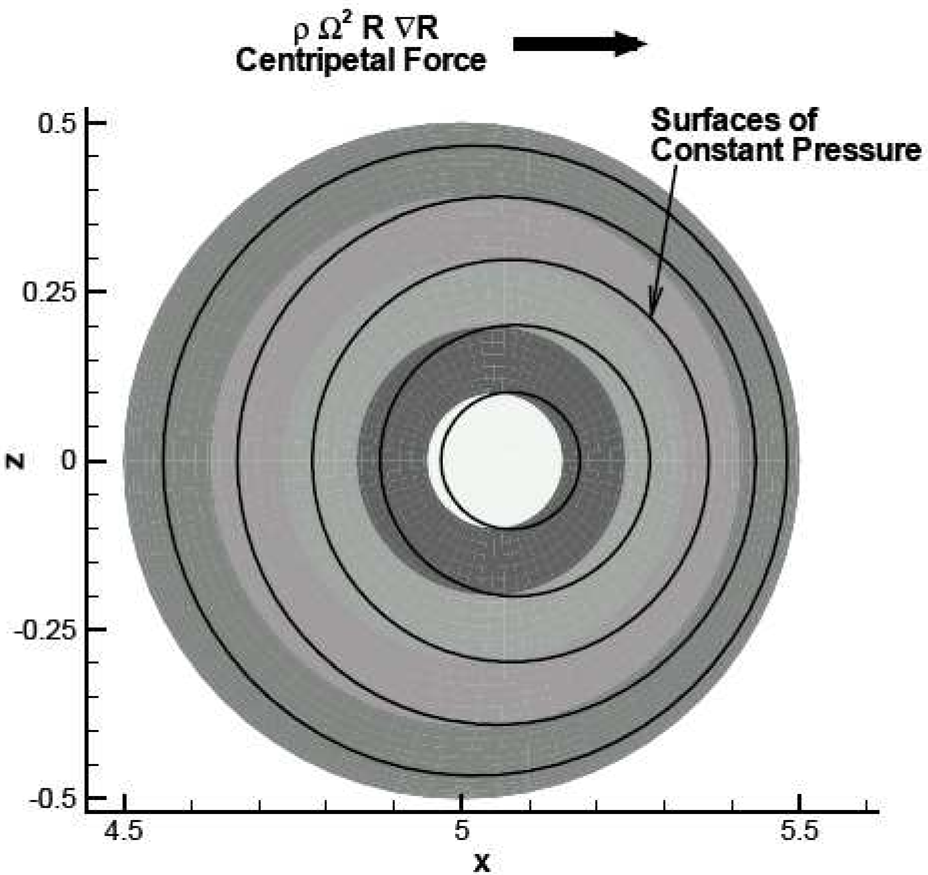
- **stabilizing effect due to equilibrium changes e.g. enhancement of pressure-curvature contribution?**
- **stabilizing effect due to flow induced de-coupling of rational surfaces?**

Summary of various stabilizing contributions on γ

Toroidal Coupling	Pressure-Curvature	Equilibrium Flow	Linear growth rate
Off	Off	Off	2.90e-3
Off	On	Off	2.85e-3
On	Off	Off	2.20e-3
On	On	Off	2.01e-3
Off	Off	On	2.84e-3
Off	On	On	2.80e-3
On	Off	On	2.11e-3
On	On	On	1.89e-3

- Stabilizing effect of pressure-curvature enhances in presence of differential flow
- Enhancement can be attributed to centrifugal force modification of equilibrium pressure surfaces
- Stabilizing effect due to flow induced de-coupling of rational surfaces

Equilibrium with toroidal flow



Constant pressure Surfaces shifted from Constant flux surfaces

Maschke & Perrin, Plasma Phys. 22 (1980) 579

$$p_0 = p_{nf}(\psi_0)\exp\left(\frac{\Gamma}{2}M_s^2(\psi_0)(\hat{R}^2 - \hat{R}_{axis}^2)\right)$$

Dispersion relation in presence of flow

- **Slab or cylinder**

$$\Delta' \Psi_s = -i\,(\omega - \Omega_s)\,\tau_L \Psi_s; \qquad \Omega_s = \vec{k} \cdot \vec{V}_0$$

$$\boxed{\gamma = \frac{\Delta'}{\tau_L}} \qquad \boxed{\Omega = \Omega_s}$$

- **Toroidal geometry**

$\Psi_{\mathbf{large}} = \Delta' \Psi$ **outer response - Δ′ matrix**

$\Delta_{ij}(\omega) = -i(\omega - \Omega_j)\tau_{Lj}\delta_{ij}$ **inner response**

$\gamma_{ii} = \Delta'_{ii}/\tau_{Li}$ individual growth rate

$\gamma_{ij} = \Delta'_{ij}/\tau_{Li}$ coupling coefficient

Dispersion relation in presence of flow - contd

Linear dispersion relation:

$$\det \begin{bmatrix} \Delta'_{11} - \Delta_{11}(\omega) & \Delta'_{12} \\ \Delta'_{21} & \Delta'_{22} - \Delta_{22}(\omega) \end{bmatrix} = 0.$$

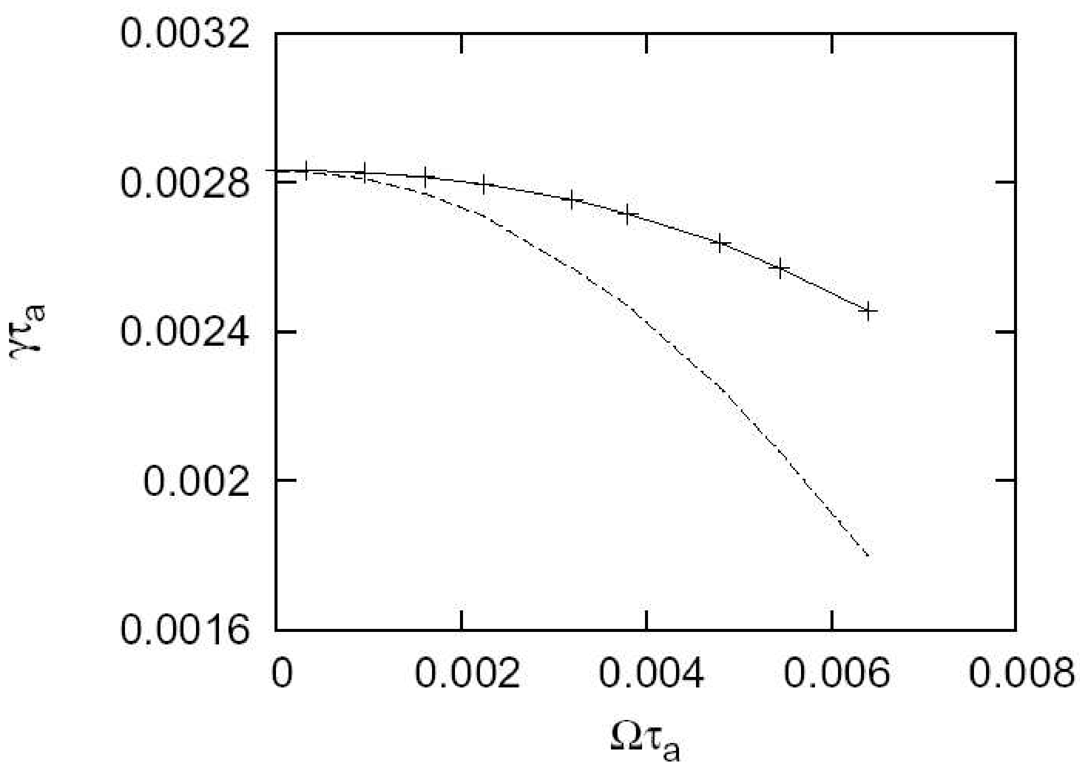
Quadratic equation:

$$\omega^2 - \omega[\Omega_1 + \Omega_2 + i(\gamma_{11} + \gamma_{22})] + \frac{[\Omega_1 + \Omega_2 + i(\gamma_{11} + \gamma_{22})]^2}{4} + \gamma_{12}\gamma_{21} - \frac{[\Delta\Omega + i(\gamma_{22} - \gamma_{11})]^2}{4} = 0$$

Ratio of eigenvectors [surface 1 → (3,1) & surface 2 → (2,1)]:

$$\frac{|\Psi_{3,1}|}{|\Psi_{2,1}|} \approx \frac{\gamma_{1,2}}{\Delta\Omega}$$

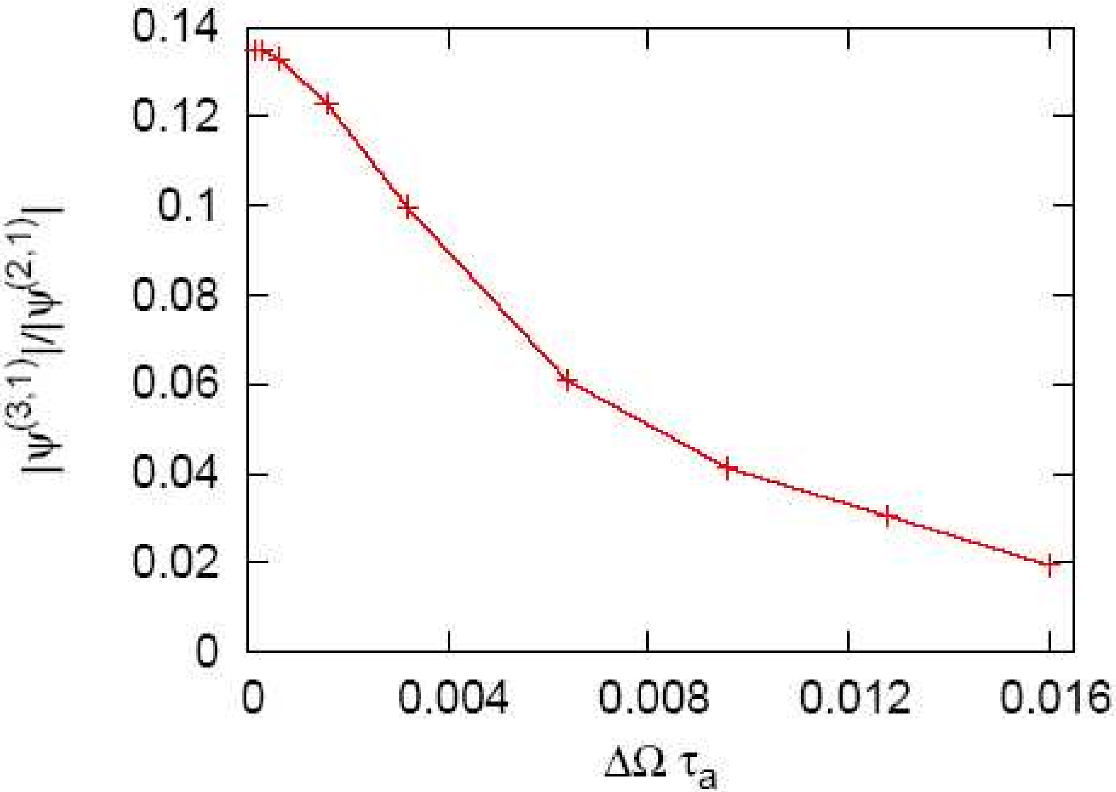
Reduction of (2,1) resistive TM growth with differential flow



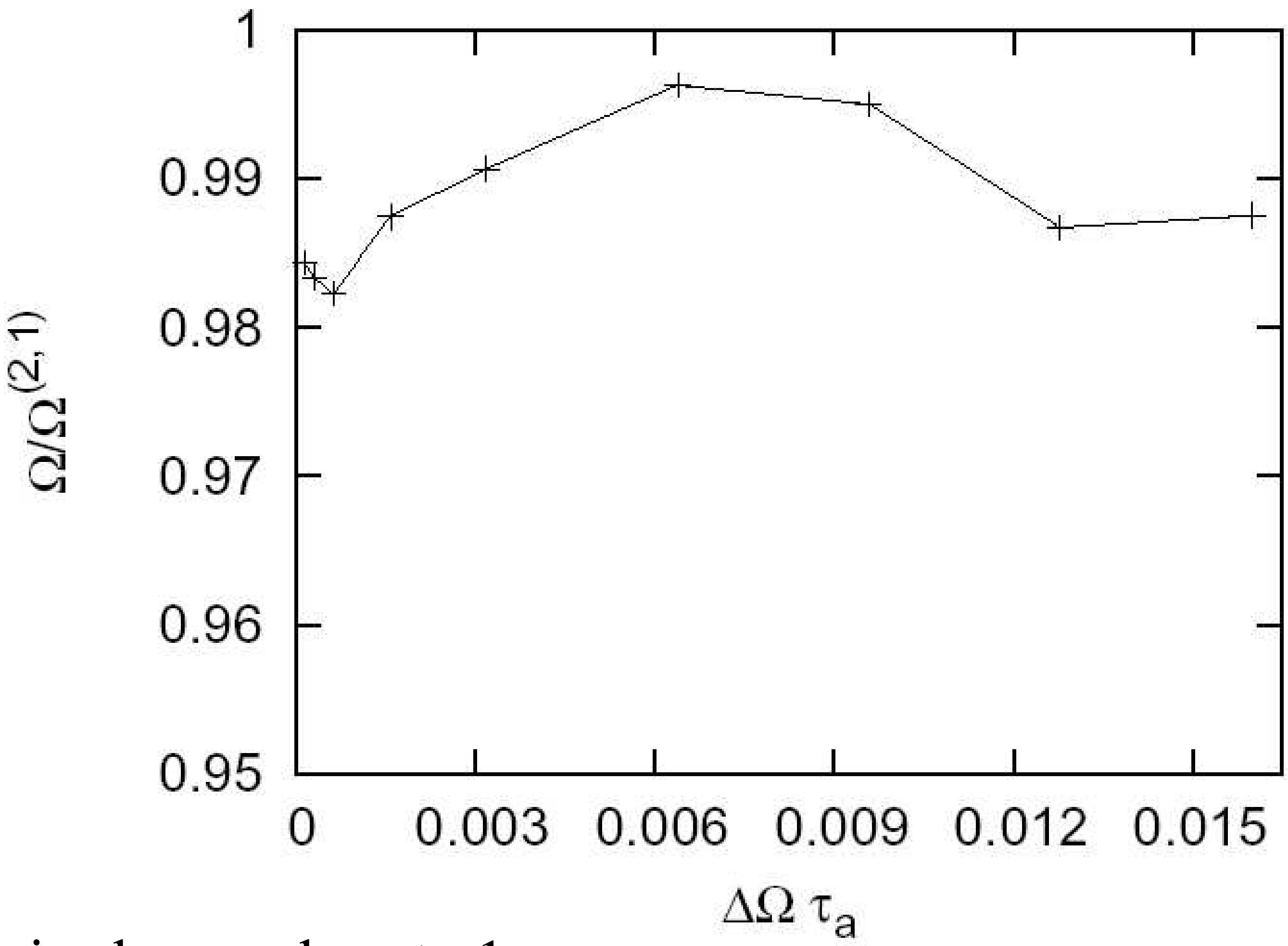
Upper curve – NEAR

Lower curve - analytic solution of quadratic equation in ω, derived from the dispersion relation

Reduced reconnection at the (3,1) surface



Ratio of mode freq to flow rotation freq at (2,1) surface



- The ratio is always close to 1

- **In the presence of finite negative flow shear the stabilization effect is smaller**

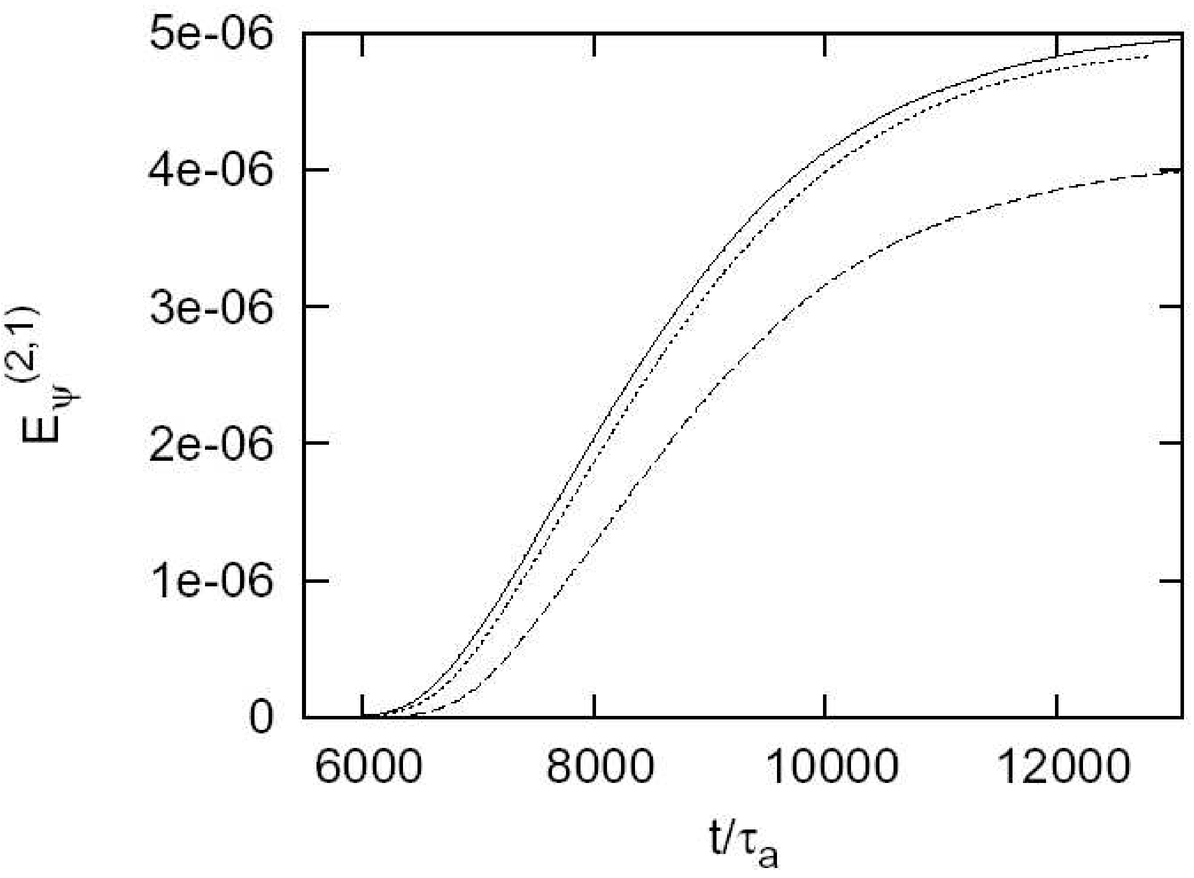
- This can be understood and explained quantitatively on the basis of linear slab theory analysis **(Chen & Morrison, PF B 2 (1990) 495)**

$$\gamma \sim \alpha^{2/5} \Delta'^{4/5} S^{-3/5} \hat{\gamma}$$

$$\hat{\gamma} = \text{flow correction} \geq 1$$

Small negative flow shear destabilizes the resistive mode through changes in the inner layer dynamics

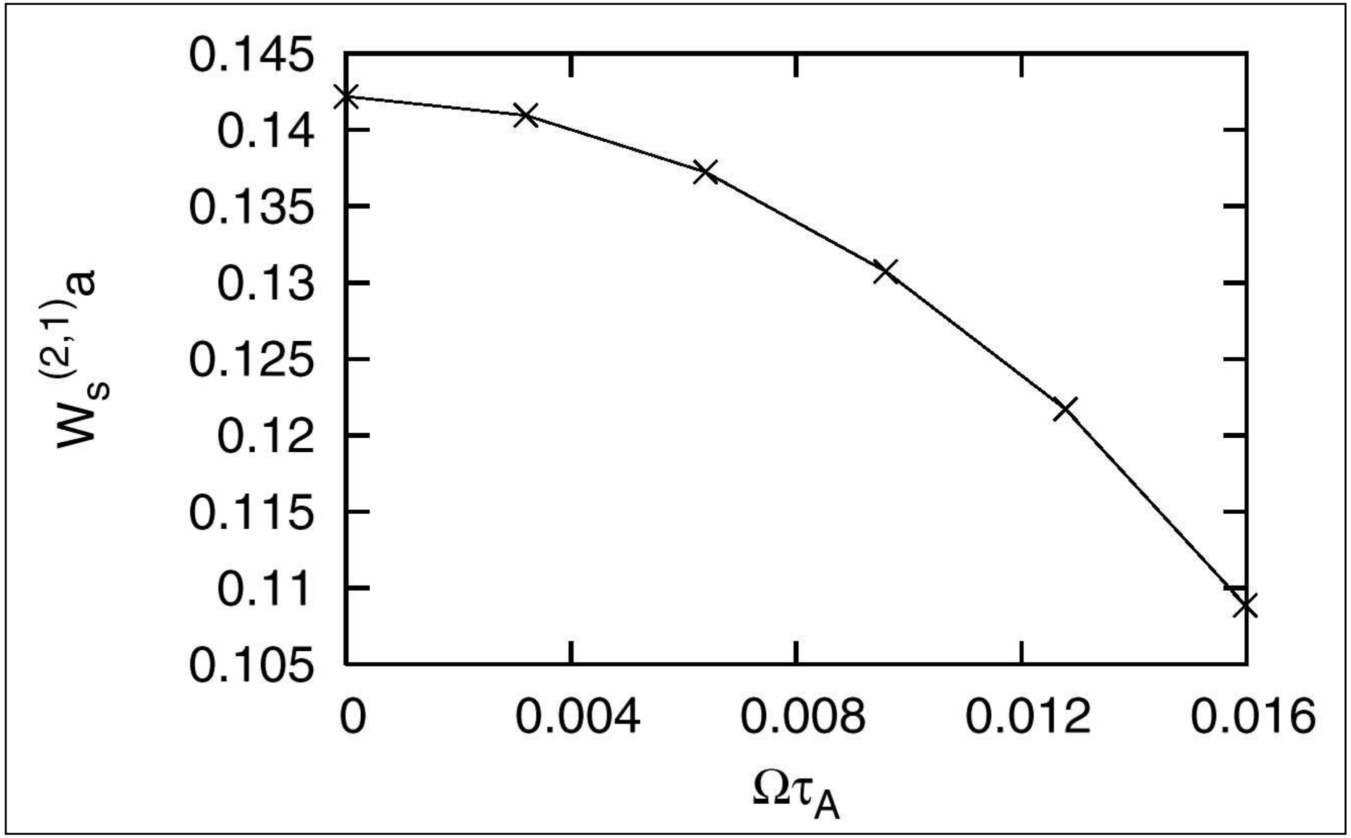
Nonlinear evolution of (2,1) resistive modes



——— No flow -------- Negative sheared flow

- - - - - Differential flow

Saturated island width decreases with differential flow



Summary of numerical results for classical TMs

- In the **linear** regime:
  - flow induces mode rotation
  - differential flow : stabilizing influence
    - modification in Mercier criterion
    - decoupling of rational surfaces
  - negative flow shear: destabilizing influence – consistent with inner layer dynamical theories

- **Nonlinear** regime

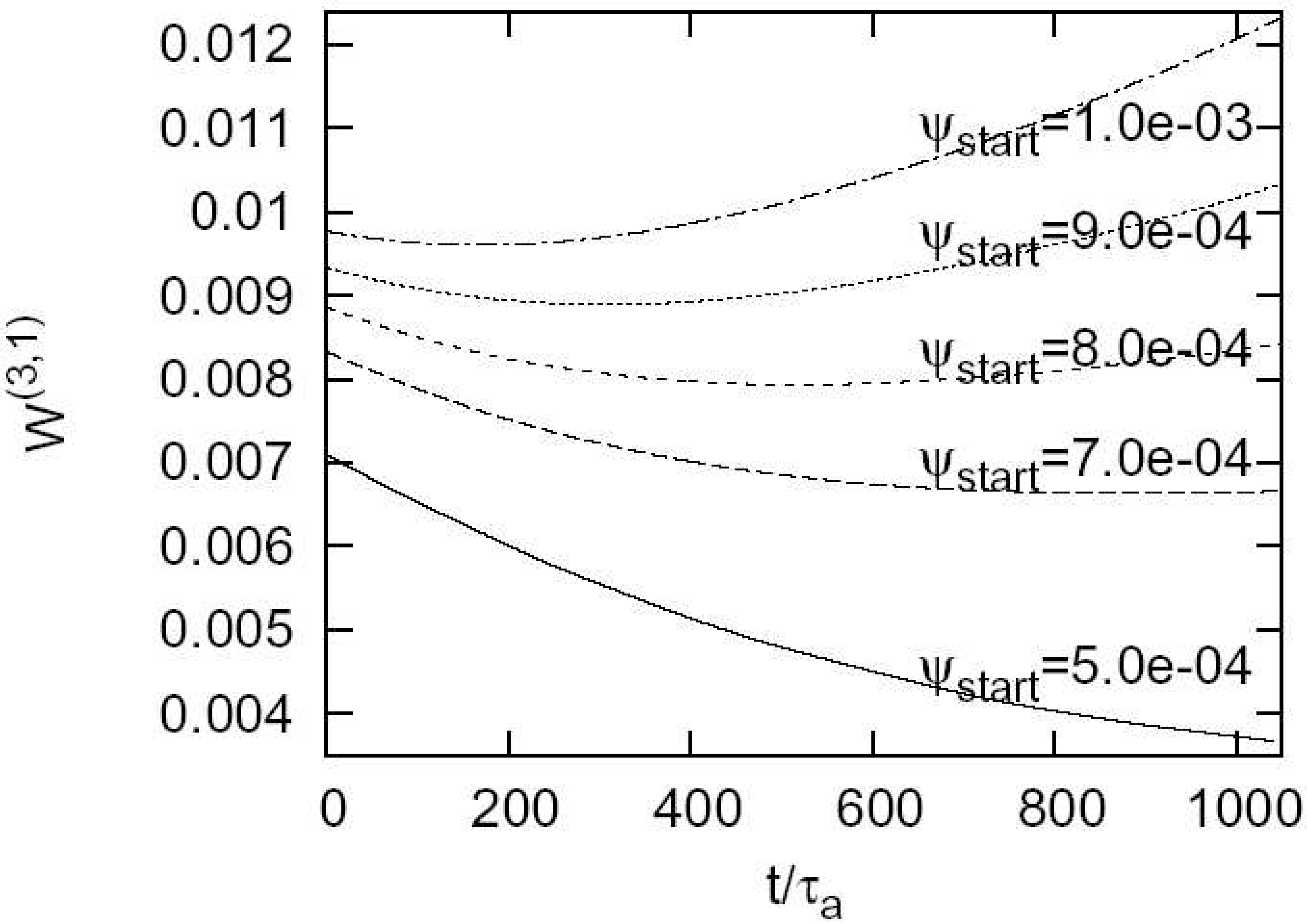
  - Above trend continues for differential and sheared flows
  - Mode acquires real frequency which asymptotes to flow frequency
  - Flow reduces the saturated island width

Neoclassical Tearing Modes

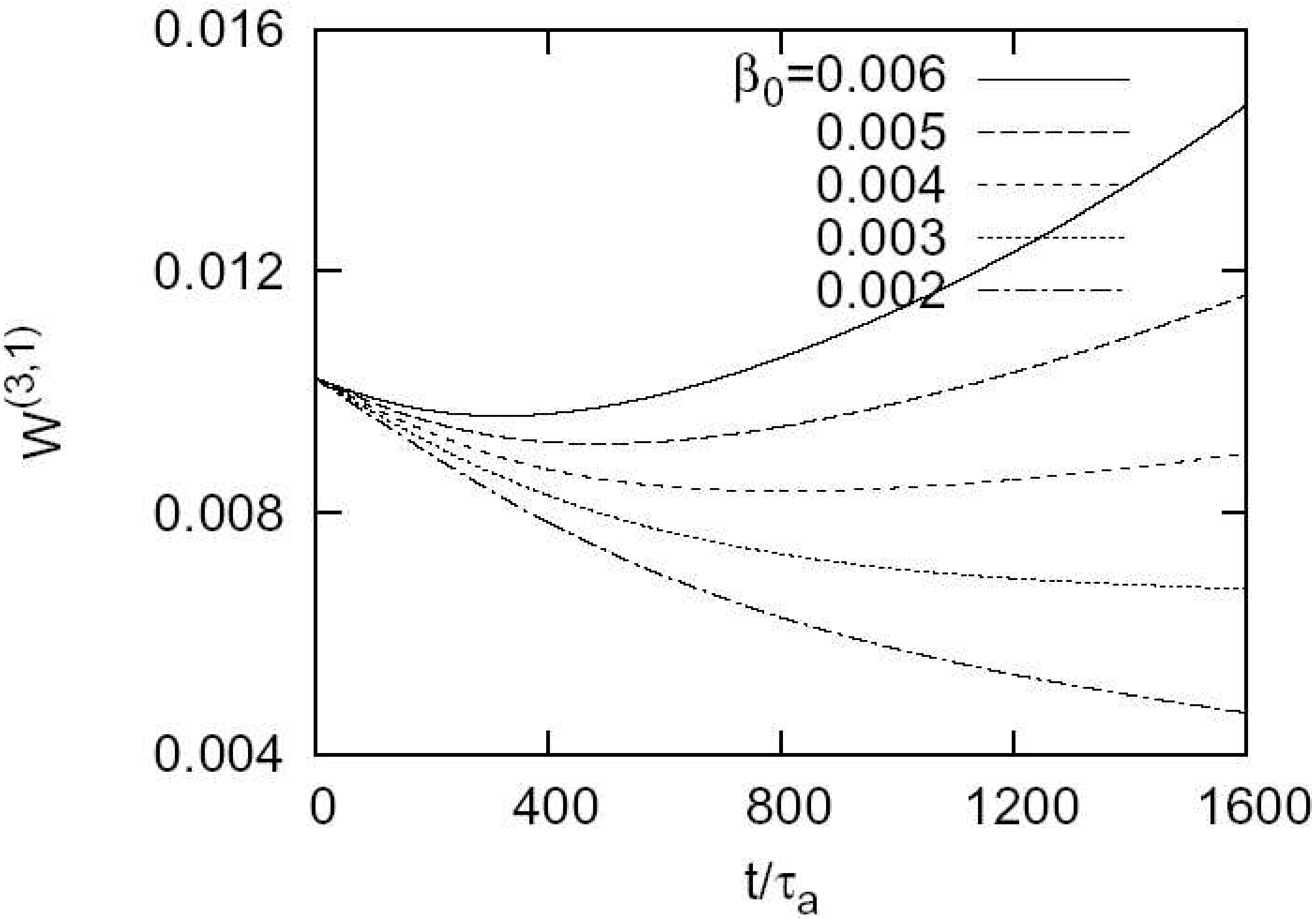
Benchmarking tests in absence of flow:

- threshold amplitude for instability
- nonlinear behavior – island saturation
- pressure equilibration

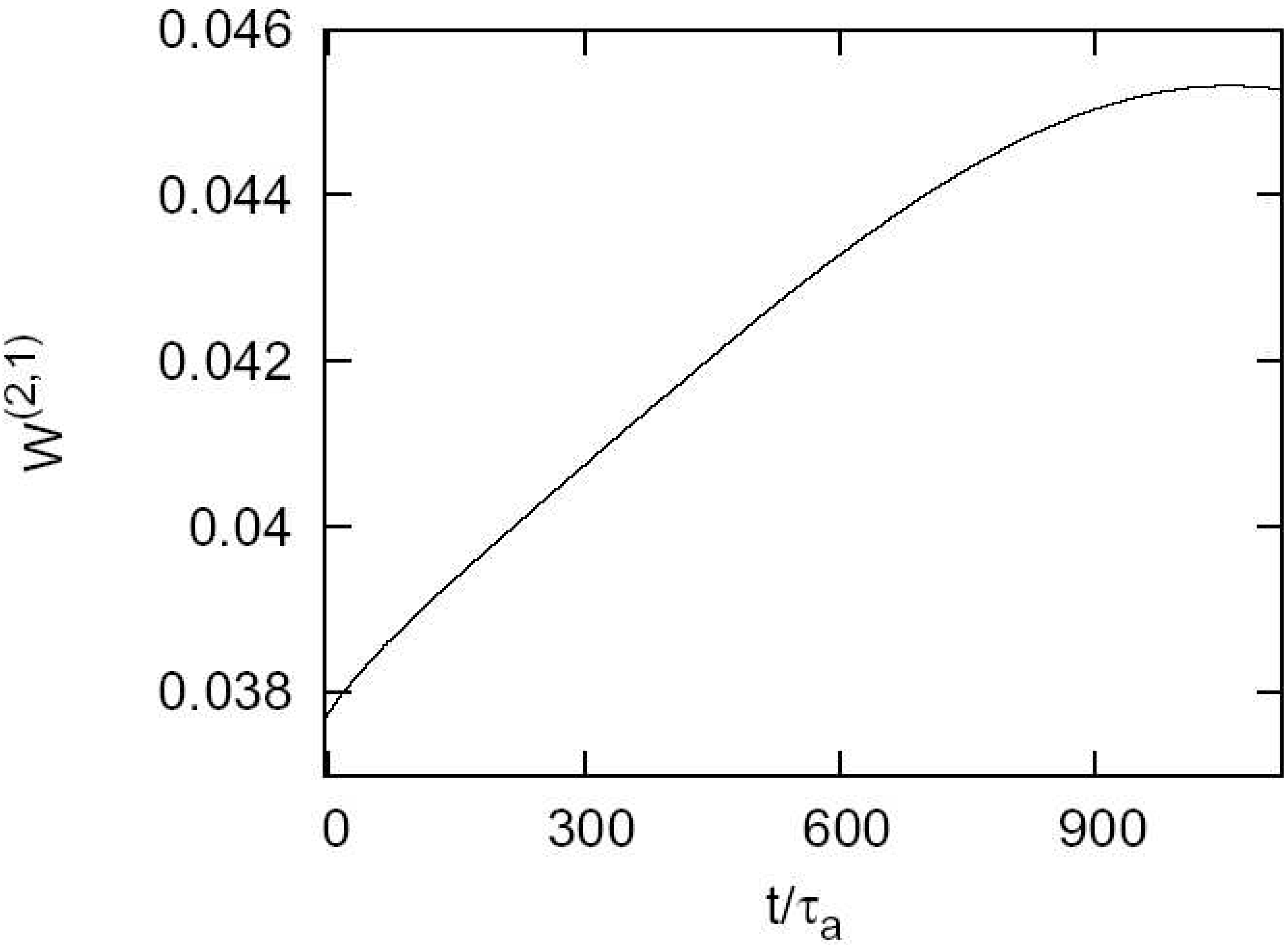
Existence of threshold amplitude for (3,1) NTM



Existence of threshold β for (3,1) NTM



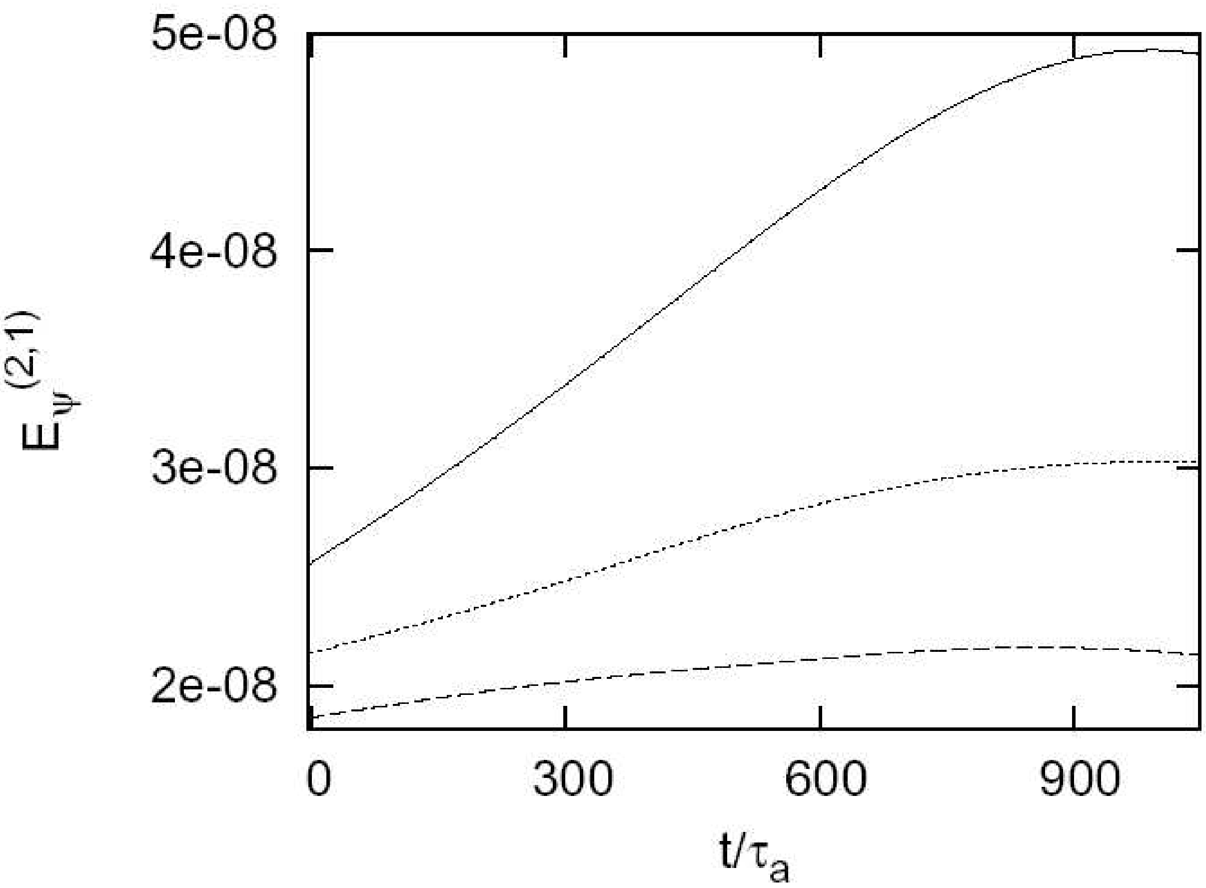
Nonlinear evolution of (2,1) NTM island width



NTM with flows

- self-consistent equilibria generated by TOQ
- two types of flow profiles – differential flow, sheared flow

Nonlinear evolution of (2,1) NTM



——— No flow

- - - - - - Sheared flow

- - - - - Differential flow

Summary of numerical results for NTMs

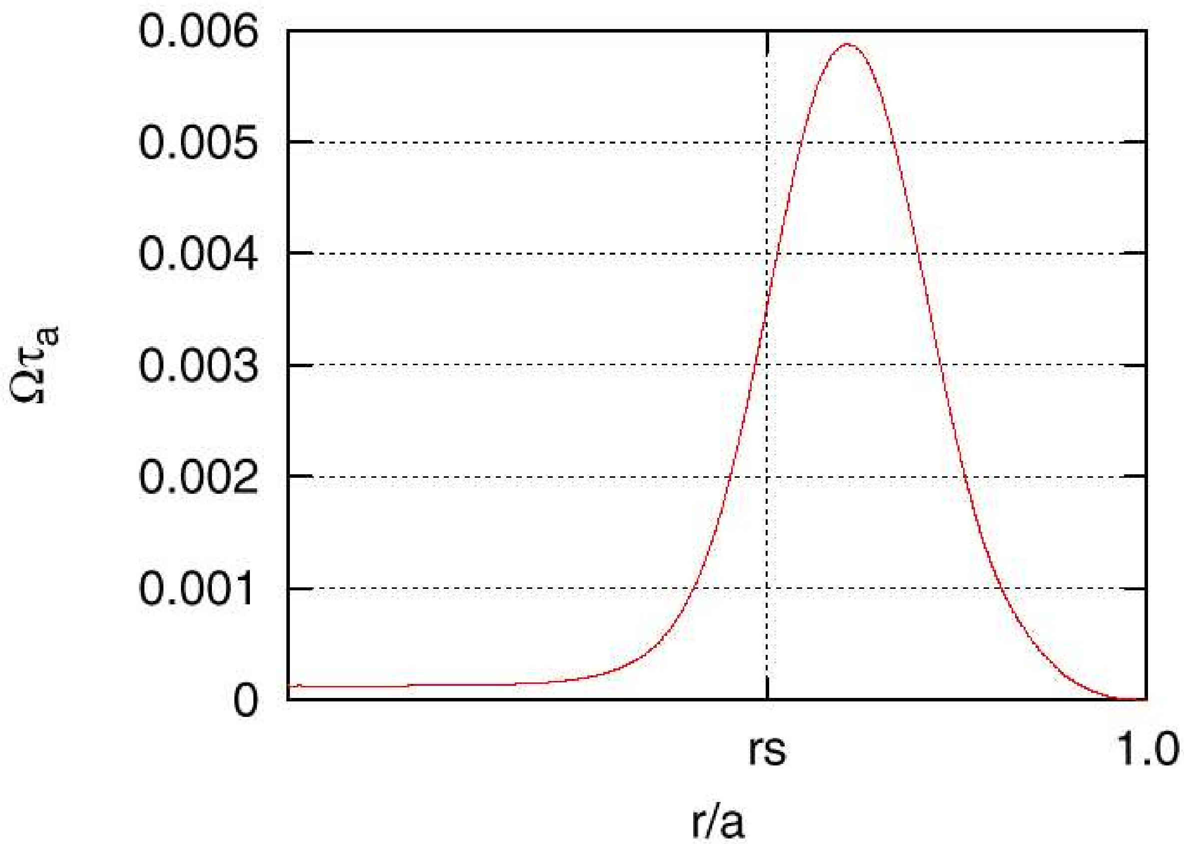
• differential flow : stabilizing influence

• negative flow shear: destabilizing influence

- Is the sign of flow shear important?

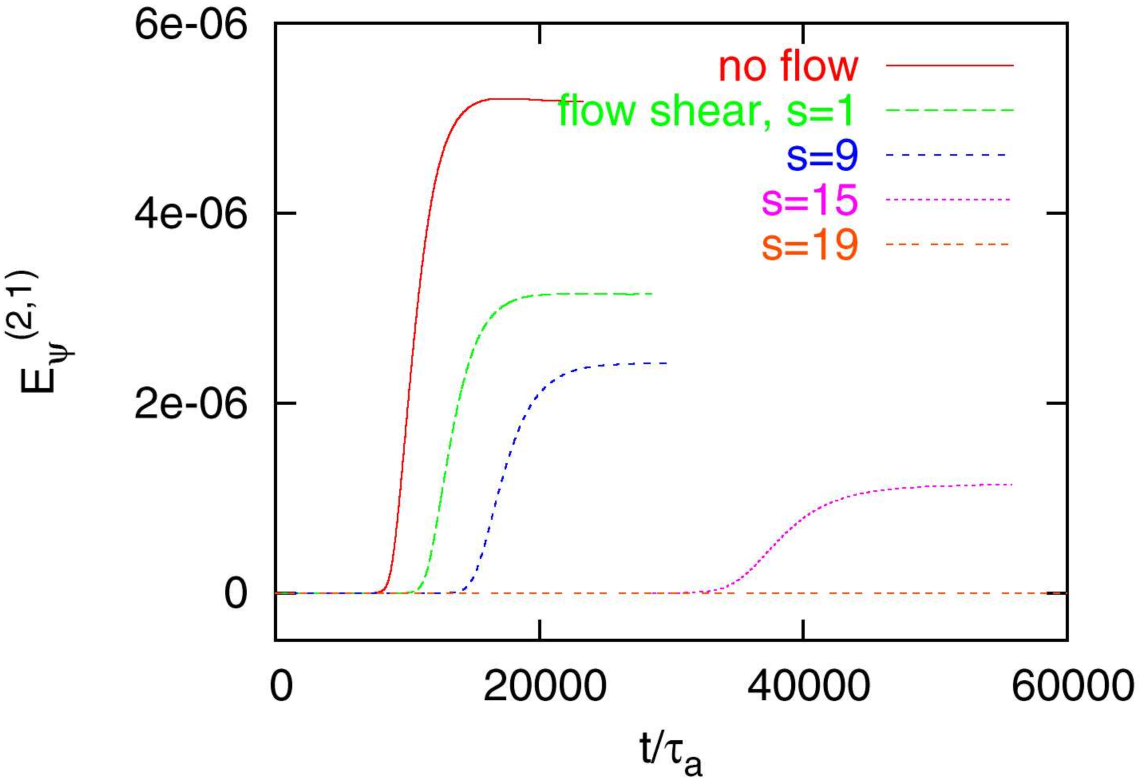
- two types of flow profiles – positive shear, negative shear
- nature of profiles kept similar so that other effects would not influence the result
- Single helicity runs

Profile with positive flow shear at (2,1) surface



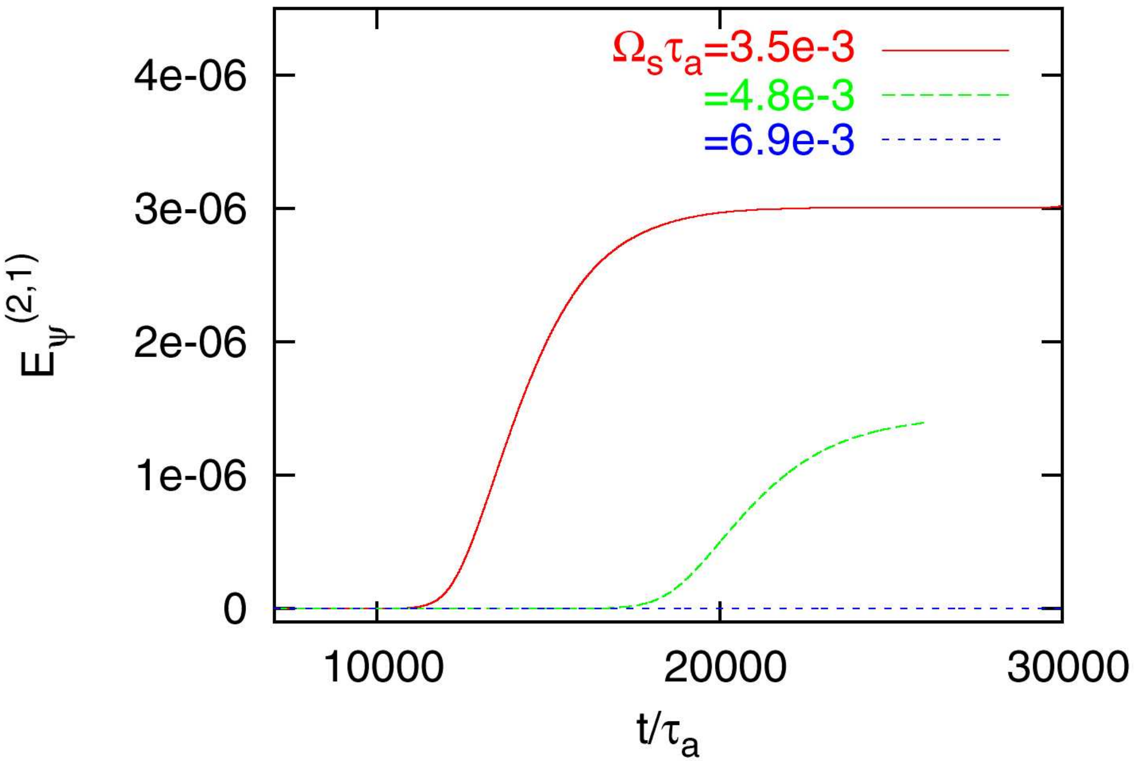
- Looked at single helicity mode dynamics

Effect of positive flow shear on (2,1) classical TMs



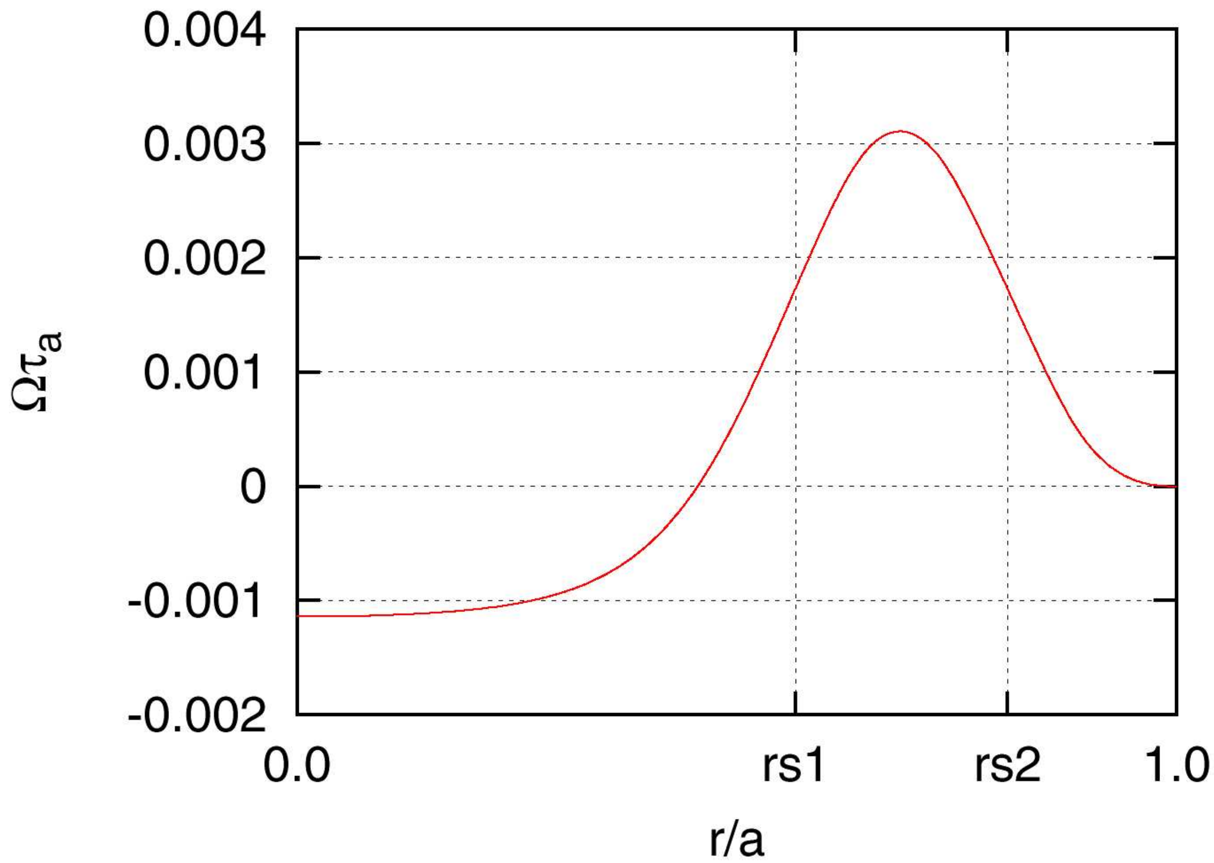
- +ve flow shear has a strong stabilizing effect
- Increase of shear for a given amount of flow can completely stabilize the CTMs

Effect of flow magnitude at positive flow shear



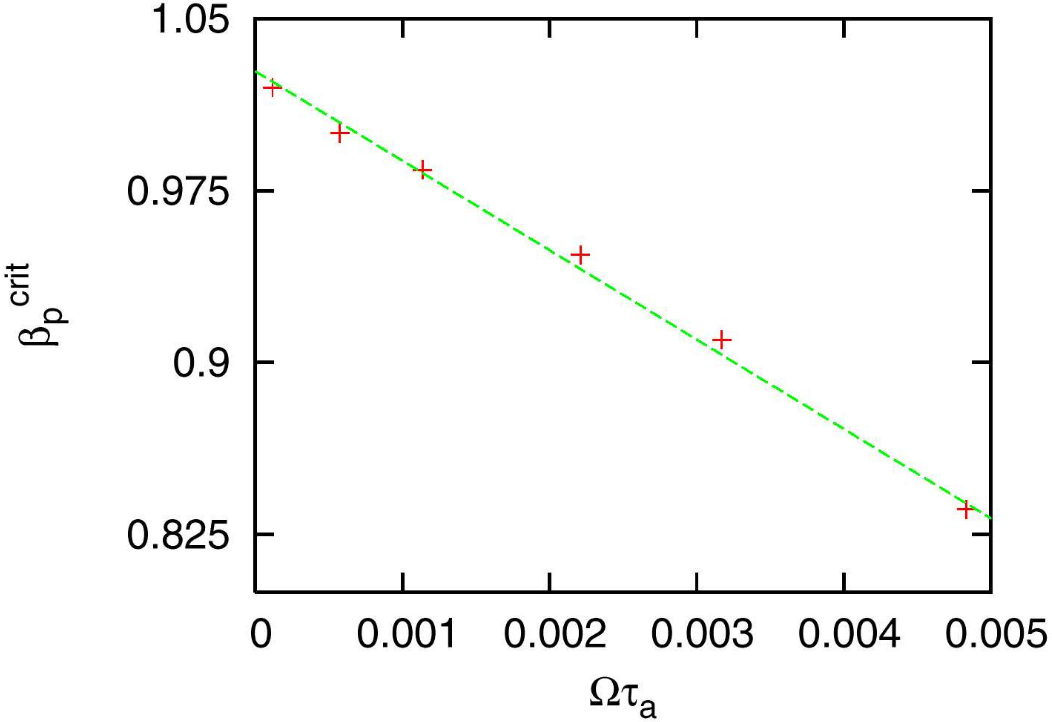
- In presence of pos flow shear, increase of flow magnitude also has a strong stabilizing effect
- Increase of flow at a given shear (s=10) can also completely stabilize the CTMs

Profile with equal flow at (2,1) & (3,1) surfaces



- coupled modes
- pos shear at (2,1) and neg shear at (3,1) surface but flow is equal

Effect of flow magnitude on the threshold- β value of (3,1) NTMs at negative flow shear



- In presence of -ve flow shear, increase of flow magnitude decreases the β threshold value
- So in tokamaks with usual -ve flow shear profile, toroidal flow can destabilize the NTMs for the same seed island

Summary of numerical results of flow gradient

- positive flow shear: stabilizing influence
- negative flow shear: destabilizing influence
- Toroidal flow with pos shear can suppress a unstable CTMs/NTMs completely
- Toroidal flow with neg shear can make a mode unstable which is otherwise stable
- Flow gradient has similar influence on CTMs and NTMs
- Toroidal flow with neg shear can destabilize NTMs for the same seed islands by reducing it's threshold- β values

Analytic model calculation

- Single helicity calculation – a la Rutherford
- Flow effects incorporated in polarization current term
- two fluid model
- neoclassical effects included in Ohm's law
- simple pressure evolution equation & parallel dynamics included in convective derivative term
- phenomenological model for GGJ effect
- use matching conditions to get island evolution equation

Generalized Rutherford Model Calculation

• Magnetic surface

$$\psi = -\frac{B_0}{L_s}\frac{x^2}{2} + \tilde{\psi} cos\xi$$

$$\xi = m\hat{\theta} - \int \omega(t')dt'$$

helical coordinate

• Electric Potential (in presence of sheared flows)

$$\phi = \phi_0' x + \phi_0'' \frac{x^2}{2} + \tilde{\phi}$$

Also from $E_{||} \approx 0 \Rightarrow$

$$\phi = \frac{B_0 \omega x}{c k_\theta} + f(\psi)$$

$$\tilde{\phi} = \frac{B_0}{c k_\theta}(\omega - \omega_E)x - \frac{B_0}{c k_\theta}\frac{\omega_E'}{2}x^2 + f(\psi)$$

$$\tilde{\phi} = \frac{B_0}{ck_\theta}(\omega - \omega_E)(x - \lambda) - \frac{B_0}{ck_\theta}\frac{\omega'_E}{2}(x^2 - \lambda^2)$$

where $$\lambda(\psi) = \frac{W}{\sqrt{2}}\left[\left(-\frac{\psi}{\tilde{\psi}}\right)^{1/2} - 1\right]$$ = 0 inside island

$\longrightarrow x$ for $x >> W$

and $$W = \left(\frac{4L_s\tilde{\psi}}{B_0}\right)^{1/2}$$ Island half-width

- Next we need the parallel current density to use in the matching conditions

$$\int_{-\pi}^{\pi} d\xi \cos\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_c \pi\tilde{\psi} \qquad \int_{-\pi}^{\pi} d\xi \sin\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_s \pi\tilde{\psi}$$

$$\nabla_{\|} J_{\|} + \nabla_{\perp} J_{\perp} = 0$$

$$\nabla_{\|} = \frac{k_\theta}{B_0}\frac{\partial \psi}{\partial x}\left(\frac{\partial}{\partial \xi}\right)_{\psi}$$

$$\nabla_{\|} J_{\|} - \frac{c^2}{4\pi v_A^2}\frac{d_0}{dt}\nabla_{\perp}^2\left(\phi + \frac{p_i}{en}\right) = 0$$

$$\frac{d_0}{dt} \equiv \frac{\partial}{\partial t} + \left(\mathbf{v_E} + \mathbf{v_{\|0}}\right)\cdot\nabla$$

inertial effects

$$J_{\|} \;=\; A(\psi)(cos\xi - < cos\xi >) + \frac{\sigma_{\|}}{c}\frac{\partial \tilde{\psi}}{\partial t} < cos\xi > - \frac{\mu_e}{\nu_{ei}}\frac{c}{B_\theta}\left\langle \frac{\partial \psi}{\partial x}\frac{\partial p}{\partial \psi}\right\rangle$$

$$A(\psi) = \frac{cB_0^4 W^2}{8\pi v_A^2 k_\theta^2 L_s^2}\frac{\partial}{\partial \psi}\left(\frac{\omega_E'}{2}\frac{\partial \lambda^2}{\partial \psi} - \overline{\omega - \omega_E - \omega_{*pi}}\frac{\partial \lambda}{\partial \psi} + \frac{k_\theta v_{\|0}}{B_0}\right)\left(\frac{\omega_E'}{2}\frac{\partial^2 \lambda^2}{\partial \psi^2} - \overline{\omega - \omega_E}\frac{\partial^2 \lambda}{\partial \psi^2}\right)$$

Neoclassical Tearing Mode Evolution Equations

$$G_1 \frac{\partial W}{\partial t} = D_R^{neo} \left[\frac{\Delta_c'}{4} + G_2 \frac{\sqrt{\varepsilon}\beta_\theta \frac{L_q}{L_p}}{W} \frac{W^2}{W^2 + W_\chi^2} - G_3 \frac{D_I}{\sqrt{W^2 + 0.65 W_\chi^2}} \right.$$

$$\left. + \frac{L_s^2}{k_\theta^2 v_A^2} \left(G_4 \frac{(\omega - \omega_E)(\omega - \omega_E - \omega_*)}{W^3} + G_5 \frac{{\omega_E'}^2}{W} \right) - G_6 \frac{L_s}{k_\theta v_A} \frac{\bar{v}_{\|0}}{v_A} \frac{\omega_E'}{W} \right]$$

$$G_W \frac{\partial}{\partial t} \left[W(\omega - \omega_E) + \frac{\omega_E'}{2} W^2 \right] = -6 G_V \frac{\mu_e}{W} (\omega - \omega_E) - \frac{1}{4\sqrt{2}} \left(\frac{n s V_A}{R^2 q} \right)^2 W^4 \Delta_s'$$

Island evolution equation with sheared flow

Pressure/curvature **Neoclassical current**

$$0.41\frac{\partial W}{\partial t} = D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right.$$

$$\left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + 0.24\frac{{\omega_E'}^2}{W}\right) - 0.77\frac{L_s}{k_\theta v_A}\frac{\bar{v}_{||0}}{v_A}\frac{\omega_E'}{W}\right]$$

differential flow **flow shear**

polarization current

$\omega_E = k_\theta c\Phi_0'(r=r_s)/B_0$; flow shear, $\omega_E' = k_\theta c\Phi_0''(r=r_s)/B_0$

$\bar{v}_{||0}$ = average value of equilibrium parallel flow

- For toroidal flow both $\bar{v}_{||0}$ and ω_E' are finite, the last term has the required structure to be in accord with our numerical findings on flow shear effects

Summary and Conclusions

- Presented numerical simulation results, using a model set of GRMHD equns. with neoclassical viscous terms and toroidal flow, for nonlinear evolution of resistive TMs and NTMs

- Differential flow has a stabilizing influence – can be understood intuitively in the linear regime as occurring from decoupling of rational surfaces - the decoupled surface appears as a conducting surface and exerts a stabilizing influence. Same trend continues in the nonlinear regime – no analytic theory exists in the nonlinear regime

- Sheared toroidal flow has a significant influence on tearing mode stability, nature of which depends on the sign of flow shear.

Summary and Conclusions (contd)

- Toroidal flow with positive flow shear has a stabilizing influence

- Toroidal flow with negative flow shear (usual tokamak scenario) has a destabilizing influence

  - It reduces the stabilizing influence of differential flow

  - Strong shear can even destabilize a stable mode

  - It can destabilize NTMs for the same seed island by reducing the threshold value of β

- In the quasi-linear regime our approximate single mode Rutherford type calculation shows a similar trend.

Open issues and future work

- Numerical : neoclassical closure issues, two fluid effects, etc.
  a major initiative in NIMROD

- Analytical: better model calculations including toroidal effects,
  multiple helicity, poloidal flows etc.

- Basic issues which also need experimental investigation

  - NTM trigger
  - interaction with flows
  - ion polarization effects
  - island rotation