Kinetic Features in Plasma Turbulence

Y. Sarazin, G. Darmet, G. Dif-Pradalier, E. Fleurence, X. Garbet, Ph. Ghendrih, V. Grandgirard

Association Euratom-CEA, CEA/DSM/DRFC Cadarache, France

P. Bertrand

LPMIA-Université Henri Poincaré, Vandœuvre-lès-Nancy, France

N. Besse, E. Sonnendrücker

IRMA, CNRS-Université Louis Pasteur, Strasbourg, France

Kinetic vs. Fluid approach

Gyro*-Kinetic description of turbulence is powerful:

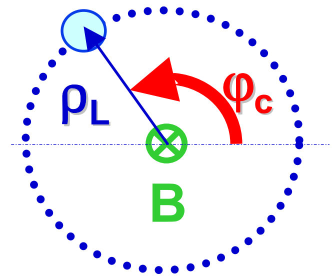
- **Landau resonances**
- **All classes of particles** (trapped & energetic)

… but CPU time & memory consuming

*Average over the rapid cyclotron phase φ_c

$\Downarrow$

5D & μ=invariant



Strategy:

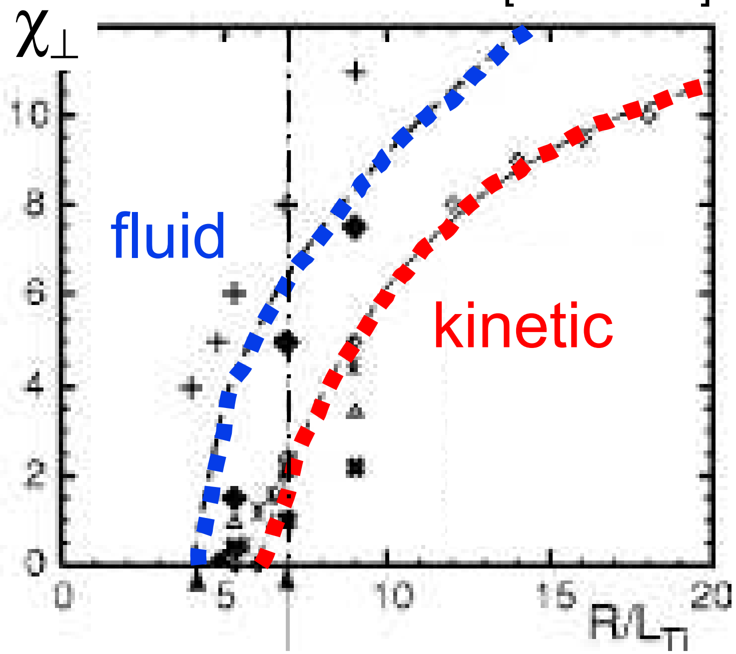
- Develop more & more complex (3D $\rightarrow$ 5D) gyro-kinetic codes
- Fluid still useful $\rightarrow$ adequate **fluid closures**

Motivations

- **Kinetics** ***vs.*** **Fluid: discrepancies** in turbulence results

  Linear thresholds - Non linear fluxes

[Dimits '00]



- **Importance of Zonal Flows**

  [Rosenbluth-Hinton, '98, Lin et al. '98]

- **New types of non collisionnal closures:**

  Non local

  [Hammet-Perkins '90, Waltz-Dominguez-Hammett '92, Beer '95, Snyder-Hammet-Dorland '97, Passot-Sulem '03]

  Non dissipative

  [Sugama-Watanabe-Horton '01,'04]

Outline :

1. Kinetic vs. fluid: non-linear comparisons of interchange turbulence
2. An alternative non-collisional fluid closure
3. Generation of Zonal Flows by $\nabla_\perp n$

2D+1D interchange instability

[Thesis Fleurence '05]

Hamiltonian: $H = v_d E x + \phi$

Curvature drift $E v_d \vec{e}_y$ - $E \approx v_\perp^2$ ($v_{//}=0$ ions) - Adiabatic electrons

Slab geometry (x,y) - Limit $k_\perp \rho_i \to 0$

Kinetics

$$\partial_t f + [\phi, f] + v_d E\, \partial_y f = 0$$ Drift kinetic eq.

$$\phi - \langle\phi\rangle - \nabla_\perp^2 \phi = \frac{1}{n_{eq}} \int_0^\infty f \, \mathrm{d}E - 1$$ Quasi-neutrality

Fluid

$$\partial_t n + [\phi, n] + v_d\, \partial_y P = 0$$

$$\partial_t P + [\phi, P] + v_d\, \partial_y Q = 0$$

Closure: $Q \equiv \int f\, E^2 \mathrm{d}E = \Upsilon P T$

$$\begin{cases} \Upsilon = 1 \Leftrightarrow \int (E - T)^2 f \, \mathrm{d}E = 0 \\ \Upsilon = 2 \Leftrightarrow \int (f - f_M)\, E^2 \mathrm{d}E = 0 \end{cases}$$

Fluid ≈ F at 2 energies

Constraint: ***same numerics to treat fluid & kinetic descriptions***

2 distributions $f_\pm(x,y)$ at energies $E_\pm = T_0 \pm \varepsilon$

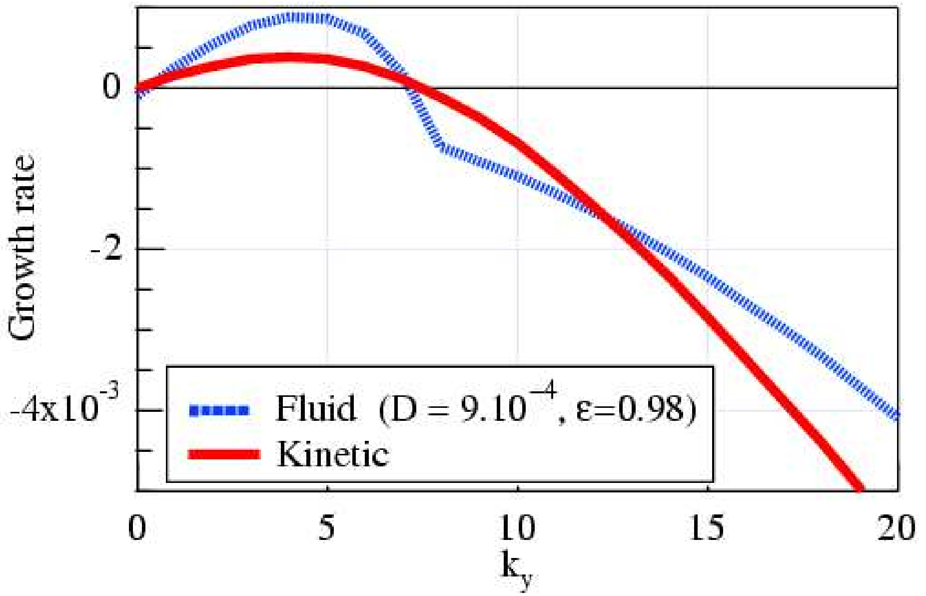
$$\partial_t f_\pm + [\phi, f_\pm] + v_d(T_0 \pm \varepsilon)\,\partial_y f_\pm = D\,\Delta_\perp f_\pm$$

$$n = f_- + f_+ \qquad P = E_- f_- + E_+ f_+ = nT_0 + \varepsilon(f_+ - f_-)$$

$$Q = E_-^2 f_- + E_+^2 f_+ = nT^2 + 4\varepsilon^2 \frac{f_- f_+}{n}$$ ⇔ ϒ=1 in the limit ε <<1

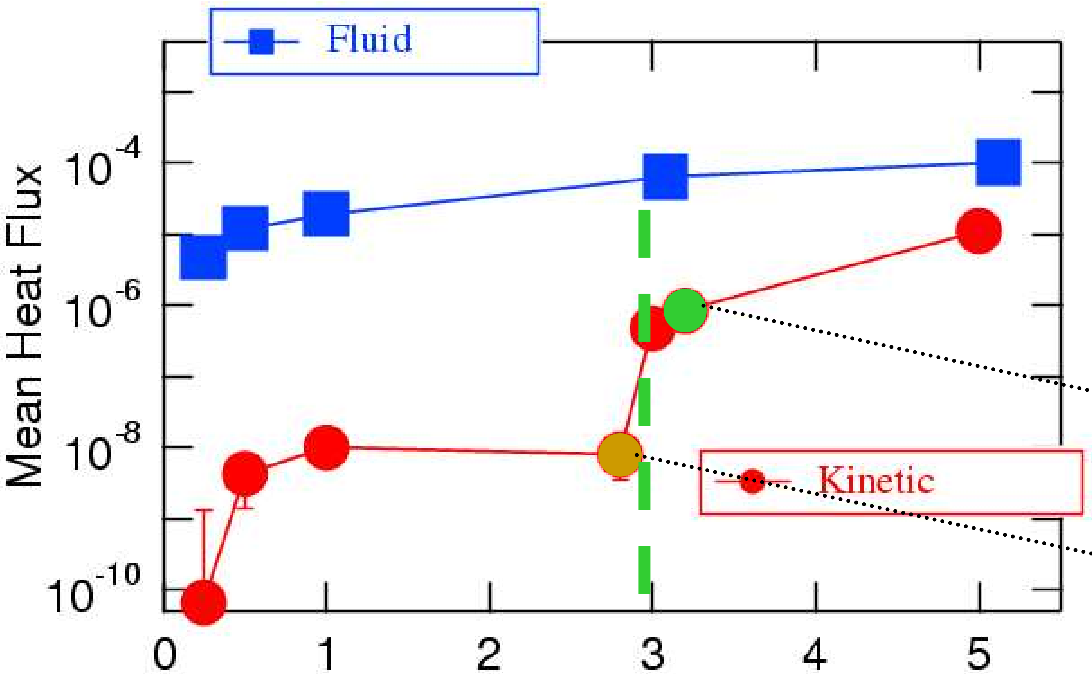
Linear fluid properties can mimic kinetic ones:

1. Linear threshold
2. Unstable spectrum width (or maximum growth rate)



Non linear discrepancy: $Q^{fl} >> Q^{kin}$

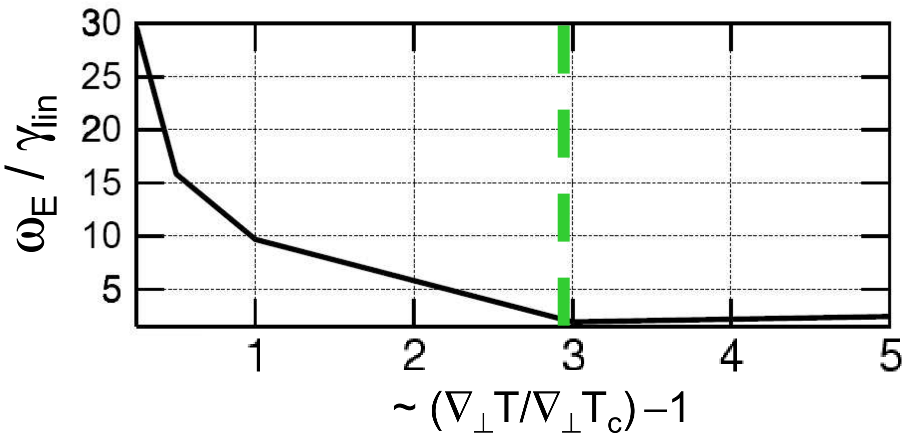
- ❑ Heat turbulent transport much larger in fluid than in kinetic ($\forall\ \nabla_\perp T$)
- ❑ Non-linear threshold in kinetics (Dimits upshift)



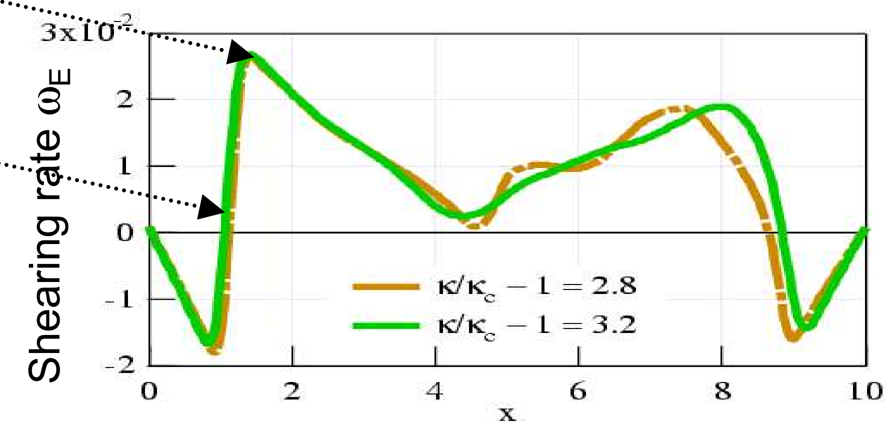
Departure from linear threshold ~ $(\nabla_\perp T/\nabla_\perp T_c)-1$

(analogous to "Dimits graph" with the same code)

- ❑ Transition when $\gamma_{lin} > \omega_E = \langle v_y \rangle'$



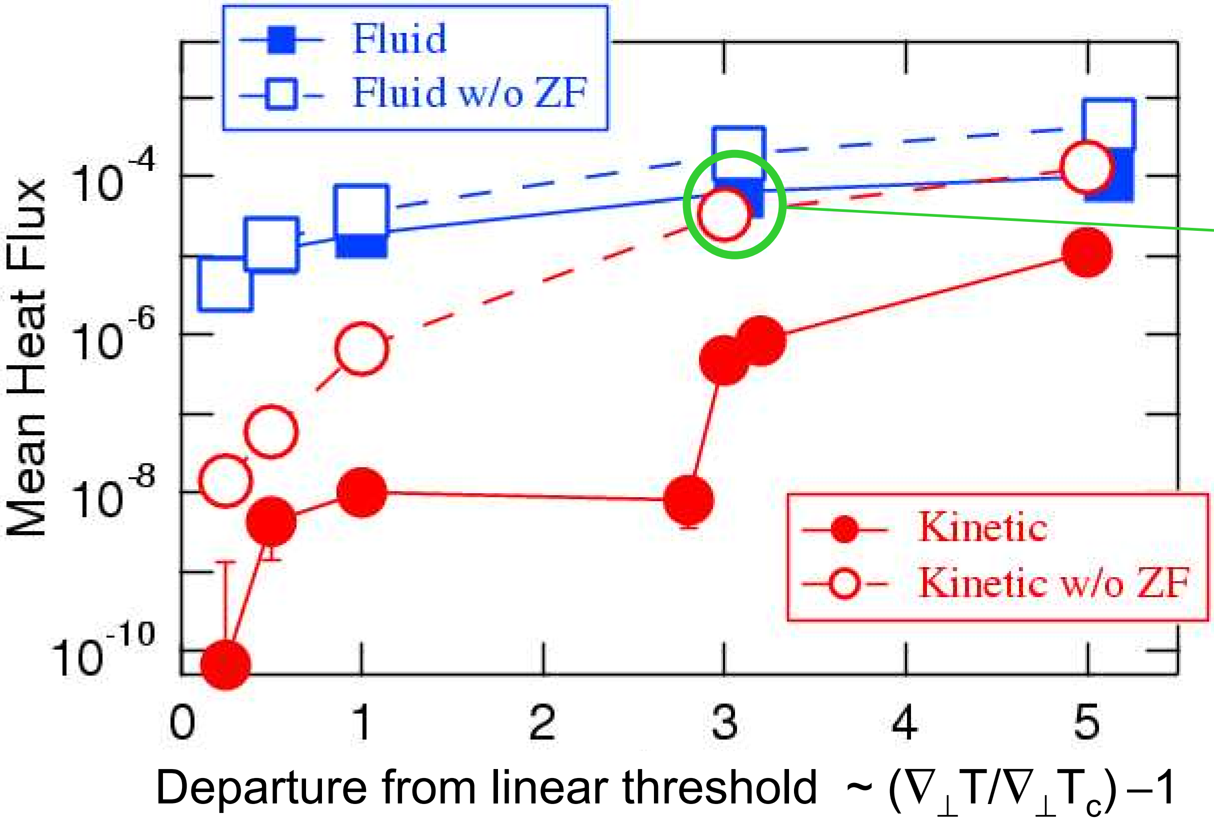
- ❑ ω_E unchanged (level & dynamics)

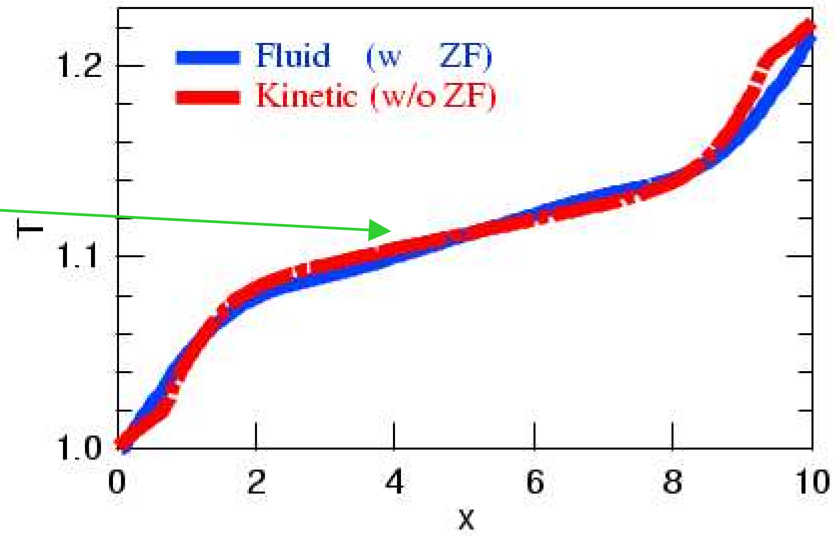


Zonal Flows DO NOT fully explain the discrepancy

When Zonal Flows are artificially suppressed:

- ❑ Turbulent flux increases (in both Kinetics & Fluid)
- ❑ **Discrepancy persists between kinetic & fluid**





Note similar T profiles
for similar fluxes

Quantifying the departure from $F_{Maxwell}$

Projection on the ortho-normal basis of Laguerre polynomials $L_p(\xi)$

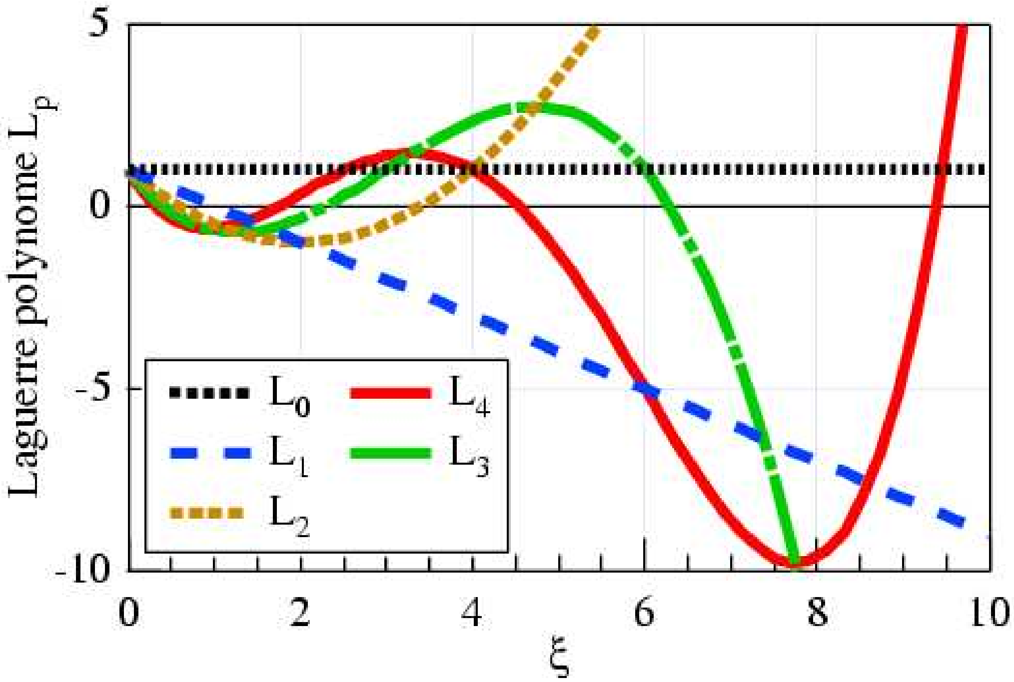
(standard approach for neoclassical transport)

$$f(x,y,E,t) = \sum_{p=0}^{\infty} \hat{f}_p(x,y,t)\, L_p(\xi)\, \mathrm{e}^{-\xi}$$

with

$$\xi \equiv E/T_{eq}$$

$$\hat{f}_p \equiv \int_0^{\infty} L_p f \, \mathrm{d}\xi$$



Correspondence

k^{th} Fluid moment $\leftrightarrow$ coefficients $f_1 \ldots f_k$

$$M_k = \sum_{p=0}^{k} c_p \hat{f}_p$$

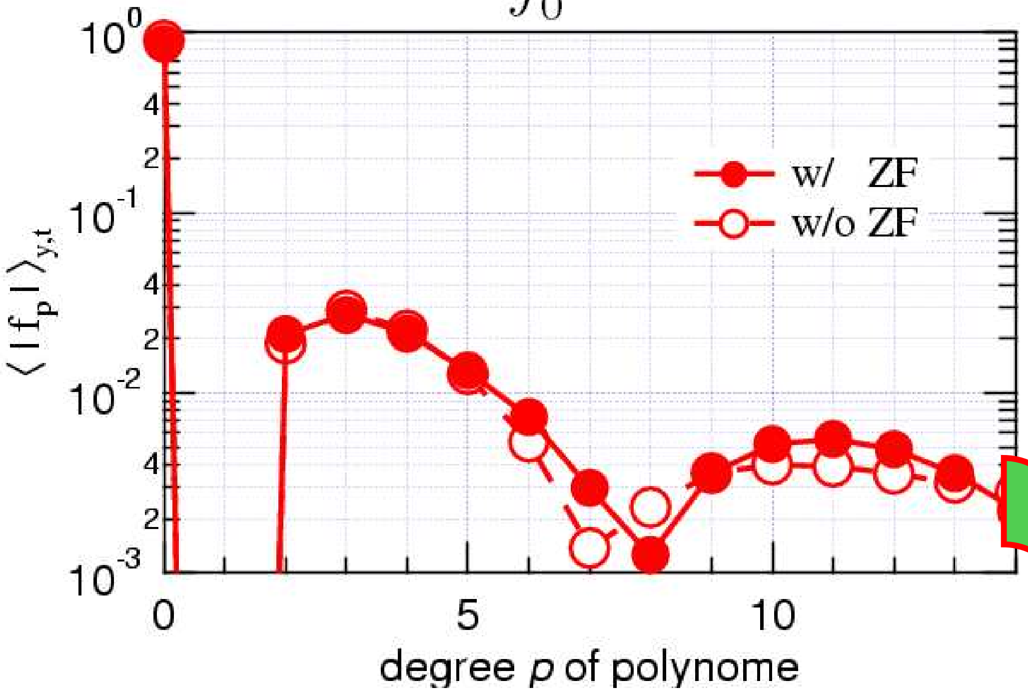
2 fluid moments are not enough

Slow convergence of f_p towards 0

Standard fluid description (couple of moments M_k) would not capture the whole dynamics

Kinetic approach justified

$$\hat{f}_p \equiv \int_0^{\infty} L_p f \, \mathrm{d}\xi$$



May explain why fluid & kinetic results are still different w/o ZF

Alternative closure: entropy production rates

Main ideas:
- $\dot{S}^{QL}$ **governed by QL transport**
- **Closure fulfils 2nd principle**

Fluid closure: $Q = \Upsilon \, (P + P_{eq} \, \sigma \,) \, T$

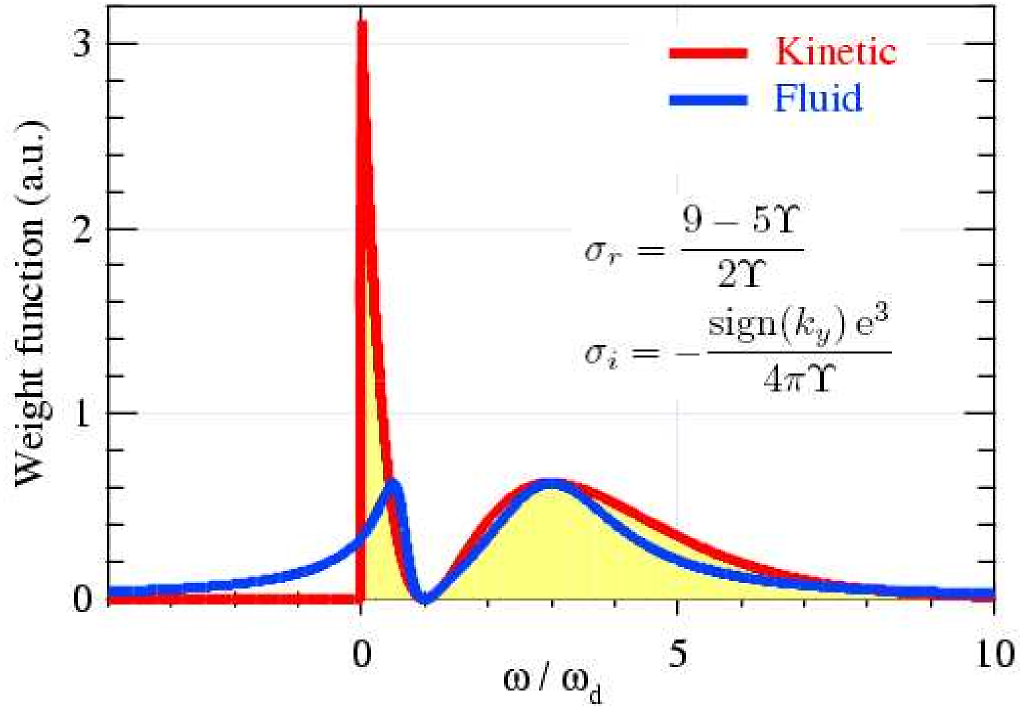
operator: $\sigma_r(k_y) + i \, \sigma_i(k_y)$

$$\frac{\mathrm{d}S^{QL}}{\mathrm{d}t} = \int \mathrm{d}x \, n_{eq} \left(\frac{T'_{eq}}{T_{eq}} \right)^2 \sum_{k,\omega} |k\hat{\phi}_{k,\omega}|^2 \, W^{QL}$$

Weights W^{QL} :

Kinetic: infinity of resonances

Fluid: only 2



3D+1D model for slab ITG

[Grandgirard et al. '04]

***B** constant along z - Limit $k_\perp \rho_i << 1$*

$v_E.\nabla_\perp f$

$$\partial_t f + [\phi, f] + v_\| \partial_z f - \partial_z \phi \, \partial_{v_\|} f = 0$$

Vlasov 4D $(r,\theta,z,v_{//})$

$$\tau(\phi - \langle\phi\rangle) - \frac{n_0{}'}{n_0}\partial_r \phi - \Delta_\perp^2 \phi = \int f \mathrm{d}v_\| - 1$$

Quasi-neutrality

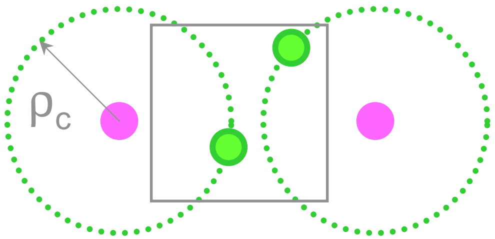
important when $k_\perp L_n \approx 1$

Adiabatic electrons

No particle transport

Fixed particle density profile $n_0(r)$

Gyrokinetic ion response

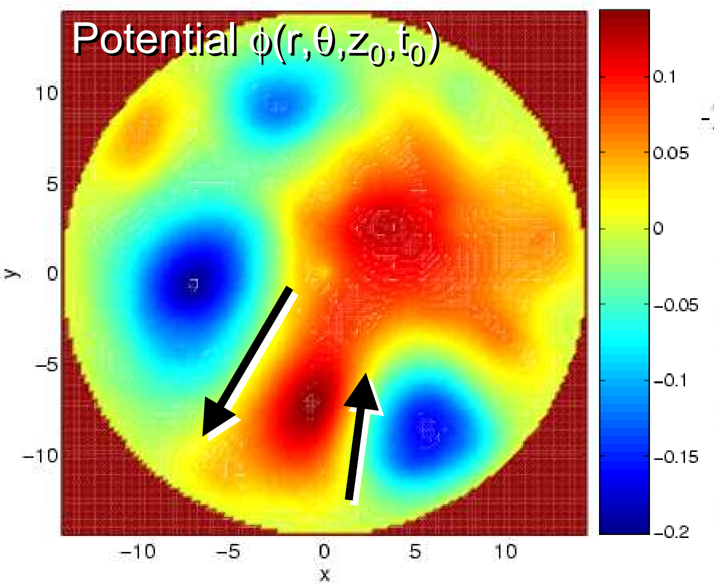


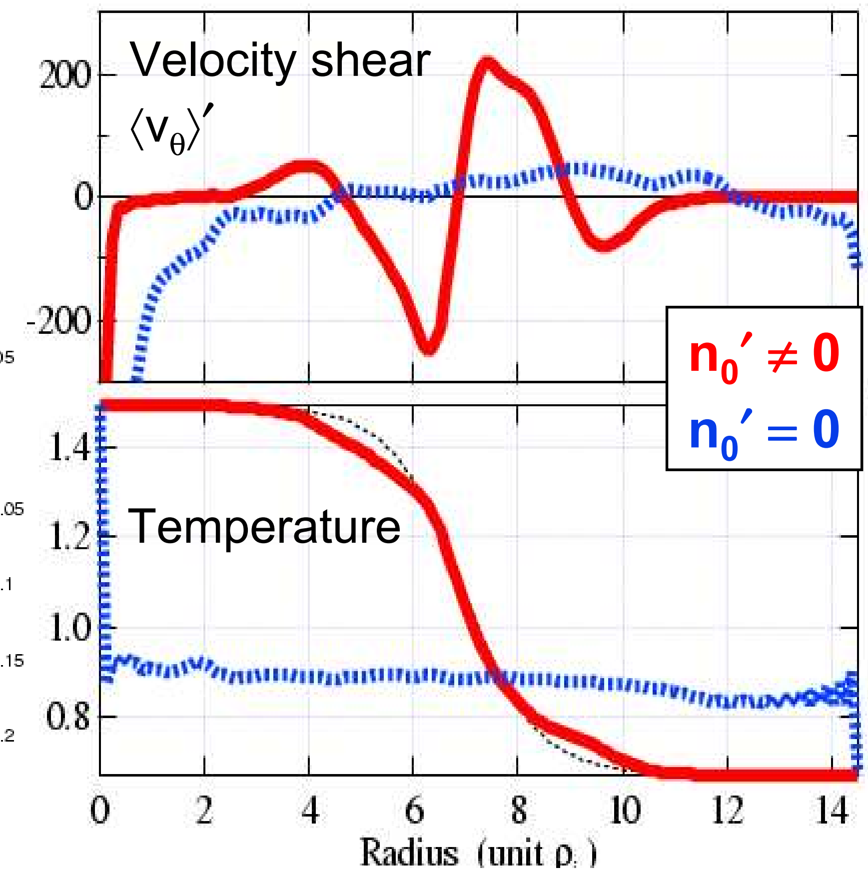
Convective cells governed by n_0'

2 simulations with different density profiles: $n_0' = 0$ & $n_0' \neq 0$

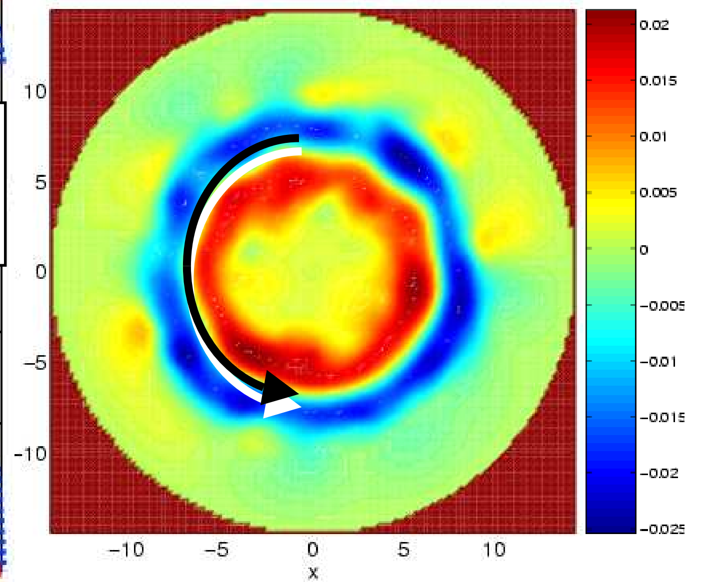
$n_0' \neq 0$ → **Zonal Flows** → **Transport Barrier** sustained

$n_0' = 0 \;\Rightarrow$ Streamers





$n_0' \neq 0 \;\Rightarrow$ Zonal Flows



Relaxation towards marginal stability

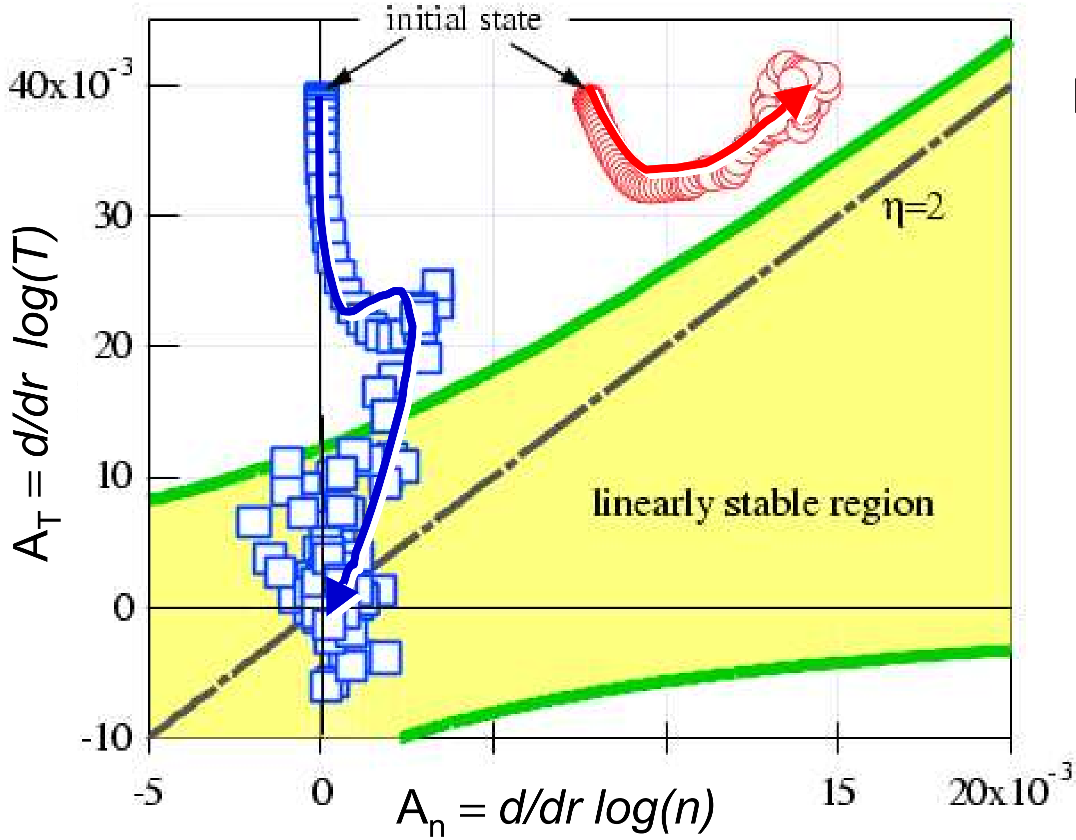
Linear stability threshold $$A_T^{*c} = A_n^* \pm \left\{ A_n^{*2} + 4\left(\frac{k_\parallel}{k_\theta \rho_i}\right)^2 (C^2 - C) \right\}^{1/2}$$

Landau resonance $\omega = k_{//} v_{//}$

Time evolution

at mid radius

[Sarazin '05]



$A_n = d_r\,log(n_G)$ computed with "guiding center density" n_G

$$n_G \equiv n_0 \left\langle \int f \mathrm{d}v_\parallel \right\rangle = n_0 - \frac{1}{r}\frac{\partial}{\partial r}\left(r n_0 \langle v_\theta \rangle\right)$$

Reynolds stress increases with density gradient

$$\frac{\partial}{\partial t}\langle v_\theta \rangle = -\Pi \frac{\mathrm{d}}{\mathrm{d}r}(\log n_0) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \Pi\right)$$

Estimation of the Reynolds stress $\Pi \equiv \langle \tilde{v}_r \tilde{v}_\theta \rangle = -\left\langle \frac{1}{r}\partial_\theta \phi \, \partial_r \phi \right\rangle$

Assumptions: *Fluid limit $\omega >> k_{//} v_{//}$ – Quasi-Linear framework*

$$\Pi = -\frac{\sqrt{3} D_B^2}{4\tau^{1/2}} \sum_{k_\theta, k_{||}} \left| k_\theta \hat{\phi}_k \right|^2 \frac{v_{T0}}{|\omega|} \frac{A_{n0}^2}{A_{T0}}$$

Density Gradient → **Reynolds stress**

$A_{n0} = n_0'/n_0$ → (source of Zonal Flows)

Conclusions

- **Kinetics & fluid descriptions of 2D interchange**

  closure $Q = \Upsilon n T^2$ - diffusive damping - same numerical tool

  – Non linear upshift in kinetics when $\gamma_{lin} > \omega_E$

  – **Fluid transport >> Kinetic transport, even when ZF suppressed**

  – Possible explanation: large number of fluid moments required

- **Alternative non-collisionnal closure** $\rightarrow$ linear OK, non-linear ?

- **Relaxation in non-linear regime depends on n_0' :**

flat n_0	$\Rightarrow$	"streamers"	$\Rightarrow$	$\nabla_\perp T_i \rightarrow 0$
shaped n_0	$\Rightarrow$	Zonal Flows	$\Rightarrow$	Transport Barrier

Density gradient is stabilising:

Linearly (threshold) & **Non-Linearly (ZF)**

Future developments in gyrokinetics

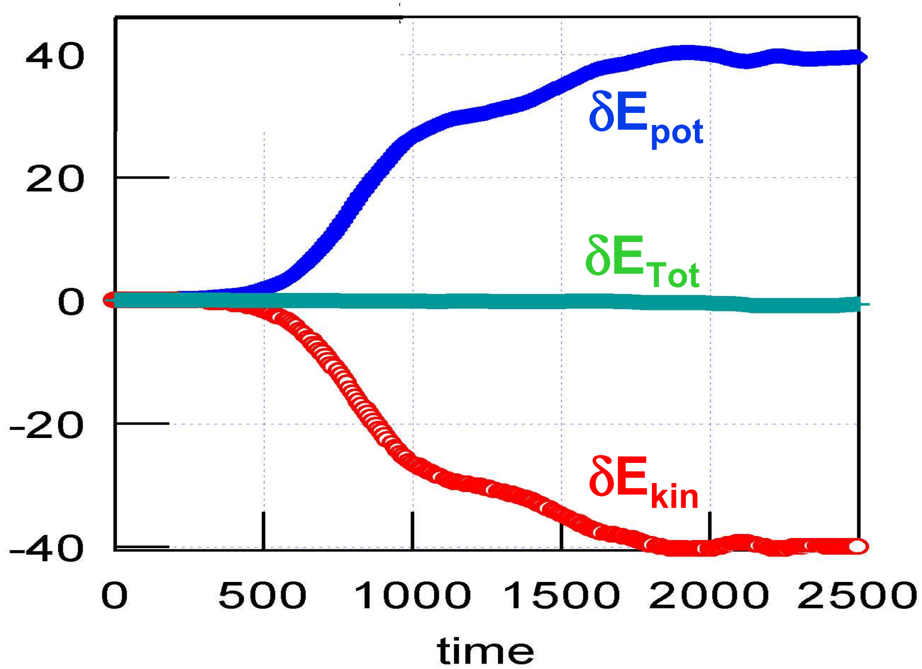
Towards a 5D gyrokinetic code for ITG turbulence

- Rotational transform → OK
- Gyro-average operator → Padé approximation
- Adiabatic invariant μ (4D→5D) → ongoing work
- Curvature drift

Good conservation properties of Semi-Lagrangian scheme:

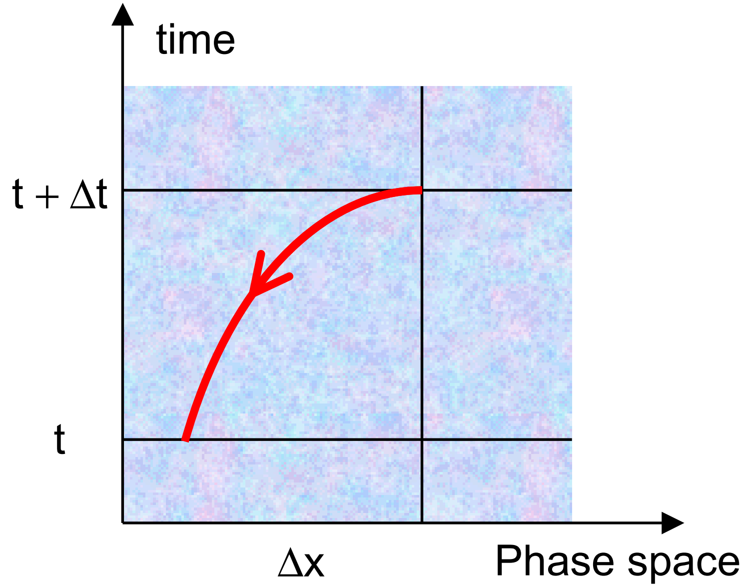
$$\delta E_{Tot} \equiv \delta E_{kin} + \delta E_{pot} \approx \mathbf{0}$$

Relative error ≈ 1%



Semi-Lagrangian numerical scheme

- **Semi-Lagrangian scheme:**
  Fixed grid in phase space
  Follow the characteristics backwards
  in time



- **Total distribution function F**
- **Global code**, F=Cst at boundaries

- **Damping at radial ends** to prevent numerical instabilities at boundaries

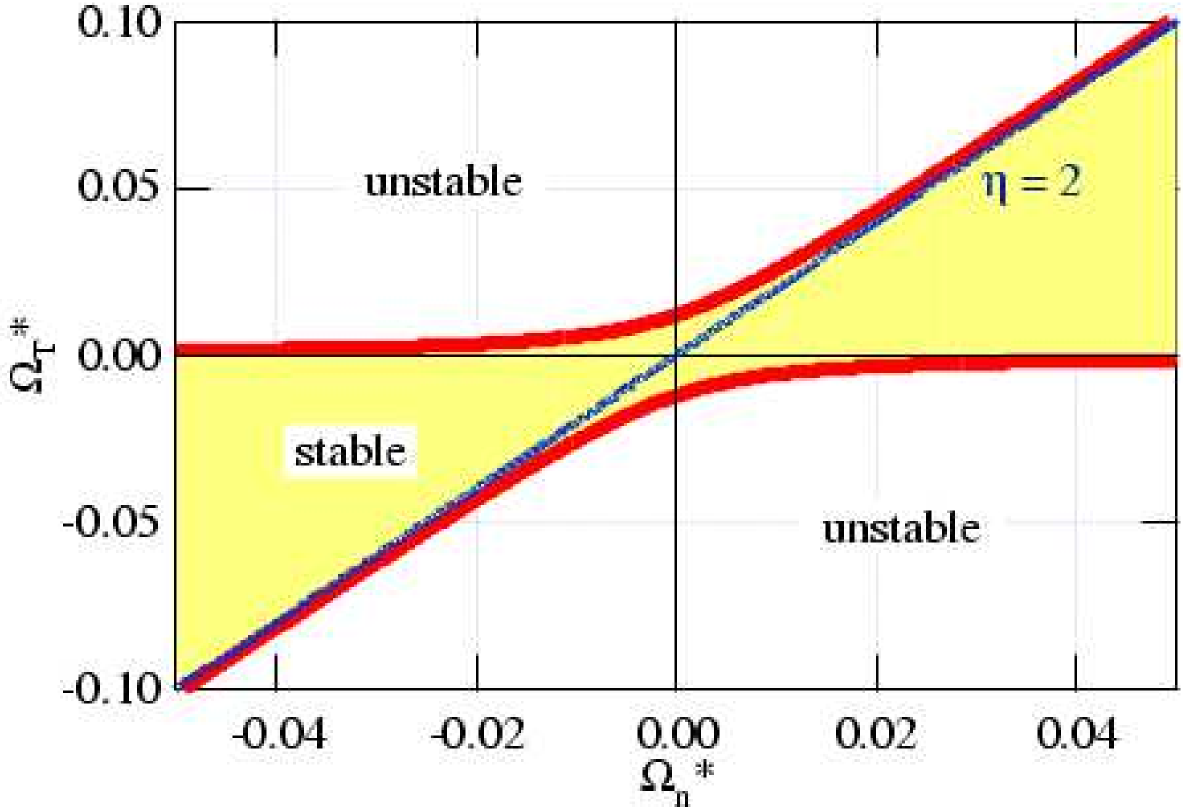
- **Good conservation properties** (*e.g.* Error on energy < 1%)

[Grandgirard et al. 2004]

Linear stability threshold

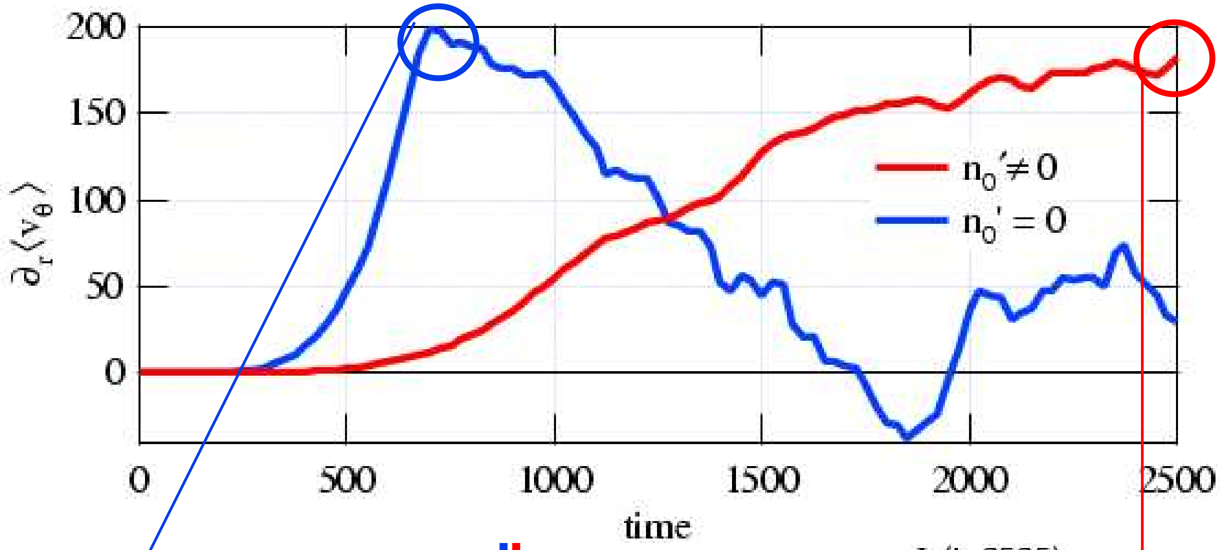
$$\Omega_T^{*c} = \Omega_n^* \pm \sqrt{\Omega_n^{*2} + 4\omega_\parallel^2 (C^2 - C)}$$

$k_\theta v_{th}^2 \, d_r \log(T)$ $k_\theta v_{th}^2 \, d_r \log(n)$ $k_{//} v_{th}$ $1 + \tau + k_\perp \rho_i$

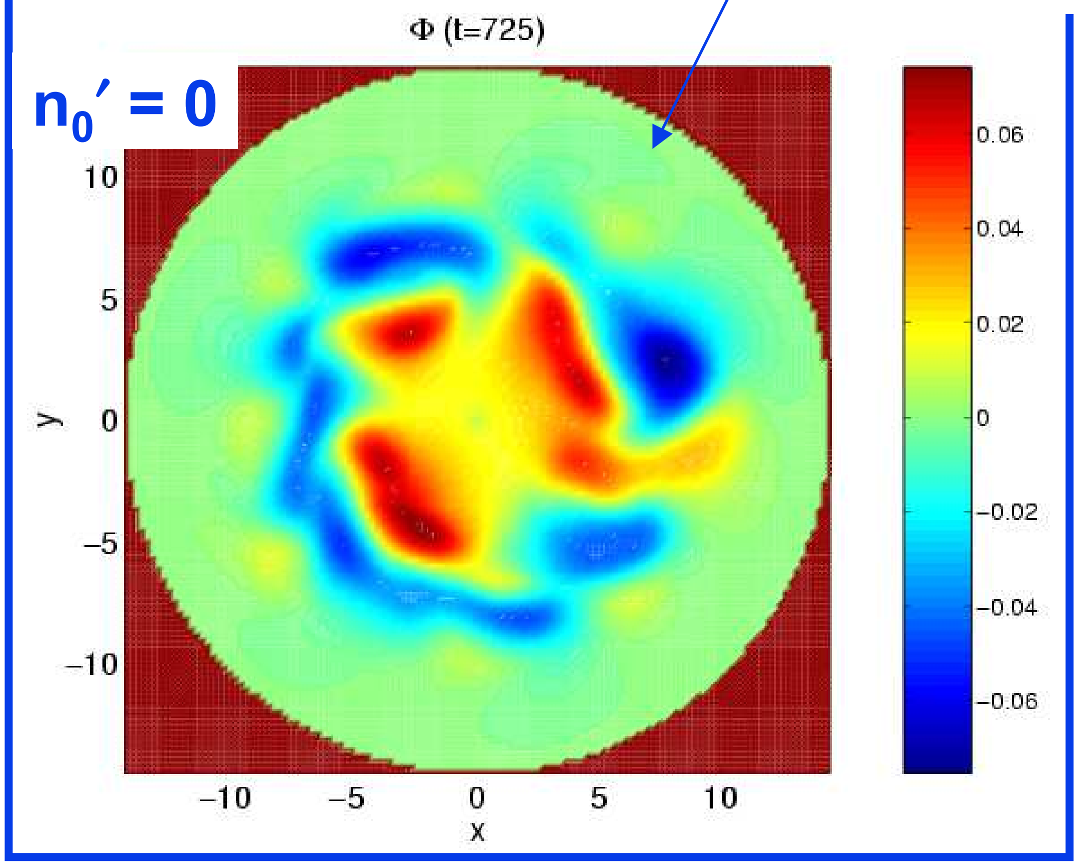


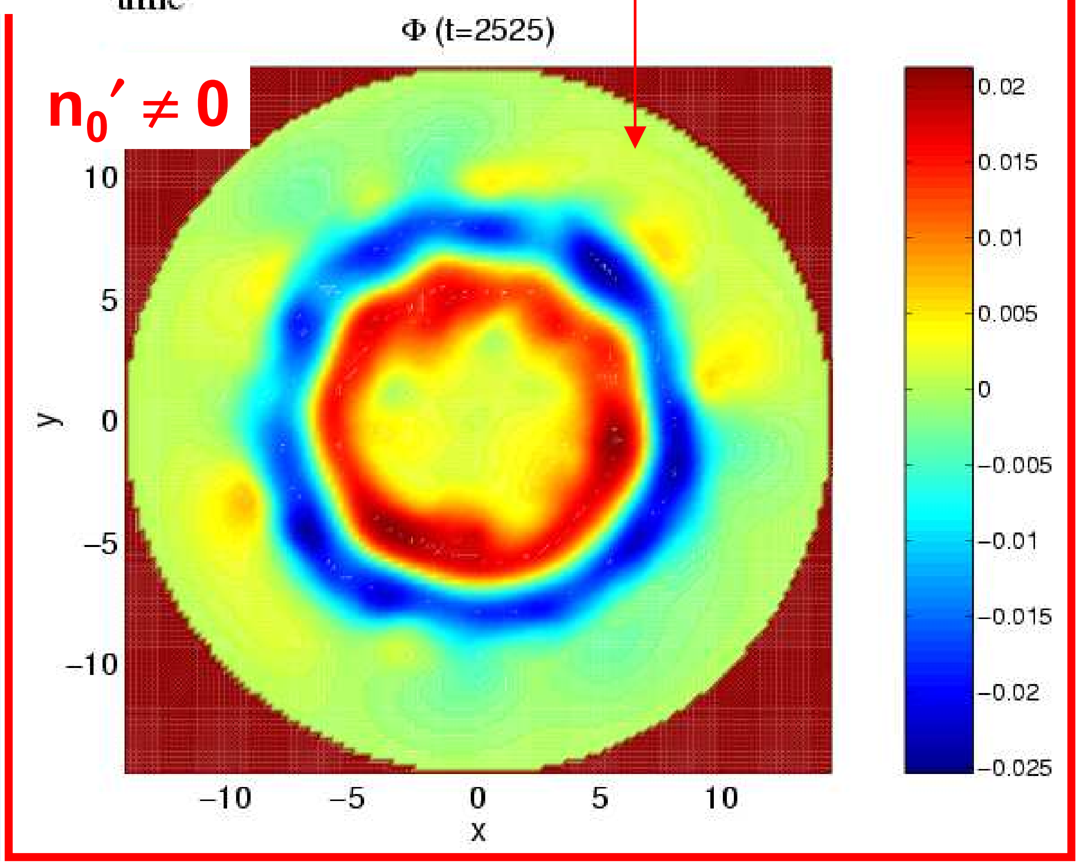
- "Slab" instability: Landau resonance $\omega = k_{//} v_{//}$
- 2 unstable regions
- **Density gradient linearly stabilising**

Velocity shear



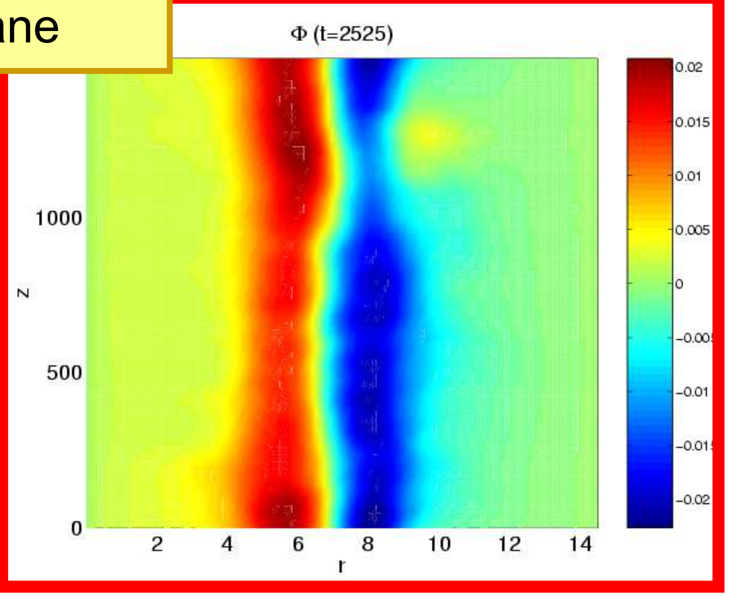
Electric potential

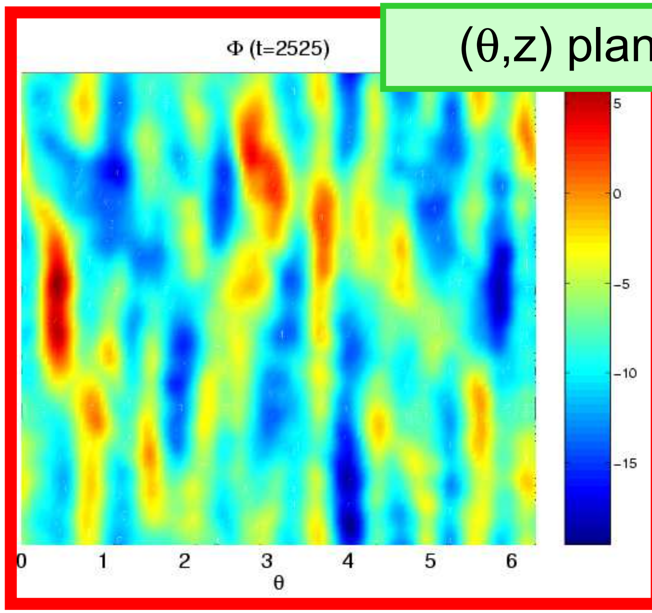




2D maps of the potential

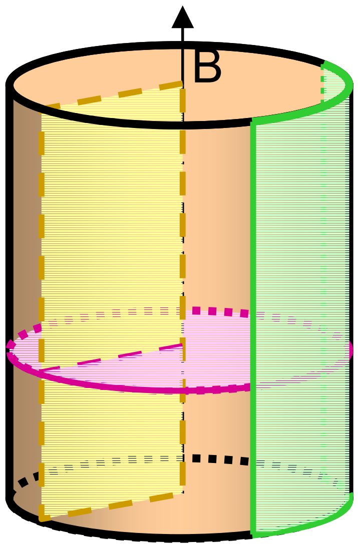
(r,z) plane



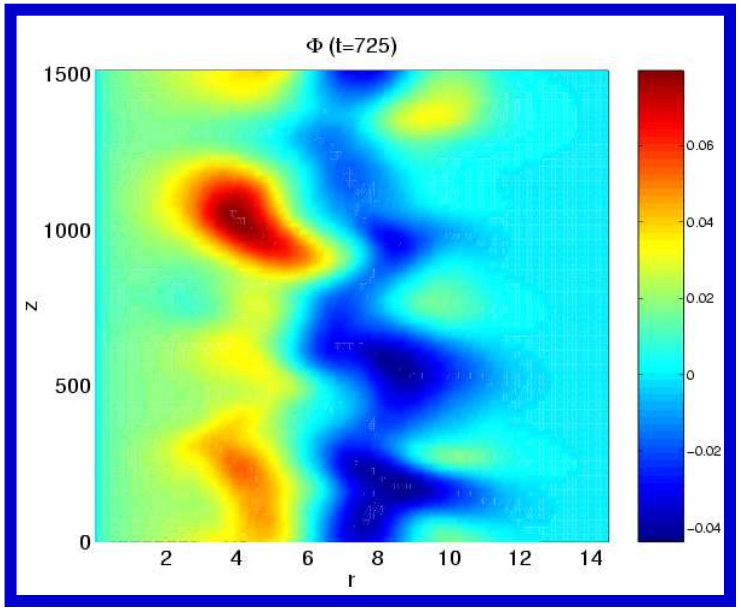


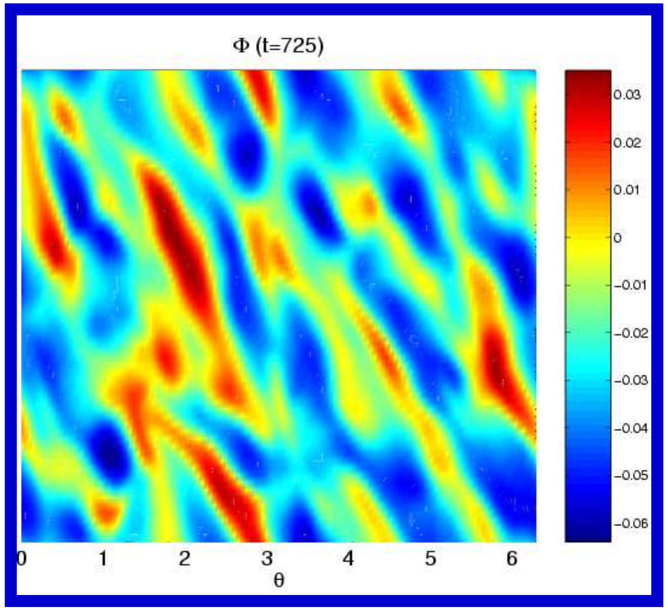
(θ,z) plane

$n_0' \neq 0$



$n_0' = 0$





Alternative closure: entropy production rates

Main ideas:
- **S^{QL} governed by QL transport**
- **Closure fulfils 2nd principle**

Fluid closure: $Q = \Upsilon\,(\,P \;+ P_{eq}\,\sigma\,)\,T$

↳ **operator:** $\sigma_r(k_y) + i\,\sigma_i(k_y)$

$$\frac{dS^{QL}}{dt} = \int dx\, n_{eq} \left(\frac{T'_{eq}}{T_{eq}}\right)^2 \sum_{k,\omega} |k\hat{\phi}_{k,\omega}|^2\, W^{QL}$$

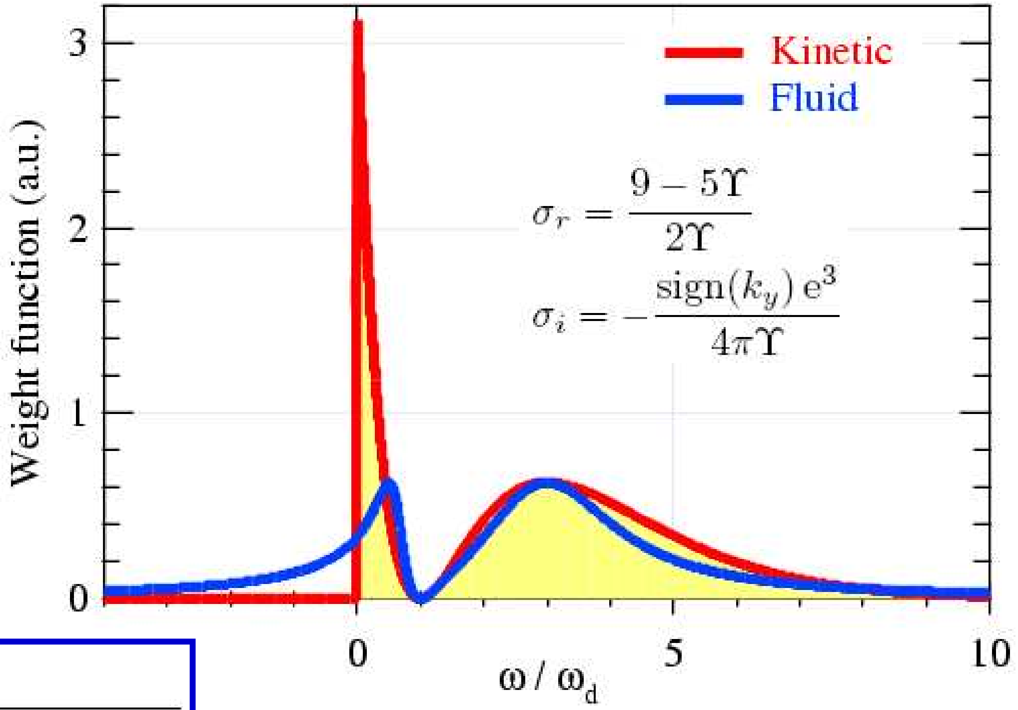
Weights W^{QL} :

Kinetic: infinity of resonances

Fluid: only 2

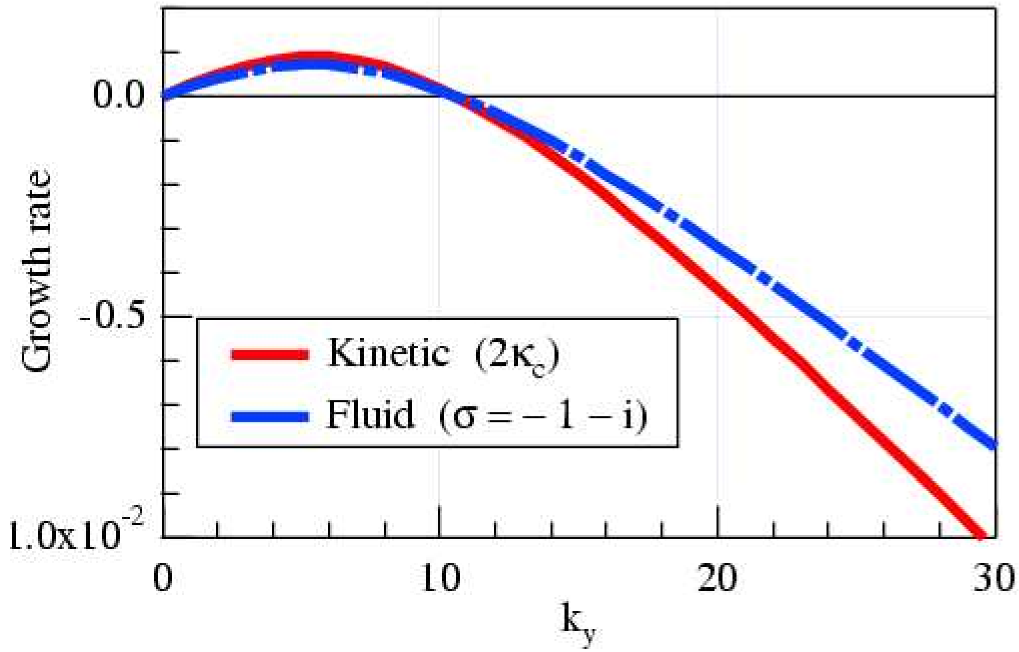
$$W^{QL}_{kin} = \frac{\pi}{|\omega_d|}\left(\frac{\omega}{\omega_d} - 1\right)^2 \exp\left(-\frac{\omega}{\omega_d}\right)$$

$$W^{QL}_{fl} = \frac{-\Upsilon\omega_d\sigma_i(\omega-\omega_d)^2}{[\omega^2 - \Upsilon(\sigma_r+2)\omega_d\omega + \Upsilon(\sigma_r+1)\omega_d^2]^2 + [\Upsilon\omega_d\sigma_i(\omega-\omega_d)]^2}$$



Similar linear behaviour w/o ad-hoc dissipation

- Same threshold **as in kinetic**: $\Omega^*{}_{Tc}{}^{fl} = k_y v_d(1+k_\perp{}^2) = \Omega^*{}_{Tc}{}^{kin}$
- Stability of small scales: **implies** $\sigma_i / k_y < 0$



Similar linear spectra → what about non linear behaviour ?