Theory of shear stabilization

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- Simple rule for the L-H transition

$$\underline{\Omega} > \underline{\Delta\omega}(r_L)$$

shearing rate — turb. decor. rate

→ Ω : stabilize linear instability

Reduce transport & turb. level

⇒ How much reduction by Ω ?

- Boedo et al '02

$$\Gamma \propto \Omega^{-\alpha} \qquad (2 \lesssim \alpha \lesssim 3.6)$$

- Terry et al '01

$$(\partial_t + \vec{u}\cdot\vec{\nabla})\, n = D\,\nabla^2 n$$

with $\vec{u} = \vec{V}_{turb} + \vec{u}_0$

$$\Rightarrow \langle n'\vec{u}\rangle \propto (u_0')^{-4}, \quad \cos\delta \propto |u_0'|^{-3}$$

$$\langle n'\vec{u}\rangle = \sqrt{\langle n'^2\rangle}\sqrt{\langle u'^2\rangle}\,\cos\delta$$
$$= -D_{tur}\vec{\nabla}\langle n\rangle$$

- Can passive scalar field model capture turbulence in real tokamaks?
- Is there a universal scaling of flux with Ω?
- Is a cross-phase ($\cos\delta$) a dominant suppression mechanism?
- Reduction of flux by mean $\vec{E}\times\vec{B}$ (Ω) and zonal flows ($\Omega_{rms} = \sqrt{\langle V_\theta'^2 \rangle}$)
- Effect of shear flow on intermittent transport carried by a coherent structure?

Outline

- Mean vs random shearing
- Turbulent transport
  - Passive scalar model
  - Interchange turbulence
- Intermittent transport

II. Coherent vs Random Shearing

$$k_x(t) = k_x(0) + k_y \int^t \Omega(t')dt' \quad [\mathbf{U} = -x\Omega(t)\hat{y}]$$

1. **Coherent shearing with constant Ω ($k_x^2 \propto t^2$)**

$$\Rightarrow \quad D\int^t dt' k_x^2(t') \propto D k_y^2 \Omega^2 t^3$$

$$\Rightarrow \quad \tau_\Delta = (\tau_\eta/\Omega^2)^{1/3} \quad [\tau_\eta = 1/Dk_y^2]$$

2. **Random shearing with τ_{ZF} ($k_x^2 \propto t$)**

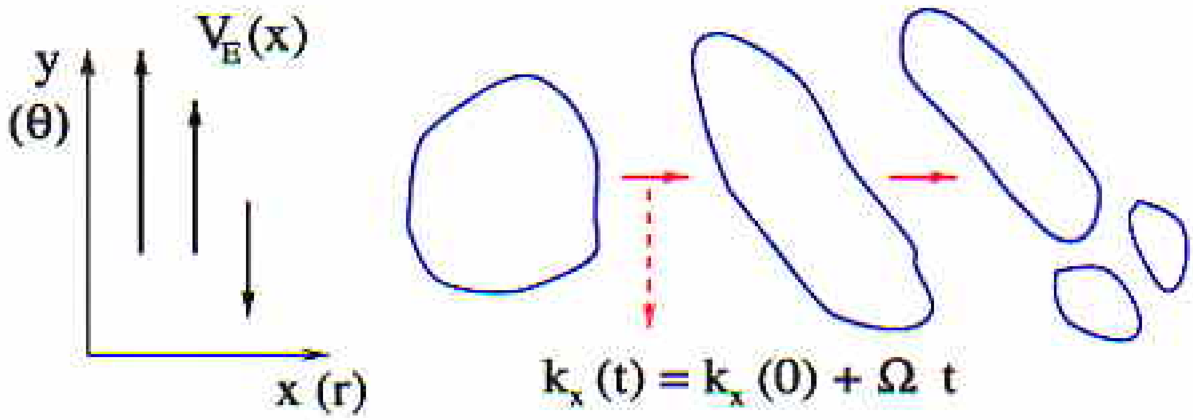
$$\Rightarrow \quad D\int^t dt' k_x^2(t') \propto D k_y^2 \tau_{ZF} \Omega_{rms}^2 t^2$$

$$\Rightarrow \quad \tau_D = (\tau_\eta/\tau_{ZF}\Omega_{rms}^2)^{1/2} = (\tau_\eta/\Omega_{eff})^{1/2}$$

- **For** $\Omega = \Omega_{rms}$, $\tau_\Delta \le \tau_D$
- **If** $\tau_{ZF} < \Omega_{rms}^{-1}$, $\Omega_{eff} = \tau_{ZF}\Omega_{rms}^2 < \Omega_{rms}$
- **For** $\tau_{ZF} \gg \tau_D$, $\Omega(t) \sim \text{const} \to \tau_D = \tau_\Delta$

Shear decorrelation

Mean $\mathbf{E}\times\mathbf{B}$ flow $\langle V_E \rangle$ | **Zonal flow $\tilde{V}_E$**



- **Mean flow (coherent shearing):**

$$\langle V_E \rangle = \langle V_\theta \rangle - \frac{B_\theta}{B}\langle V_\phi \rangle - \frac{1}{eB_z n}\frac{\partial p_i}{\partial r} + \tau$$

- **Zonal flows (random shearing):**

$$\partial_t \phi_{ZF} = \langle \tilde{v}_x \tilde{v}_y \rangle - \nu \phi_{ZF}$$

III. Turbulent transport

[Kim & Diamond '03;'04]

1. Passive scalar field n

$$(\partial_t + \mathbf{u}\cdot\nabla)n = D\nabla^2 n$$

- Quasi-linear analysis with

$\Rightarrow$ $\tau_\Delta^{-1} \gg \tau_\eta^{-1}$ as $\Omega \to \infty$ (BDT '90)

$$\mathbf{u} = \mathbf{U} + \mathbf{v}, \quad n = n_0(x) + n'$$

  - $\mathbf{v}$: Given (prescribed) turbulent flow
  - $U(x,t) = -x\Omega(t)$ [mean or zonal flows]

- Solve for fluctuation

$\Rightarrow$ $\tau_0^{-1} \gg \tau_\eta^{-1}$ as $\Omega_{rms} \to \infty$

$$(\partial_t - x\Omega\partial_y)n' = -v_x\partial_x n_0 + D\nabla^2 n'$$

- Compute $\langle n'^2\rangle, \langle n' v_x\rangle = -D_T^{xx}\partial_x n_0$ (D_T is the turbulent diffusivity)

NOTE : $\Omega = 0 \Rightarrow D_T^{xx} \sim v l$

Let

$$n'(\mathbf{x}, t) = \frac{1}{(2\pi)^3} \int d^3k \tilde{n}(\mathbf{k}, t) e^{i(k_x(t)x + k_y y + k_z z)}$$

where

$$k_x(t) = k_x(0) + k_y \int^t dt_1 \Omega(t_1)$$

and similarly for $\mathbf{v}$

- Consider $(\tau_D, \tau_\Delta) \gg (\tau_c, \tau_\Omega)$
- Flux $\Gamma = \langle n' v_x \rangle = \sum_{\mathbf{k}} |n'(\mathbf{k})||v_x(-\mathbf{k})| \cos \delta_{\mathbf{k}}$

Time scales

- τ_{ZF}: correlation time of zonal flows
- τ_c: correlation time of turbulent flow $\mathbf{v}$
- $\tau_\Omega = \Omega^{-1}, \Omega_{rms}^{-1}$: shearing time scale
- τ_Δ, τ_D: decorrelation time due to coherent and random shearing

- **For mean flow or zonal flow with** $\tau_D \ll \tau_{ZF}$ $(\Omega_{rms} \sim \Omega)$

	$\tau_c < \tau_\Omega$	$\tau_\Omega < \tau_c$
$\langle n' v_x \rangle$	Ω^0	Ω^{-1}
$\langle n'^2 \rangle$	$\tau_\Delta \propto \Omega^{-1}$	$\tau_\Delta \Omega^{-1} \propto \Omega^{-5/3} D^{-1/3}$

- **For zonal flow with** $\tau_c < \tau_{ZF} \ll \tau_D$ **and Gaussian PDFs:**

	$\tau_c < \tau_\Omega$	$\tau_\Omega < \tau_c$
$\langle\langle n' v_x \rangle\rangle$	Ω^0_{rms}	Ω^{-1}_{rms}
$\langle\langle n'^2 \rangle\rangle$	$\tau_D \propto \Omega^{-1}_{rms}$	$\tau_D \Omega^{-1}_{rms} \propto \Omega^{-2}_{rms} D^{-1/2}$

Note: $\langle\langle n'^2 \rangle\rangle / \langle n'^2 \rangle = \tau_D / \tau_\Delta > 1$

Conclusions from passive scalar fields

- Flux ($\Gamma \propto \Omega^{-1}$ or Ω_{rms}^{-1}) is weakly reduced

- Cross phase $\cos\delta$ ($\propto \Omega^{-1/6}$) is very weakly reduced [cf Terry et al '01: $\Gamma \propto \Omega^{-4}, \cos\delta \propto \Omega^{-3}$]

- Effect of random shearing of zonal flows on transport and fluctuation levels depends on correlation time τ_{ZF}

- $\langle\langle n'^2 \rangle\rangle_{ZF} \propto \tau_D \Omega_{rms}^{-1} \propto \Omega_{rms}^{-2} (> \langle n'^2 \rangle \propto \tau_\Delta \Omega^{-1} \propto \Omega^{-5/3})$ is due to LONGER decorrelation time ($\tau_D > \tau_\Delta$) induced by finite τ_{ZF}

- Exact scaling with Ω or Ω_{rms} depends on the property of given turbulent flow

- Limitation of scalar field model: turbulent flow is arbitrary GIVEN (i.e., No shearing effect on turbulent flow)

$\Rightarrow$ Scalings of Γ and Q in a self-consistent model (Effect of Ω on $|v_x|$)?

3. Particle transport in interchange turbulence

[Kim, Diamond, & Hahm '04; Kim '05]

$$(\partial_t + U\partial_x)n = -v_x \partial_x N_0 + D\nabla^2 n + S$$

$$(\partial_t + U\partial_x)\omega = -g\frac{\partial_y n}{N_0} + \nu\nabla^2\omega$$

where g is effective gravity; S is noise

- $U = -x\Omega(t)$
- $\omega\hat{z} = \nabla \times \mathbf{v}$
- $D = \nu$
- Total noise $f = S - v_x\partial_x N_0 + ...$ (corr. time τ_f)
- Consider $(\tau_D, \tau_\Delta) \gg (\tau_f, \tau_\Omega)$

Time scales

- τ_f: correlation time of total noise f
- $\tau_\Omega = \Omega^{-1}, \Omega_{rms}^{-1}$: shearing time scale
- τ_Δ, τ_D: decorrelation time due to coherent and random shearing

	$\tau_D \ll \tau_{ZF}$		$\tau_D \gg \tau_{ZF}$
	$\tau_f < \tau_\Omega$	$\tau_f > \tau_\Omega$	$\tau_f < \tau_\Omega$
$\langle\langle n v_x \rangle\rangle$	$\Omega^{-2} \ln(\tau_\Delta \Omega)$	$\Omega^{-3} \ln \Omega$	$\tau_D \Omega_{eff}^{-1} \propto \Omega_{rms}^{-3}$
$\langle\langle n^2 \rangle\rangle$	$\tau_\Delta \propto \Omega^{-2/3}$	$\Omega^{-5/3}$	$\tau_D \propto \Omega_{rms}^{-1}$
$\langle\langle v_x^2 \rangle\rangle$	Ω^{-3}	Ω^{-4}	$\tau_D \Omega_{eff}^{-2} \propto \Omega_{rms}^{-5}$
$\langle\langle v_x v_y \rangle\rangle$	$-\Omega^{-3} \ln \Omega$	$-\Omega^{-4} \ln \Omega$	

- **Strong reduction in the flux due to severe reduction in** $\langle\langle v_x^2 \rangle\rangle$
- **Reduction in cross-phase is very weak (**$\propto \Omega^{-1/6} \ln \Omega$**)**

  [Falchetto and Ottaviani, '04]
- **Reynolds stress is reduced by shearing**
- **Significant reduction by random shearing by zonal flows**

IV. Intermittent Transport [Kim '05]

Passive scalar field n

1. Coherent structure $U_s(y)\hat{x} = |U_s|\cos(p_y y + \omega_s t)\hat{x}$

$$[\partial_t + U_s(y)\partial_y]n = D\nabla^2 n$$

$\Rightarrow n = n_0(x) + n_s(y)$

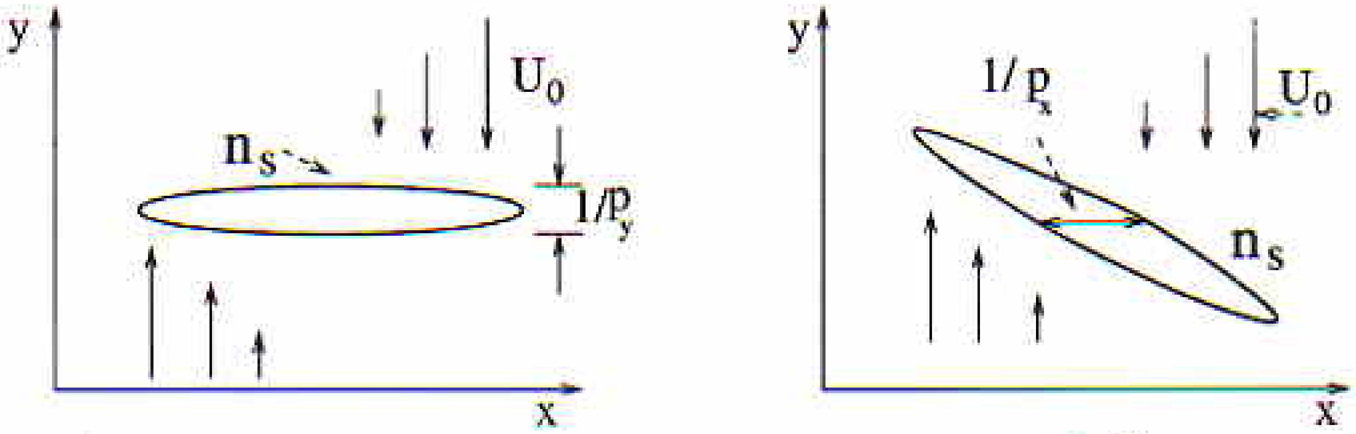
$\Rightarrow \langle n_s U_s \rangle$ gives $D_{eff} = DU_s^2 p_y^2/[\omega_s^2 + (Dp_y)^2]$ [Zeldovich '82]

2. Coherent structure (n_s, U_s) + turbulence + mean shear flow $U_0(x)\hat{y} = -x\Omega\hat{y}$

- **Turbulence:** $D \to D_T$
- **Shearing by Ω:**

$$D_T \propto \Omega^{-1}, \langle n_s U_s \rangle \propto D_T \Omega^{-2}$$

$$\Rightarrow \boxed{D_{eff} \propto \Omega^{-3}}$$



t=0

(a)

t≠0

(b)

VI. Conclusions

- Zonal flows trigger L-H transition while mean flows maintain H-mode after the transition

- Model dependent reduction in the flux and turbulence amplitude $\rightarrow$ Stronger reduction in interchange turbulence due to the suppression of velocity amplitude

- In all cases, cross phase $\cos\delta$ is very weakly reduced

- Effect of random shearing of zonal flows on transport and fluctuation levels depends on correlation time τ_{ZF}

- Random shearing can lead to significant reduction in interchange turbulence (larger transport as compared to coherent shearing)

- Significant reduction in intermittent transport

- Determination of τ_{ZF} and study on transport vs τ_{ZF}
- Effects of flow shear on blobs
- Effects of magnetic shear, toroidal geometry
- Generation and effects of zonal fields
- PDFs for intermittent transport (Kim et al '02;'03) $\Rightarrow$ interaction among coherent structures, PDFs of L-H transition
- Incorporation of spatial information: pedestal, front propagation