Intermittency like phenomena in Plasma Turbulence

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Outline

- Turbulence: (hydrodynamic fluid and plasma turbulence)
- Concept of Intermittency
- Intermittency in hydrodynamic fluids: Observations, phenomenological models, exact results, statistical results ~ the current status!!
- “Intermittency like” phenomena in plasmas: fluid models, notion of waves and wave- wave interactions, problem of natural scales , example of coherent structures with wave trapping
- Conclusions

Turbulence

Seemingly erratic flow interspersed with patterns of various sizes.

Presence of structures linked with the concept of intermittency.

Spatial and Temporal Randomness

Turbulence: A frontier research problem in theoretical physics

- State with many scales.
- All scales are important (right from energy injection to energy dissipation scales!).
- Even intermediate scales influence the dynamics.
- Not been solved as yet, in spite of the efforts made for the last century or so!!

Search for a description …

Random phenomena: Hence a statistical description, which involves exact specification of PDF's, moments, spatial and temporal correlation functions.

Expectation : Sum of a large number of independent random variables follows a Gaussian distribution

$$\phi(x) = \int a(k) exp(ikx + i\beta_k) dk$$

β_k : random phases $[0,2\pi]$ of the Fourier modes

Observation : PDF's are significantly non Gaussian

→ Presence of phase correlations and some amount of order!!

Intermittency
(A first glimpse)

Qualitatively:

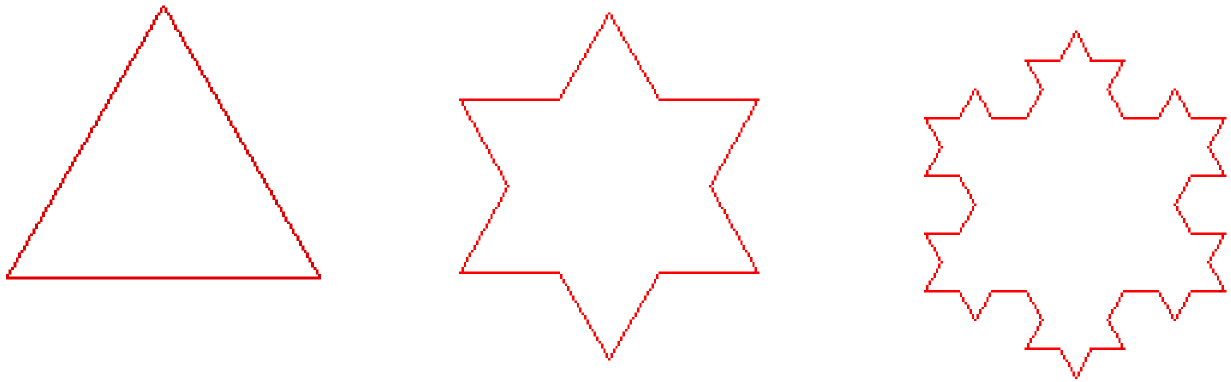
- Intermixing of randomness and coherence in the turbulent state.
- Deviations from Gaussian statistics.

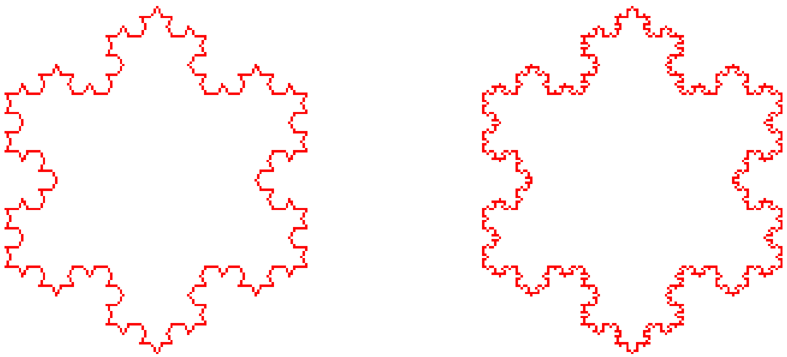
Problem of many scales: successful approach in Physics

Self similarity of scales.

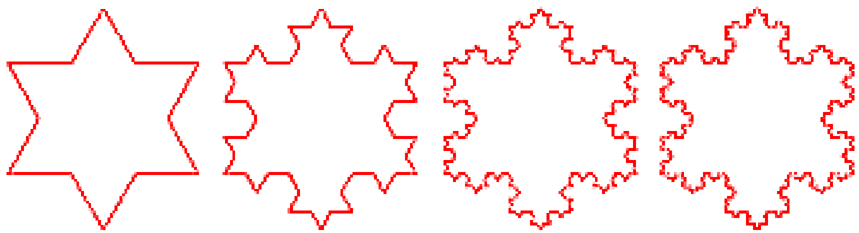
Rules connecting various scales can be constructed.

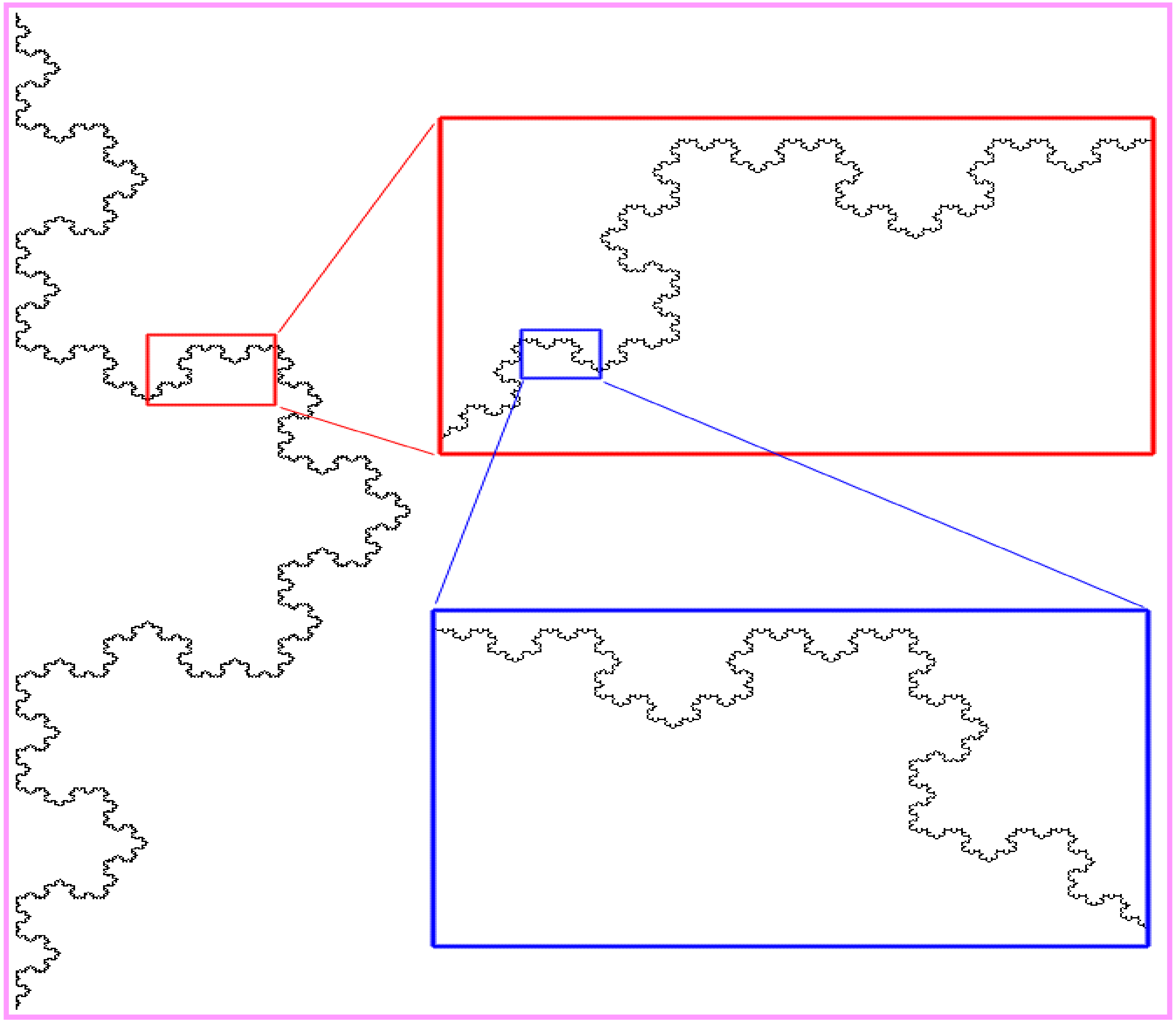
Successful in the context of phase transitions and critical phenomena



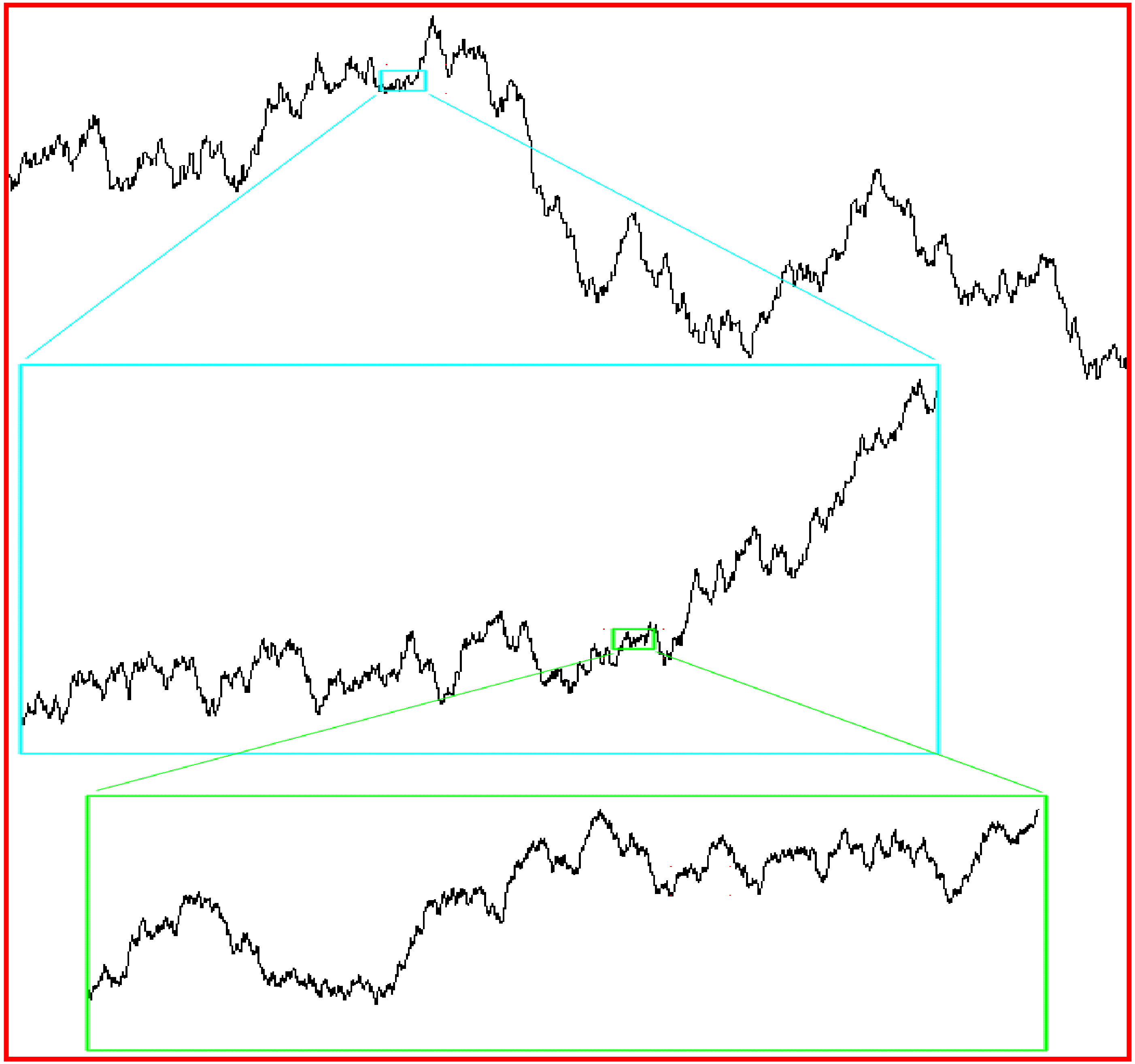


Simple construction of complex self similar structures!!





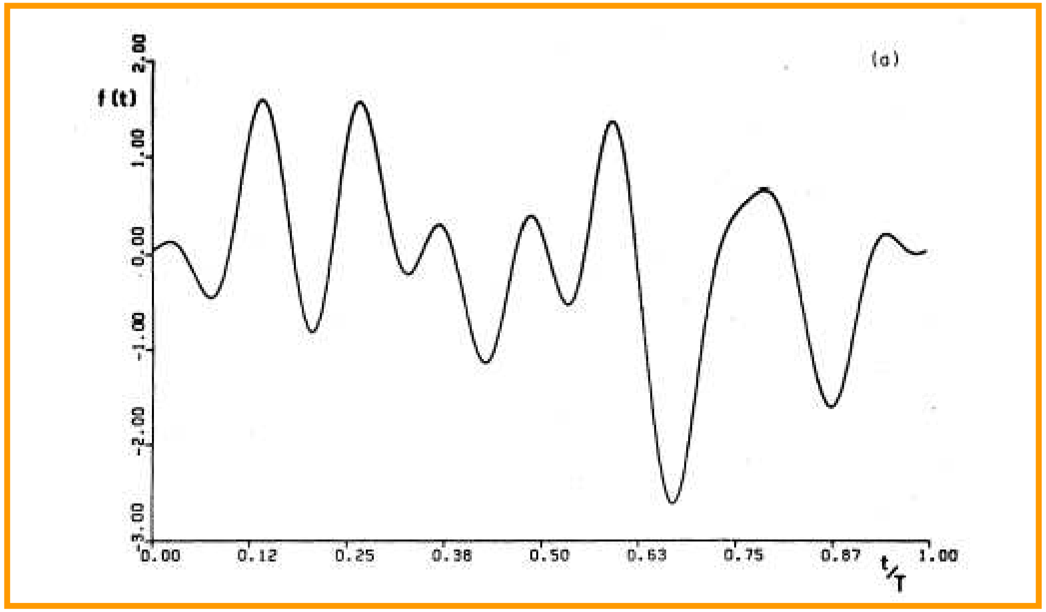
Natural self-similarity : Gaussian Noise

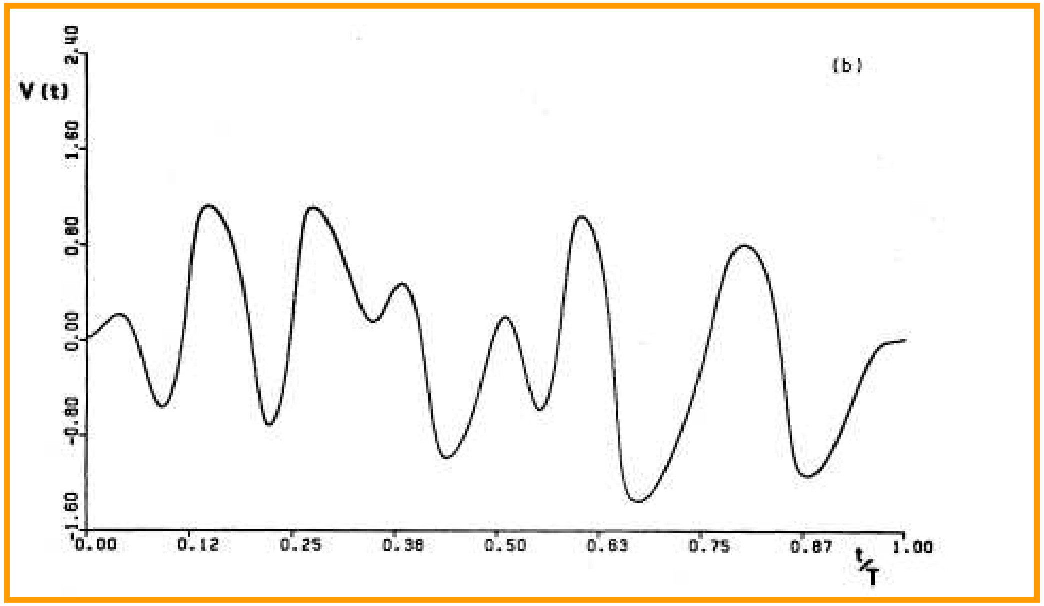


Many scales still …

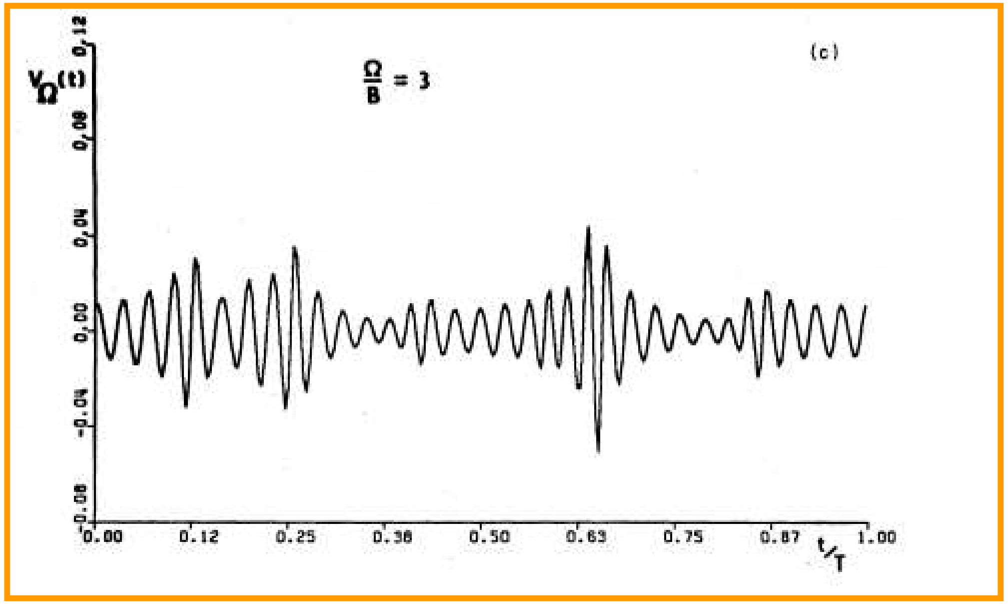
- Simple mathematical rules connect them.
- No special scale. (Scale invariant system)
- Using such a simple concept of scale similarity Kenneth Wilson developed the **Renormalization Group Theory** which successfully interpreted the scaling exponent in the phenomena of phase transitions.
- Problem with turbulence : Self similarity doesn't quite work…..

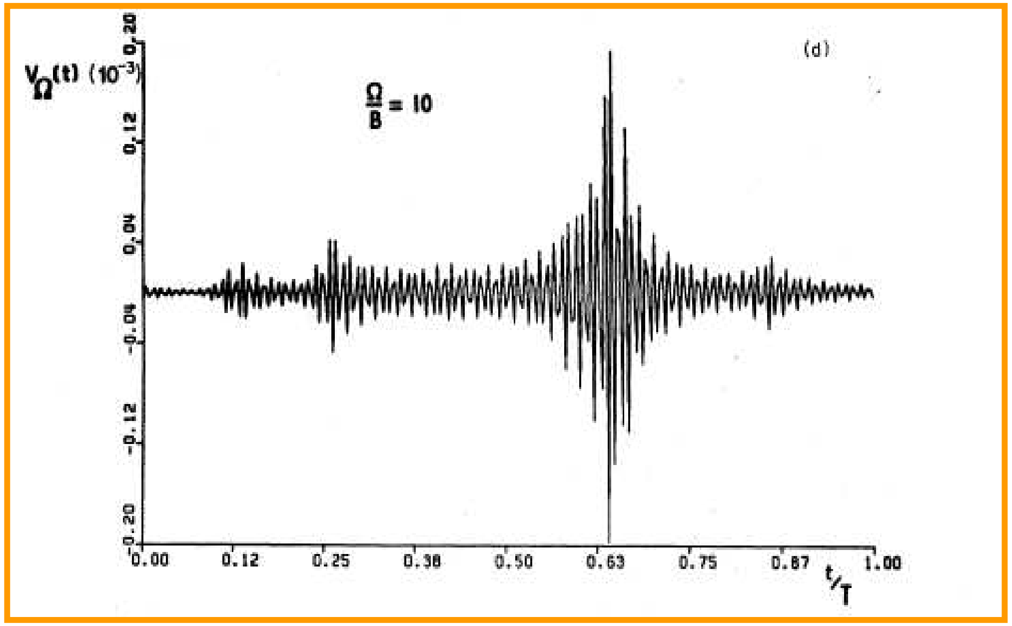
Numerical studies in turbulence





(a) One period of the forcing function.
(b) Unfiltered periodic solution for velocity.





Numerical evidence in turbulence (contd.)

(c) High pass filtered solution. Filter frequency 3 times the maximum excited mode in (a).

(d) Filter frequency is 10 times.

No Self Similarity

Intermittency: increased flatness of PDF

Observation of quiescent period. Activity gets restricted over limited time regime.

Let γ be the fraction of time for which the signal is on.

$$\langle v_\gamma^2 \rangle = \gamma \langle v^2 \rangle$$
$$\langle v_\gamma^4 \rangle = \gamma \langle v^4 \rangle$$

$$F_\gamma = \frac{\langle v_\gamma^4 \rangle}{\langle v_\gamma^2 \rangle^2} = \frac{1}{\gamma} \frac{\langle v^4 \rangle}{\langle v^2 \rangle^2}$$

Clearly $F_\gamma > F$

Why Turbulence continues to remain an unsolved Problem

- For turbulence, “scale invariance” ideas are useful but of limited validity.
- Scale invariance implies power law (e.g. ~ l^a) dependence. So that as the scales are stretched, an amplification factor alone takes care of the changes.
- In the inertial range Kolmogorov predicted a power law scaling for the energy spectrum for turbulence $E(k) \sim C\, k^{-5/3}$ based on self similarity, locality of interactions in wave number space and dimensional arguments.
- Observed spectrum very close but not quite the same.

Kolmogorov's similarity hypothesis

Navier Stokes Equation for fluids:

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V} = -\nabla P + \nu \nabla^2 \vec{V} + F$$

Characterized in terms of dimensionless Reynold's number **Re=VL/ν**; large **Re** → implies nonlinear turbulent state.

K41:

- Energy input at large length scales L.
- Energy dissipation at small length scales l_d.
- Energy cascade in the intermediate inertial range due to nonlinear terms, where there is no natural scale !

K41 (Contd.)

- Inertial range: energy transfer through local interaction of wavenumbers with a rate that is same for all scales.
- $\varepsilon = \varepsilon_k = v_k^2/\tau_k$ = external forcing rate = energy transfer rate in each scale. (ε homogeneous in space and time)

$$E = \int E(k)\,dk;$$

$$E \sim L^2 T^{-2};\ E(k) \sim L^3 T^{-2};\ \varepsilon \sim L^2 T^{-3}$$

$$\varepsilon = dE/dt$$

$$E(k) = \varepsilon^{\alpha} k^{\beta}$$

$$\alpha = \frac{2}{3};\ \beta = -\frac{5}{3}$$

K41 (Contd.)

- Observations indicate deviations from 5/3rd law. $E(k) \sim k^{-5/3+\mu}$, μ is known as the intermittency coefficient.
- Deviations prominent in higher order structure functions.

Structure functions are defined as

$$S_p(r) = \left\langle [\delta v(\vec{x}, t, r)]^p \right\rangle$$

$$\delta v(\vec{x}, t, r) = [\vec{v}(\vec{x}, t) - \vec{v}(\vec{x} + \vec{r}, t)] \bullet \vec{r} / r$$

Structure functions and Intermittency

Note that

$$\left\langle (\delta v_r)^2 \right\rangle = \frac{\int (\delta v_r)^2 d^3x}{\int d^3x} = \int |v_k|^2 k^2 dk = \int E(k) dk$$

⇒

$$\left\langle (\delta v_r)^2 \right\rangle \sim |v_k|^2 k^3 \sim E(k) k \sim k^{-2/3} \sim r^{2/3}$$

$$\left\langle (\delta v_r)^p \right\rangle \sim \left\langle \left[(\delta v_r)^2 \right]^{p/2} \right\rangle \sim r^{p/3}$$

Structure fn of order p (↑ $\left\langle (\delta v_r)^p \right\rangle$)

Possible only for a Gaussian ensemble (↑ $\left\langle \left[(\delta v_r)^2 \right]^{p/2} \right\rangle$)

Thus any deviation from linear p/3 scaling of p^{th} order structure function is a measure of Intermittency

Relationship between bivariate gaussian and scaling of S_p

Let $\phi_1 = \phi(x)$ and $\phi_2 = \phi(x+r)$; their joint bivariate gaussian PDF is given by: (here $\langle \phi_1^2 \rangle = \langle \phi_2^2 \rangle = a$ and $\langle \phi_1 \phi_2 \rangle = b$)

$$P(\phi_1, \phi_2) \sim exp\{-\phi^T \Sigma^{-1} \phi\} = exp\left\{-\frac{\left[a\left(\phi_1^2 + \phi_2^2\right) - 2b\phi_1\phi_2\right]}{\left(a^2 - b^2\right)}\right\}$$

$$\left\langle (\delta\phi)^2 \right\rangle = \left\langle (\phi_1 - \phi_2)^2 \right\rangle = 2(a-b) = f(r) = r^\beta$$

Self similarity

Homogeneity

Gaussianity and scaling of S_p

$$P(\delta\phi) = \int P(\phi_1, \phi_1 - \delta\phi)\, d\phi_1 = exp\left\{ -\frac{(\delta\phi)^2}{2(a-b)} \right\}$$

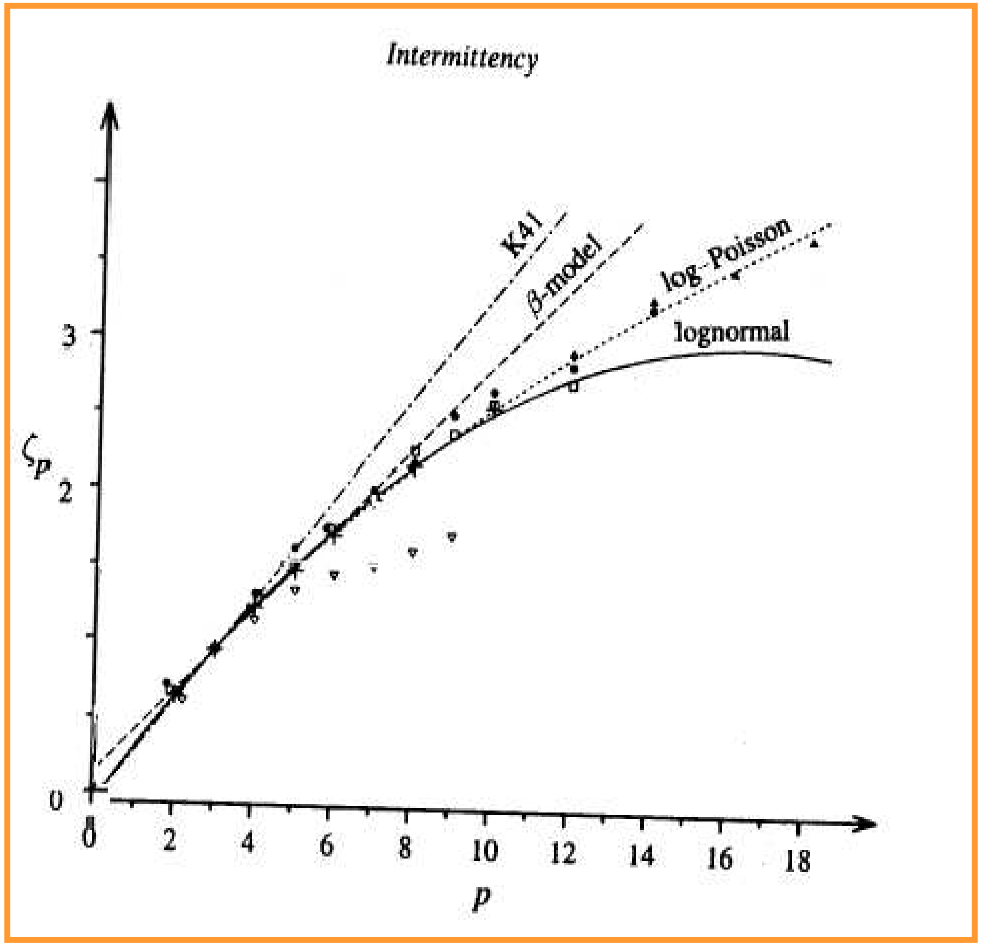
$$S_p = \left\langle (\delta\phi)^p \right\rangle = \left\langle (\delta\phi)^2 \right\rangle^{p/2} \sim \left(r^{\beta} \right)^{p/2}$$

Complete solution : specification of a multivariate PDF and the study of its deviations from multivariate gaussianity. Structure functions capture the essence.

Quantitatively

Intermittency: is quantified by deviation of ς_p from p/3

$$S_p(r) \sim r^{\varsigma_p}$$



Physical Mechanism of Intermittency

In reality the dissipation ε is a statistical quantity

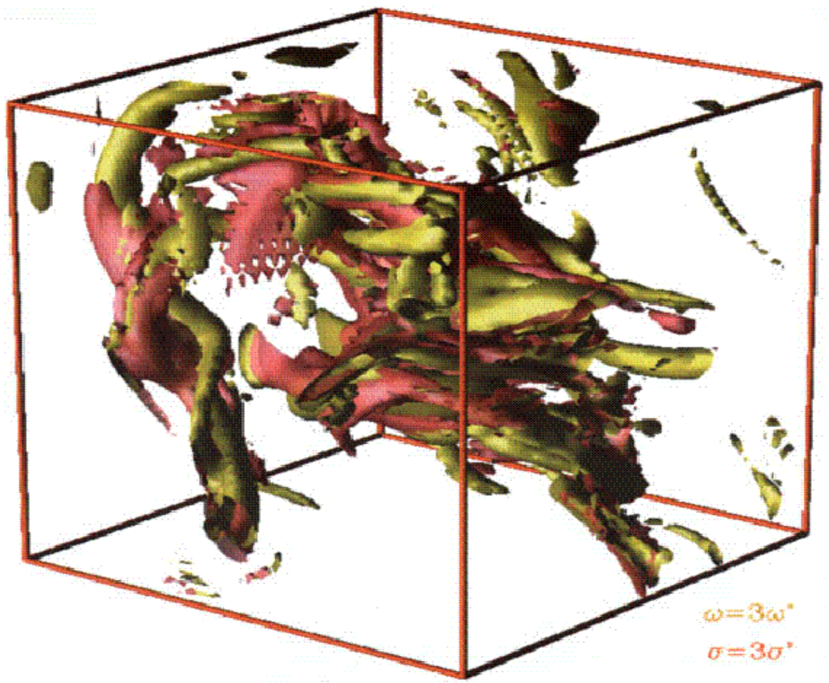


FIG. 5. (Color) Demonstration that vorticity at large amplitudes, say greater than 3 standard deviations, organizes itself in the form of tubes (shown in yellow), even though the turbulence is globally homogeneous and isotropic. Large-amplitude dissipation (shown in red) is not as organized, and seems to surround regions of high vorticity. Smaller amplitudes do not possess such structure even for vorticity. In principle, the multifractal description of the spiky signals of Fig. 4 is capable of discerning geometric structures such as sheets and tubes, but no particular shape plays a central role in that description. The dynamical reason for this organization of large-amplitude vorticity is unclear. The ubiquitous presence of vortex tubes raises a number of interesting questions, some of which are mentioned in the text. At present, elementary properties of these tubes, such as their mean length and scaling of their thickness with Reynolds number, have not been quantified satisfactorily; nor has their dynamical significance.

Concept of Intermittency in short

- In reality ε is a statistical quantity. Can be diagnosed by the presence of patchy, bursty dissipation and transport. → "Intermittency"
- Departure from maximal randomness → Non Gaussian statistics and presence of structures. Extreme events are more probable than gaussian.
- Presence of structures → strong self interaction, local in physical space and non- local in k space.
- Leads to deviation from Kolmogorov's scaling. Deviations are more pronounced for higher order structure functions.

Some Exact Results

- From symmetry considerations of Navier Stokes equation it has been possible to obtain the exact value for $\zeta_3=1$.
- The scaling exponents ζ_p of the structure functions S_p of a passive scalar advected by a velocity field which is self similar, gaussian white in time (δ correlated) has been obtained exactly for all 'p' for space dimension d of 2 and above and for velocity scaling exponent ξ lying between 0 and 2.

$$\left\langle (\delta u(r,t))^2 \right\rangle \sim r^{\xi}$$

Some Exact Results (Contd.)

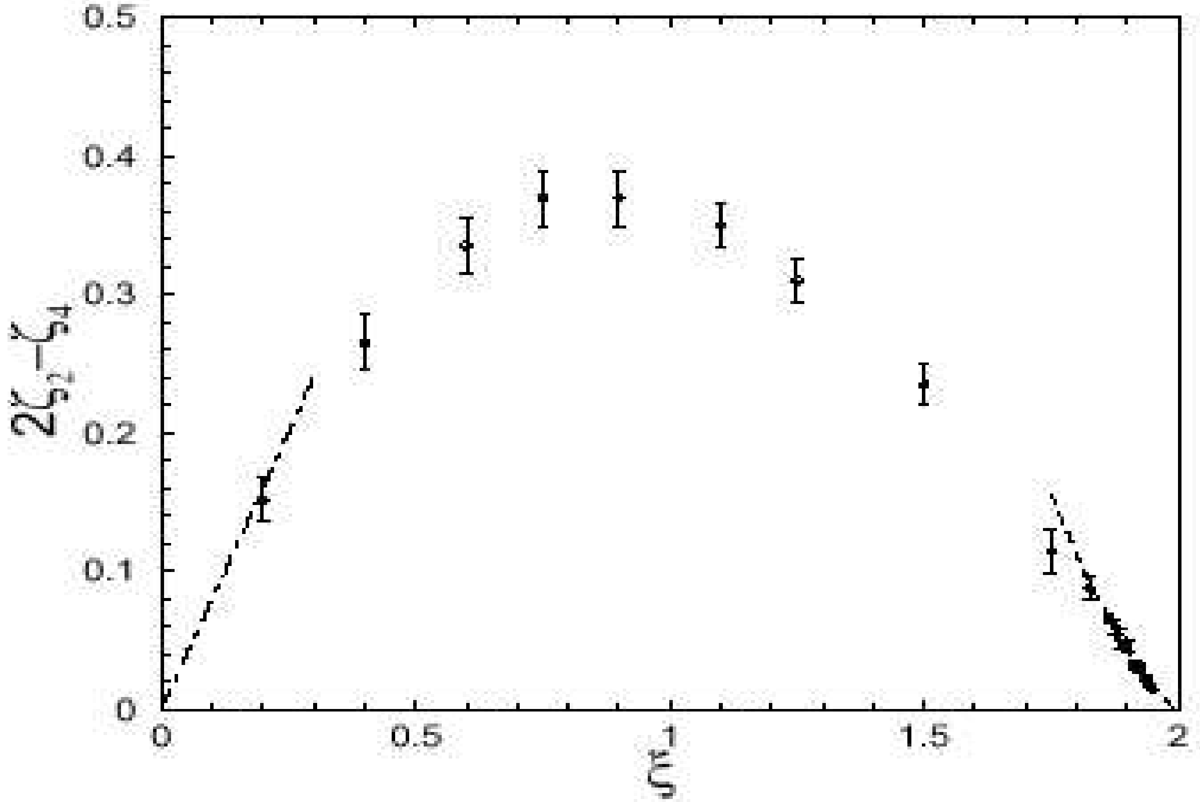


FIG. 5. The anomaly $2\zeta_2 - \zeta_4$ for the fourth-order structure function in the three-dimensional Kraichnan model.

Models for intermittency

Incorporate statistical fluctuations of ε.

- Log normal distribution.

  Kolmogorov, JFM 13, 82 (1962).

Geometrical structure of dissipation region

- Beta model.

  Novikov and Stewart; U Frisch JFM 87, 719 (1978).

- Multifractal model.

  Meneveau and Sreenivasan, JFM, 224, 429 (1991).

Currently the most favored

- Log Poisson process.

  She and Levegue, PRL 72, 336 (1994)

Lognormal Model

Kolmogorov and Obukhov assumed that ε being a statistical positive definite quantity has a log normal distribution with

$$\langle \varepsilon(x)\varepsilon(x+r)\rangle \sim r^{-\mu}$$

Leading to

$$\varsigma_p = \frac{p}{3} - \mu\,\frac{p(p-3)}{18}$$

Note the value of ς_3 as unity has been obtained exactly from the equations.
Any model ought to satisfy this constraint !!

A good fit to observed numerical and experimental data was obtained upto p = 10 for a value of μ = 0.2

Beta Model

Each level of cascade an eddy of scale l_n splits into $2^D\beta$ eddies of scale $l_{n+1} = l_n/2$. Here D is the dimensionality of space and β lies in between 0 and unity defining the fraction of space which is filled at each subsequent scale by the turbulent activity.

$$\varsigma_p = \frac{p}{3} - \frac{\delta}{3}(p-3)$$

Here $\beta = 2^{-\delta}$; and so D-δ is the fractal dimension of the region of activity

Greatest drawback :Linear scaling does not agree with observations.

Multifractal Model

Unequal distribution of energy into cascade eddies: half eddies get fraction n and the other half the remaining fraction $1-n$.

$$\zeta_p = \left(\frac{p}{3} - 1\right) D_p + 1$$

$$D_p = \log\left[n^p + (1-n)^p\right]^{\frac{1}{(1-p)}}$$

Agreement with experimental data pretty good for n = 0.7

She Levegue Log Poisson Model

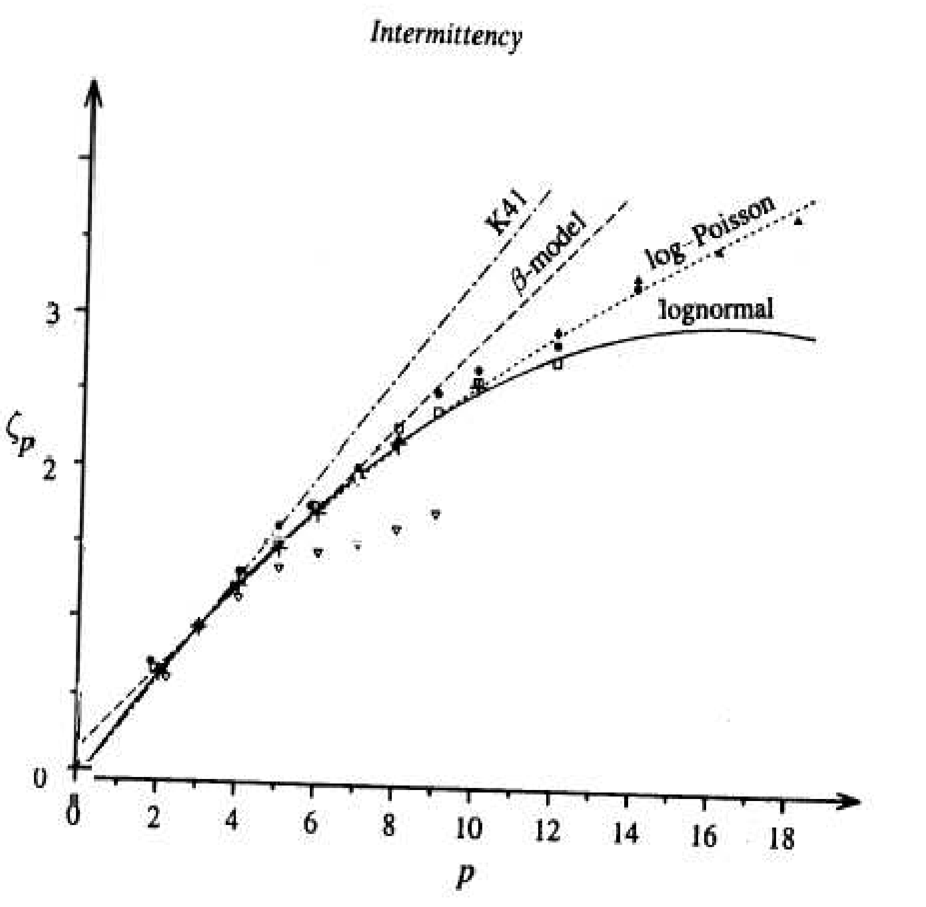
- Involves hierarchy of fluctuating structures.
- Requires no adjustable parameter
- Good agreement with data
- Wide acceptance

$$\zeta_p^{SL} = \frac{p}{9} + 2\left[1 - \left(\frac{2}{3}\right)^{\frac{p}{3}}\right]$$

Quantitatively

Intermittency: is quantified by deviation of ς_p from p/3

$$S_p(r) \sim r^{\varsigma_p}$$



To sum up Intermittency in fluid turbulence is…..

- A manifestation of unexpected deviations from self similarity in a regime where no natural scales exist

- Related to patchiness of dissipation rate ε in space-time.

- Associated with self-interacting coherent structures (i.e. nonlocal interactions in k space) giving non-gaussian pdf's.

Plasma - neutral fluid turbulence: similarities and differences

- Physics analogies: many scales excited, strong mixing, cascade..and so on.
- Fluid models of plasma: MHD, Hall MHD, EMHD, Hasegawa-Mima etc.
- Existence of dispersive waves propagating with finite group speed in addition to eddy like motion.
- Natural scales in the equations ρ_s , d_i , d_e
- Additional fields: Electric and magnetic fields, currents etc.
- Strong anisotropy in turbulence.

  At times leads to a reduction in dimensionality (3d to 2d).

Plasma Intermittency: General

- When fluid like models without natural scales are appropriate for description of turbulence in plasmas, it is clear that fluid like intermittency may occur.

- However, often there are natural scales like larmor radius, skin depth etc in the plasma fluid equations. These can lead to their own impact on scale similarity and hence plasma intermittency like phenomena.

- Similarly, the scales at which one fluid model becomes inapplicable and another one takes over can also lead to interesting effects.

Plasma Intermittency …contd.

•Plasmas abound in dispersive waves which propagate in and out of unstable regions by group propagation as wave packets.

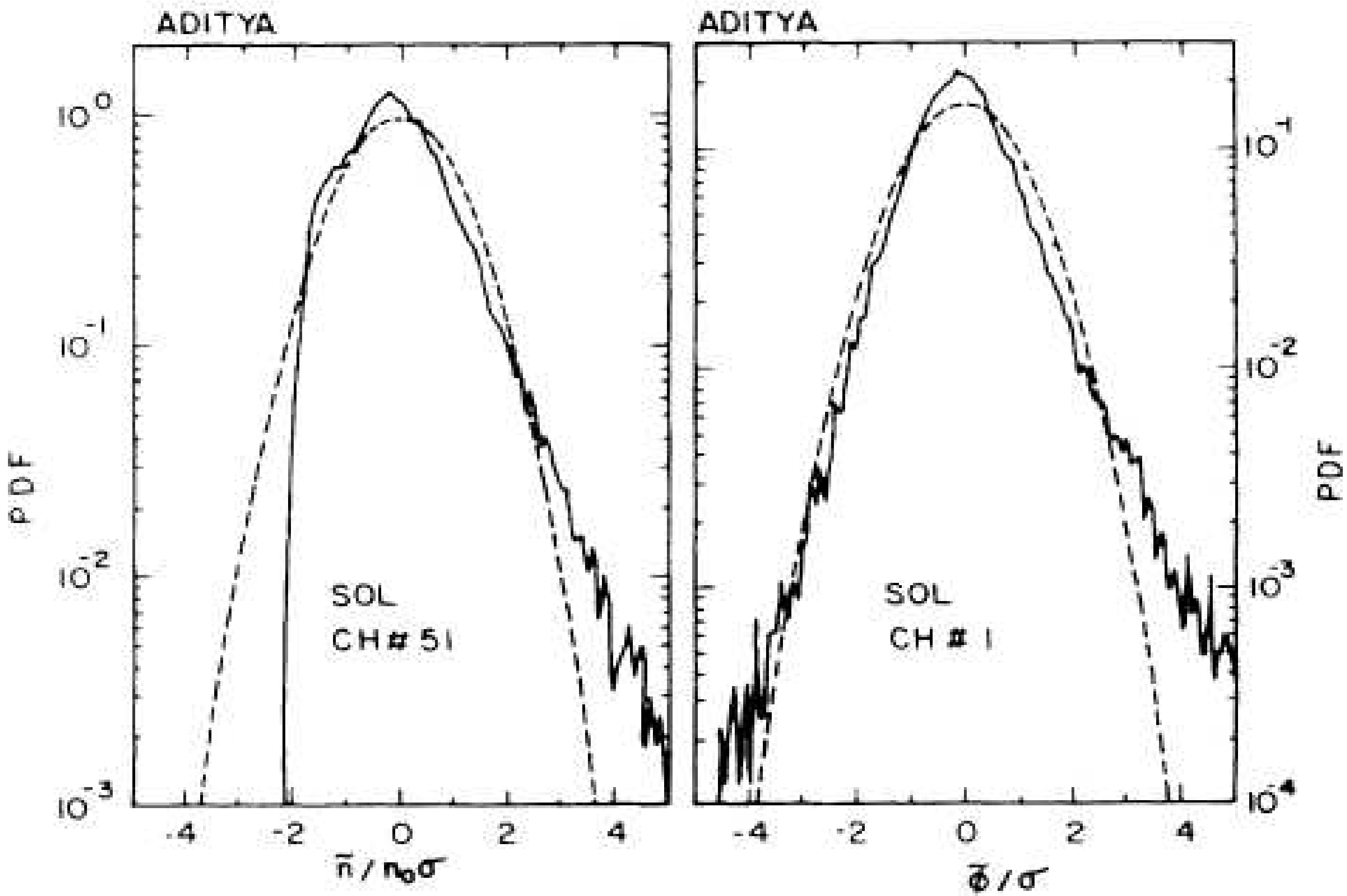
•Wave-wave interaction phenomena can modify the eddy interaction physics producing changes in energy spectra. They can also emphasize interaction through disparate scales leading to coherent structures and intermittency like phenomena.

•Thus plasma intermittency has much richer content and is still waiting for appropriate definitions and classifications.

•Here we illustrate attempts at elucidating plasma intermittency phenomena with a few examples.

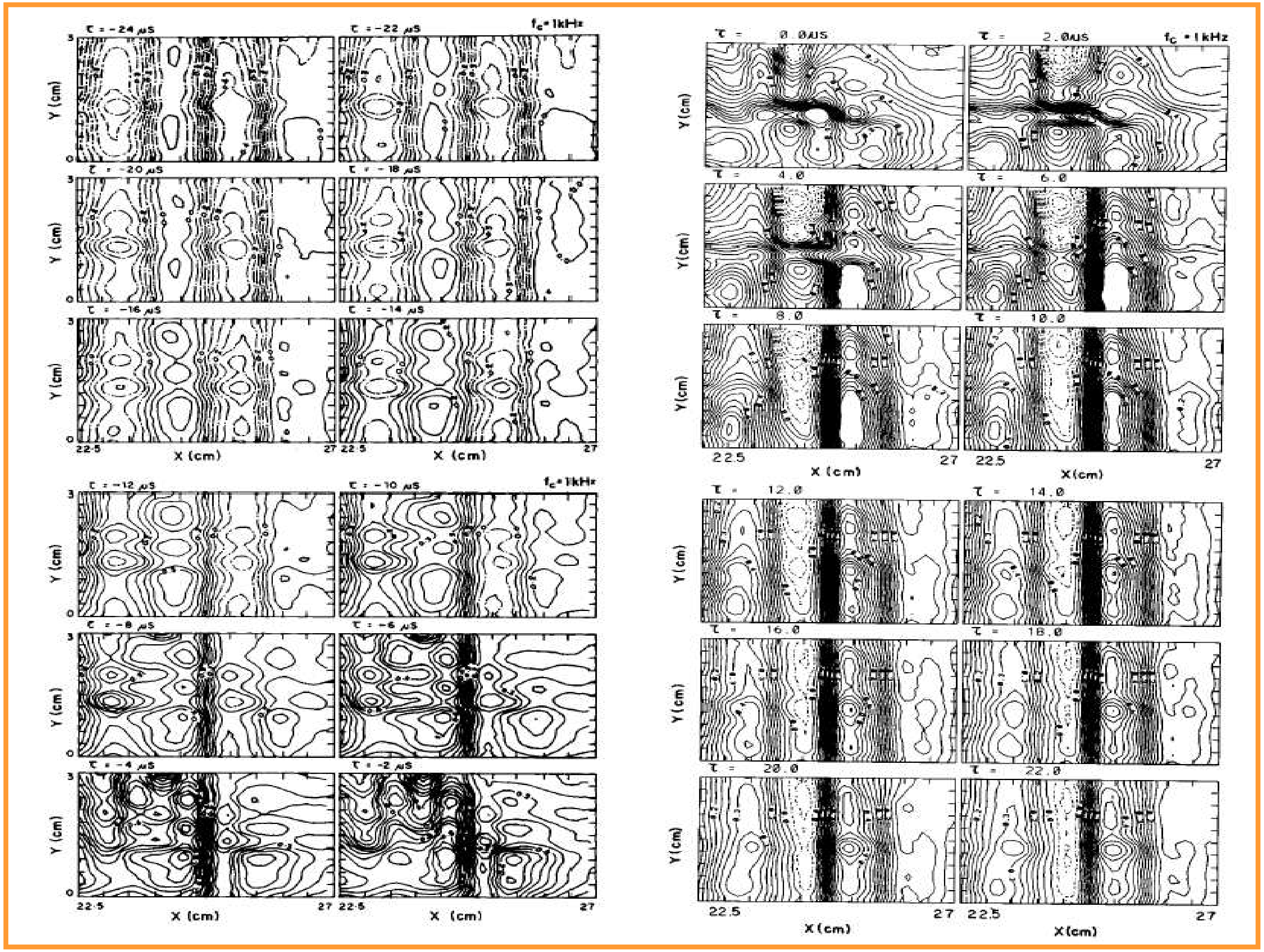
FIRST SOME OBSERVATIONS .

PDF of density and potential fluctuations in ADITYA
Jha et al Phys Rev Letters (1992)



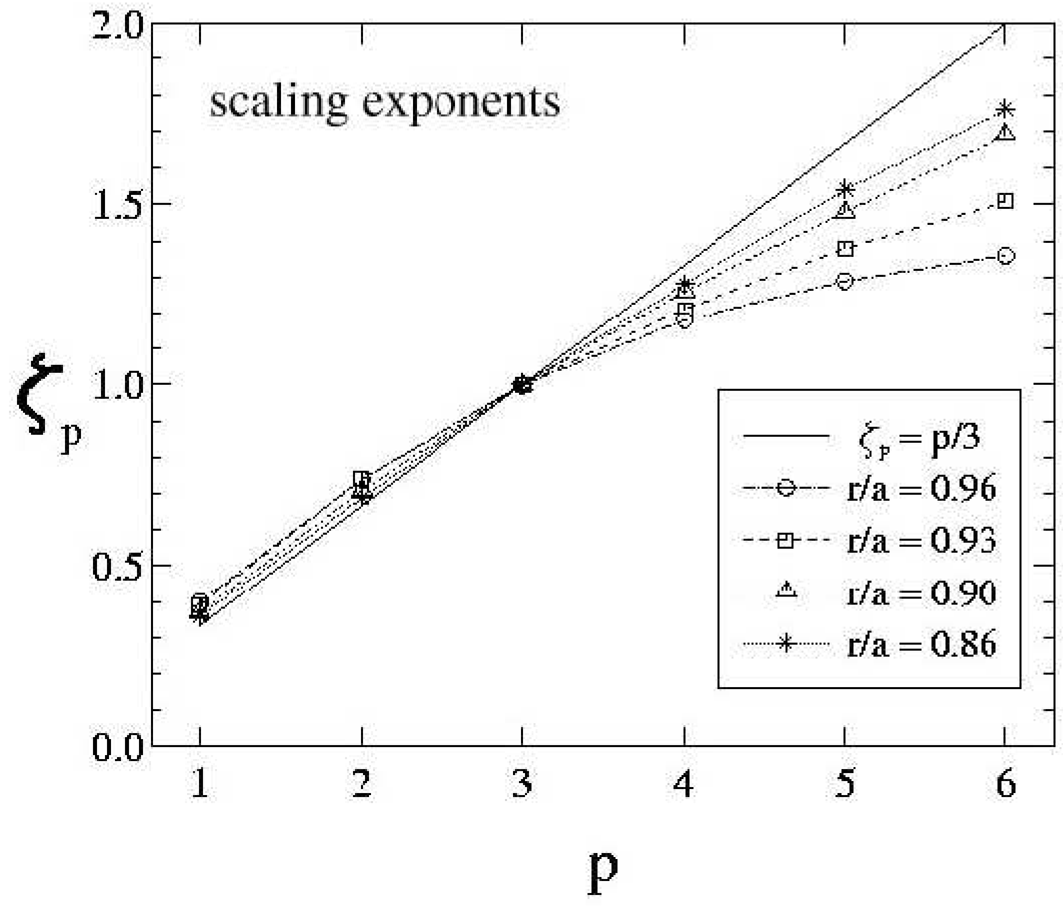
Also seen in Alcator CMOD, TJ II, TS, ATF, DIIID etc

Coherent Potential Structures in ADITYA edge plasma.
Joseph et al Phys Plasmas (1997)

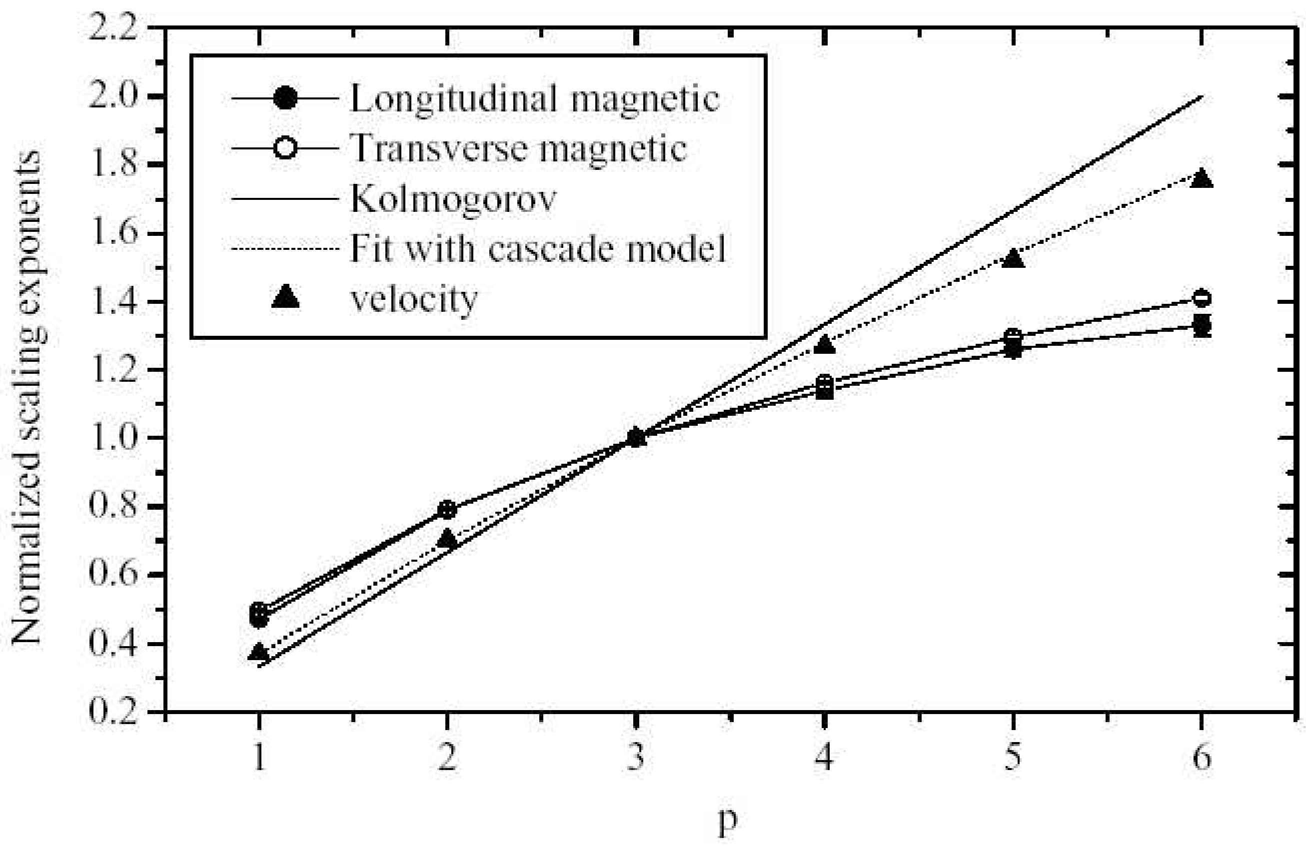


Seen also in ASDEX, Caltech Tok, NSTX, Alc CMOD

Scaling exponent of RFX magnetic field data – Carbone et al.



Scaling exponents of solar wind data – Carbone et al.



Extensive work by Burlaga et al using satellite data

Limitations of Laboratory Plasma Studies

Plasma typically inhomogeneous and affected by boundaries

Data lengths are short and not statistically stationary because of pulsed nature of experiments

No clear inertial range because excitation scales are in middle of measurement domain and not too many decades of frequency/wavenumber available because of limitations of instrument resolution .

All the same what we have seen shows that there is some experimental evidence that intermittency like phenomena may be present in plasma turbulence.

WE NOW TURN TO SOME MODELS

Intermittency studies in Plasma

(Fluid Approach)

- Most extensive study on MHD model.
- MHD Model supports Alfven waves.

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \bullet \nabla \vec{v} = \mu \nabla^2 \vec{v} - \nabla p + \vec{J} \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{v} \bullet \nabla \vec{B} = \vec{B} \bullet \nabla \vec{v} + \eta \nabla^2 \vec{B}$$

Alfven effect!!!
Modifies the energy turn over rate from $\tau_l^{-1} \sim (V/L)$ to $(\tau_{Alf} / \tau_l)\, \tau_l^{-1}$.

Change in Kolmogorov scaling $k^{-5/3}$ to Iroshnikov-Kraichnan scaling $k^{-3/2}$. Difference difficult to gauge numerically.

Generalized SL scaling

$$\varsigma_p = \frac{p}{g}(1-x) + C\left[1-(1-x/C)^{p/g}\right]$$

Here: g defines the basic scaling of the relevant field $\delta z_r \sim r^{1/g}$.
x energy transfer time $\tau_{eff} \sim r^x$
C=3-D; D dimension of dissipative structures.

Neutral Fluids
z = v, g =3, x =2/3,
D=1 so C = 2

MHD Fluid
(w/o Alfven effect)
z = Elsasser fields
g =3, x =2/3, D=2
so C = 1

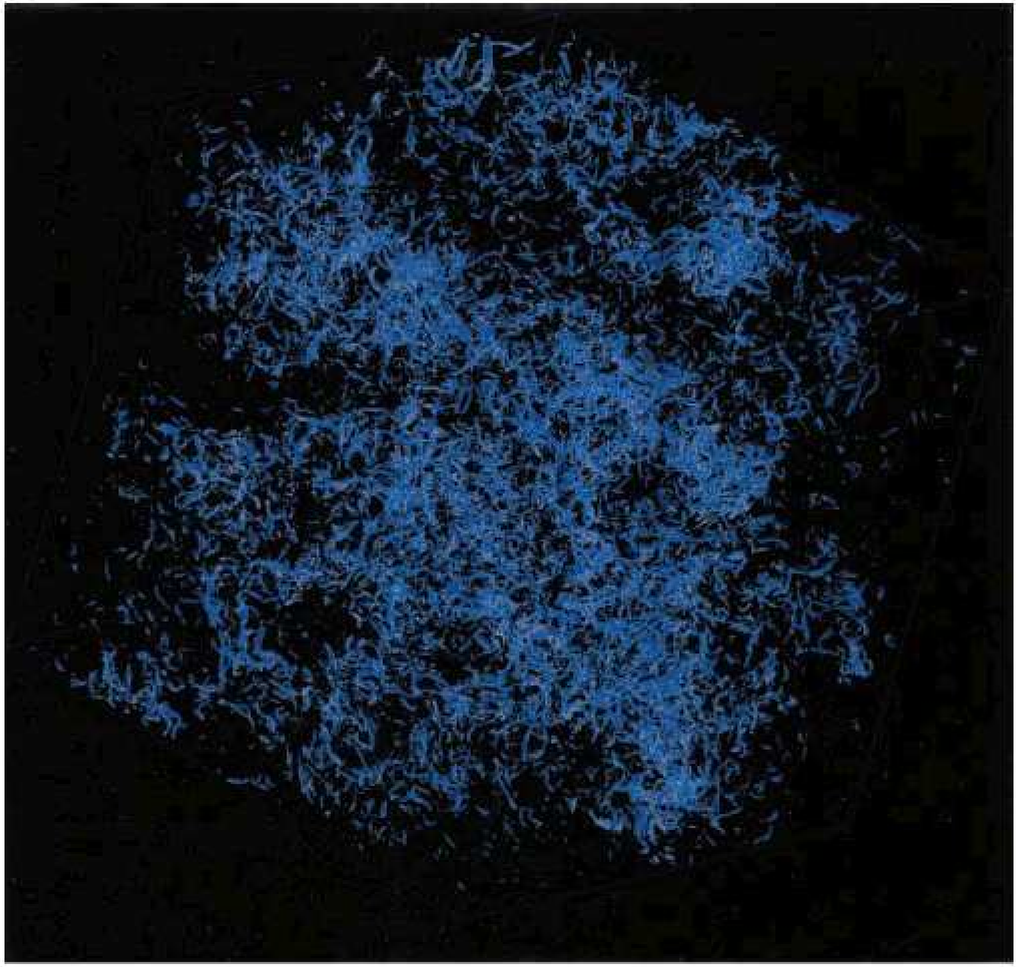
MHD Fluid
(with Alfven effect)
z = Elsasser fields
g =4, x =1/2, D=2
so C = 1

(Biskamp, Mueller 2000)

Current density isosurface MHD

(a)

Vorticity isosurface for NS



(b)

Comparison with simulation : isotropic case

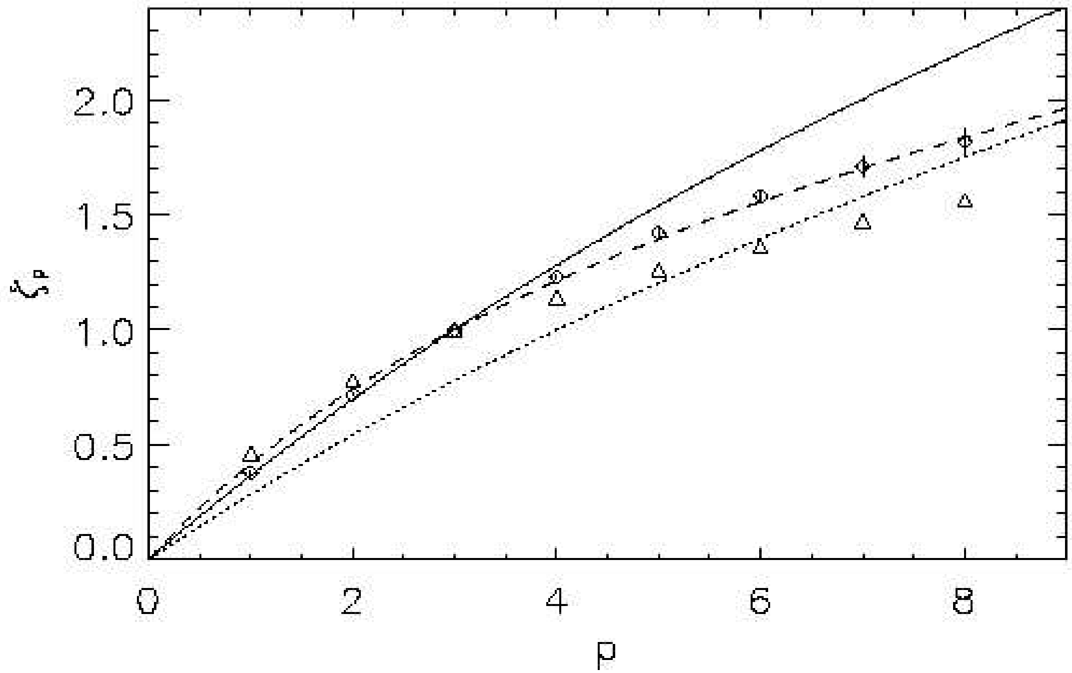


FIG. 4. Scaling exponents ζ_p^+ for 3D MHD turbulence (diamonds) and relative exponents ζ_p^+/ζ_3^+ for 2D MHD turbulence (triangles). The continuous curve is the She-Leveque model ζ_p^{SL}, the dashed curve the modified model ζ_p^{MHD} (7), and the dotted line the IK model ζ_p^{IK}.

(Biskamp, Mueller 2000)

MHD Intermittency: Anisotropic (B_0 finite)

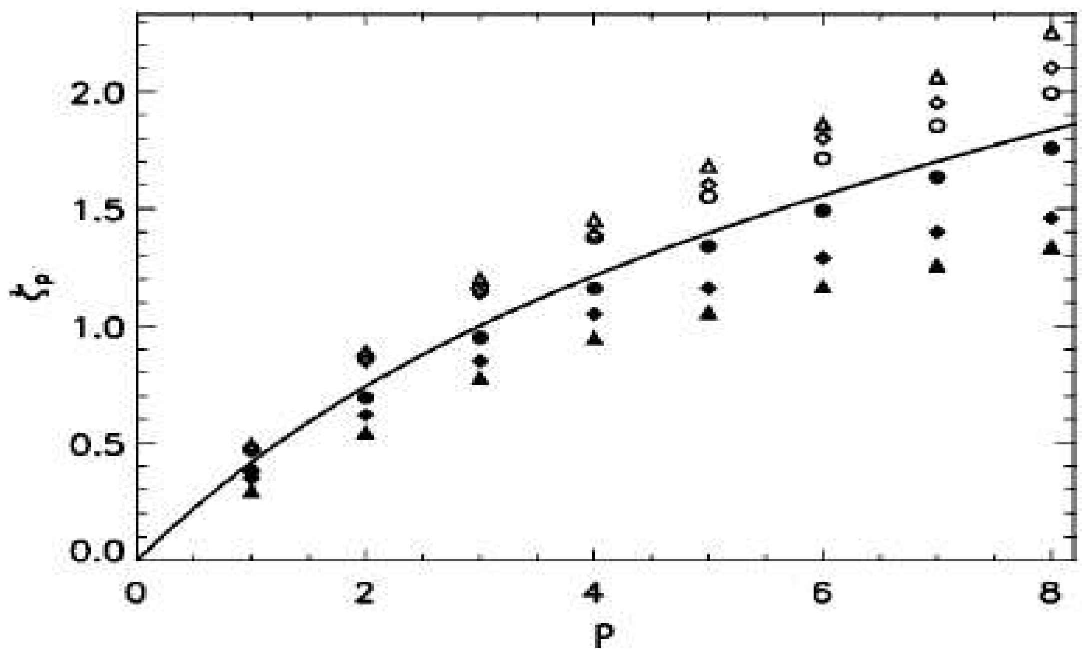


FIG. 1. Scaling exponents ζ_p of perpendicular (filled symbols) and parallel (open symbols) structure functions $S_p(\ell) = \langle |\delta z_\ell|^p \rangle$ for $B_0 = 0, 5, 10$ (circles, diamonds, triangles) together with isotropic scalings from 3D-DNS (solid line, [30]). Error bars are given in Fig. 3.

Mueller, Biskamp 2003

For large B_0 parallel intermittency is weaker and perpendicular intermittency is stronger and closer to Iroshnikov-Kraichnan

MHD Intermittency

- For isotropic 3D MHD the energy spectra are given by Kolmogorov $k^{-5/3}$ law and the intermittency is described by the Generalized She Levegue formula.

- For anisotropic MHD with large mean B_0, the energy spectra are closer to Iroshnikov-Kraichnan $k^{-3/2}$ law. The intermittency in parallel direction reduces and perpendicular intermittency also increases consistent with IK.

- Ideas consistent with dominance of wave interaction effects and anisotropy at large B_0 (Goldreich and Sridhar 1995)

Electron Magnetohydrodynamics(EMHD)

Another plasma fluid model

Governing Equations:

- Curl of Electron Momentum equation

$$\frac{\partial}{\partial t}(\nabla \times \vec{P}) = \nabla \times \{\vec{V}_e \times (\nabla \times \vec{P})\} - m_e \nu \nabla \times \vec{V}_e$$

$$\vec{P} = \gamma_e m_e \vec{V}_e - e\vec{A}/c$$

here γ_e is the relativistic factor!

- Expression for current (immobile ions)

$$\vec{J} = -en_e V_e$$

- Ampere's Law

$$\nabla \times \vec{B} = 4\pi\vec{J}/c$$

Non relativistic limit

(where $d_e^2 = c^2/\omega_{pe}^2$)

$$\Rightarrow \nabla \times \vec{P} = e\left(d_e^2 \nabla^2 \vec{B} - \vec{B}\right)/c;$$

EMHD Model in 2 dimensions

For two dimensional variations $\vec{B} = b\hat{z} + \hat{z} \times \nabla \psi$

EMHD equations reduce to following coupled set of equations for scalar fields b and ψ

$$\frac{\partial}{\partial t}\left(\psi - \nabla^2 \psi\right) + \hat{z} \times \nabla b \bullet \nabla\left(\psi - \nabla^2 \psi\right) = \eta \nabla^2 \psi$$

$$\frac{\partial}{\partial t}\left(b - \nabla^2 b\right) - \hat{z} \times \nabla b \bullet \nabla \nabla^2 b + \hat{z} \times \nabla \psi \bullet \nabla \nabla^2 \psi = \eta \nabla^2 b$$

Normalizations:

Length: skin depth (c/ω_{pe})

Magnetic field: B_0 Typical amplitude

Time: Corresponding electron gyroperiod ω_{ce}^{-1}

Studies on EMHD model

- Presence of special characteristic scale d_e.
- Supports dispersive whistler waves.
- 'Whistler' effect on scaling

Kolmogorov analysis

- $E_k = \varepsilon^{2/3} k^{-5/3}$ ($kd_e >> 1$)
- $E_k = \varepsilon^{2/3} k^{-7/3}$ ($kd_e << 1$)

Analysis with whistler effect

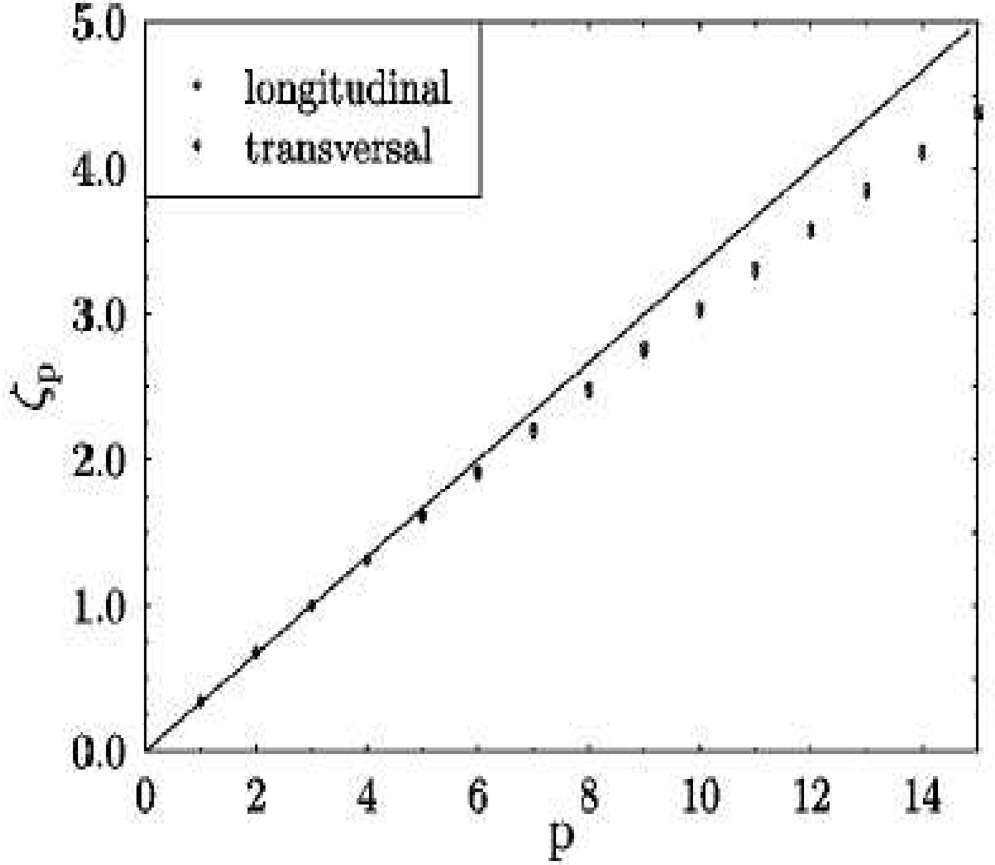
- $E_k = \varepsilon^{2/3} k^{-5/3}$ ($kd_e >> 1$)
- $E_k \sim k^{-2}$ ($kd_e << 1$)

- Biskamp et al. rule out whistler effect merely on the basis of energy spectra scaling!! Large B_0 not attempted !
- Dastgeer, Das, Kaw and Diamond PoP (2000) show that cascade is influenced by whistlers though it may not influence the scaling of spectra directly.

Studies on EMHD model (contd.)

- Intermittency in 2d EMHD has been studied recently by Germaschewski et al.
- The structure function index ζ_p for both b and ψ fields (upto p = 14) shows deviations from linearity.
- Fitting parameters (in terms of x, g and C) employed to fit the result with generalized SL expression.
- The fitting parameters were different for the two fields.
- No justification as given in the context of MHD.

EMHD Intermittency



Germaschewski et al 1999

Other evidence for Intermittency in EMHD

Boffetta et al.(PRE vol 59 3724(1999)) measured the scaling exponent of various powers of energy dissipation function numerically and showed a nontrivial scaling of the exponent τ_p with p.

$$\left\langle \varepsilon(r)^p \right\rangle = \left\langle \left[\frac{1}{V(r)} \int d^3x\, \varepsilon(x) \right]^p \right\rangle \sim r^{\tau_p}$$

Open Questions for EMHD

- What is the topology of dissipative structures?

• In $kd_e >> 1$ EMHD is like hydrodynamics and so dissipative structure should be like 1d vortex ropes.

•Whereas for $kd_e << 1$ it is like magnetized fluid and so dissipative structures should be dominated by current sheets.

•Do some coherent structures form near the natural scale d_e?

More numerical simulations required !!

Transition from MHD to EMHD

- As direct cascade brings energy to shorter scales in MHD, one encounters first the ion skin depth d_i and the Hall MHD description and then the electron skin depth d_e and the EMHD description.
- Since nonlinear transfer rates are different in different regimes, interesting structures leading to intermittency like effects may form near the transitions .
- Needs numerical studies !!

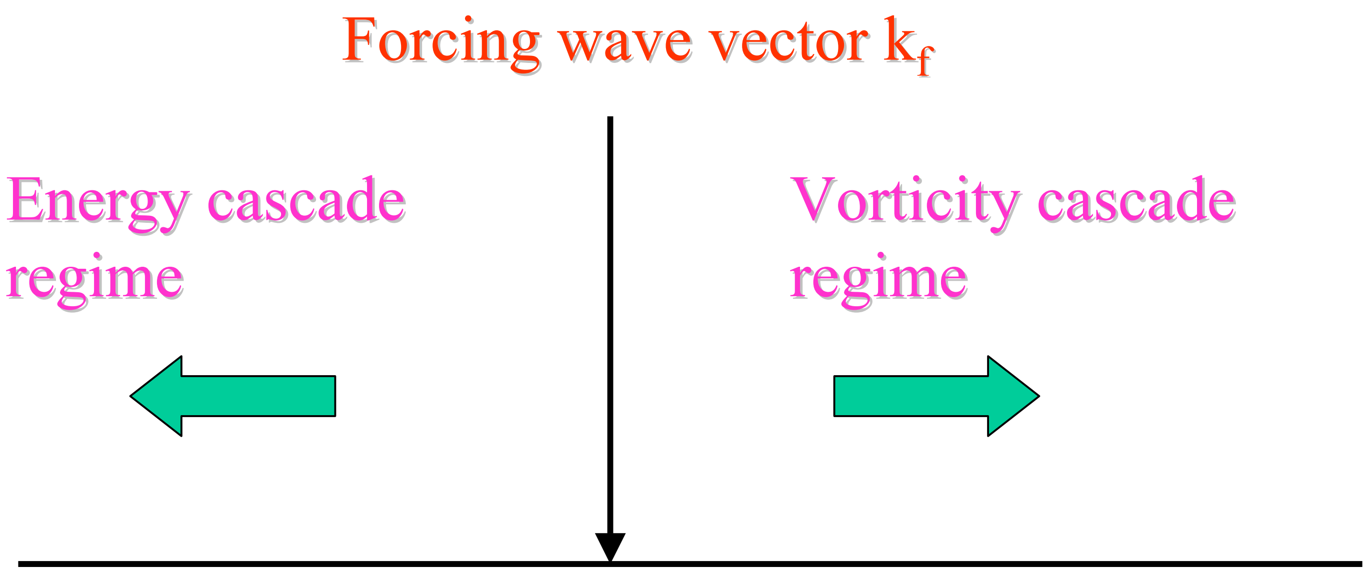
Other fluid models
(Electrostatic models)

- Hasegawa Mima (HM), ITG, ETG etc.
- HM has a characteristic scale ~ larmor radius.
- Waves : Drift waves.

$$\frac{\partial}{\partial t}\left(\phi - \nabla^2 \phi\right) - \hat{z} \times \nabla \phi \bullet \nabla \nabla^2 \phi + v_d \frac{\partial \phi}{\partial y} = -\mu \nabla^2 \nabla^2 \phi$$

- Supports two invariants.
- Hence energy (inverse) as well as vorticity (direct) cascade regimes.
- Two wavevector regimes $k\rho_s << 1$ and $k\rho_s >> 1$

Dynamics of Energy Transfer in Hasegawa-Mima Equation



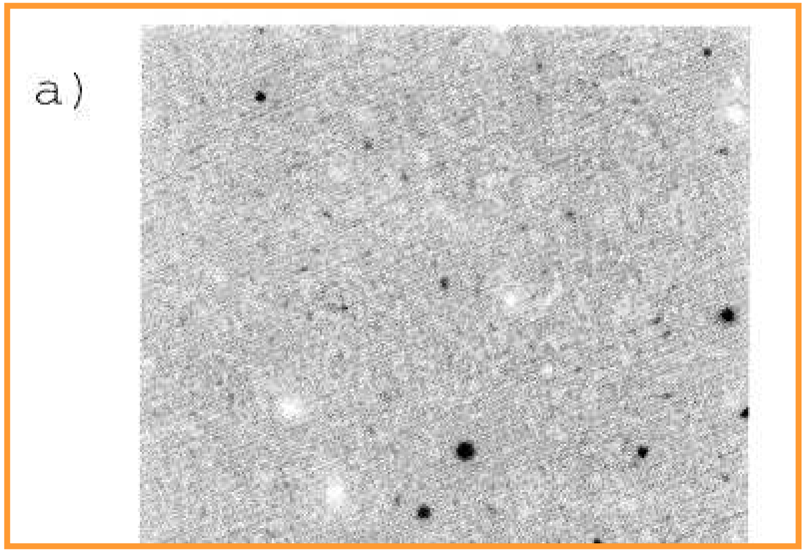
$k_f \rho_s >> 1$ energy cascade hits $k \rho_s \sim 1$ boundary
$k_f \rho_s << 1$ vorticity cascade hits $k \rho_s \sim 1$ boundary

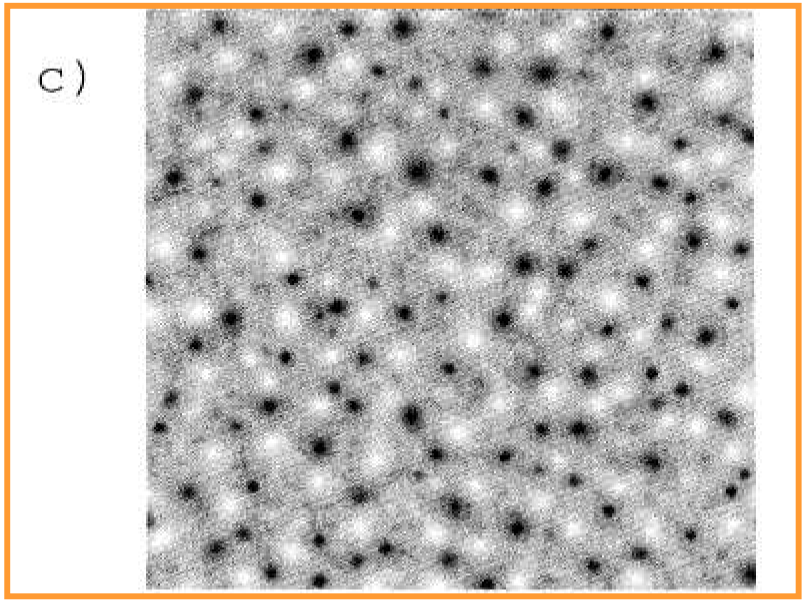
Scalings

- $k\rho_s >> 1$, equations identical to 2d hydrodynamic fluid $E_k \sim k^{-5/3}$ in energy cascade and $E_k \sim k^{-3}$ in vorticity cascade.
- $k\rho_s << 1$, $E_k \sim k^{-11/3}$ in energy cascade and $E_k \sim k^{-5}$ in vorticity cascade.
- Interesting feature in $k\rho_s << 1$ regime, reduced eddy turn over time.

$$\hat{z} \times \nabla \phi \bullet \nabla\nabla^2$$

Novel feature: Reduction in eddy turn over time results in accumulation of power in the boundary and formation of quasistationary crystalline structures. Kukharkin et al. 1995 (Oscillatory structure functions). ~ 'Intermittency like'





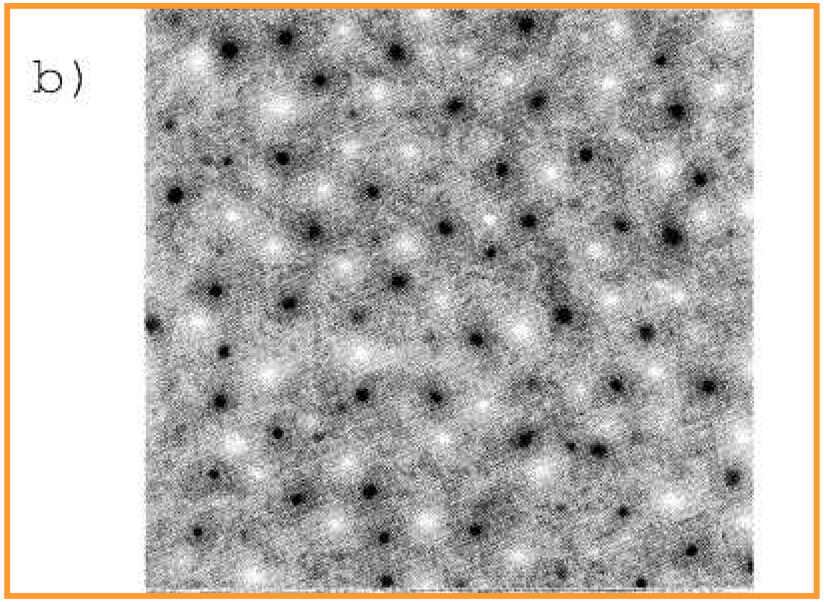
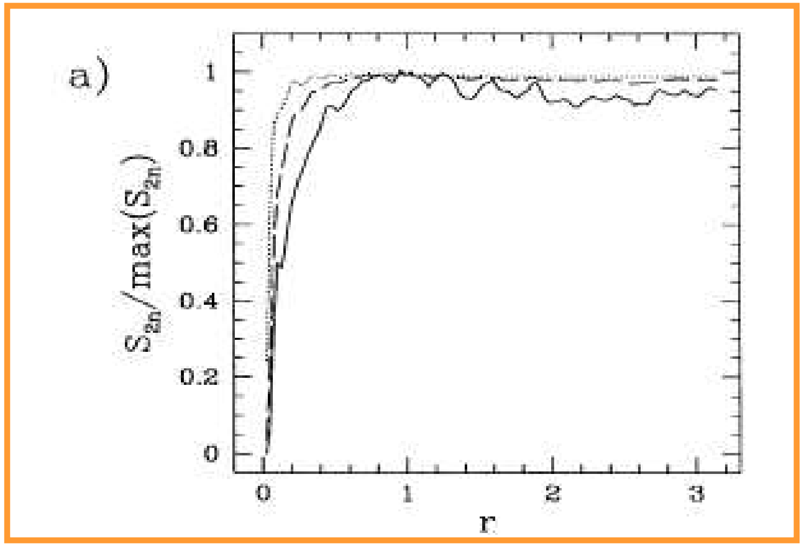
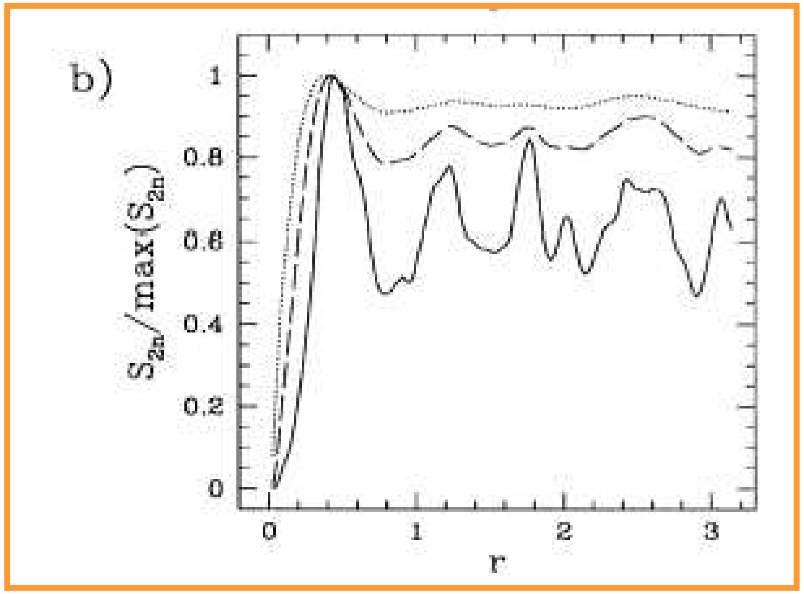


FIG. 1. The potential vorticity, $\xi = \nabla^2\phi - \lambda^2\phi$, field at $N_\lambda = 400$ for the NS and the HM equations: (a) $\lambda = 0$ (NS), (b) $\lambda = 20$, and (c) $\lambda = 40$.

$$\frac{\partial}{\partial t}(\nabla^2\phi - \lambda^2\phi) + J(\phi, \nabla^2\phi) = D + F,$$





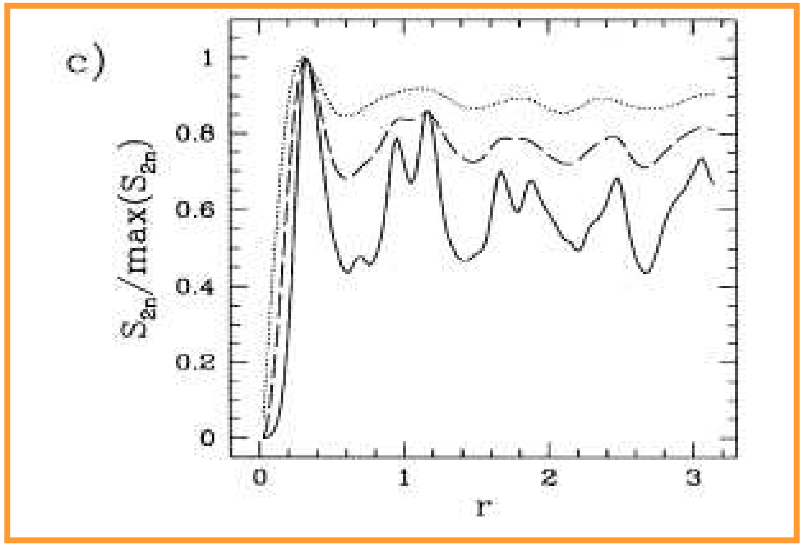
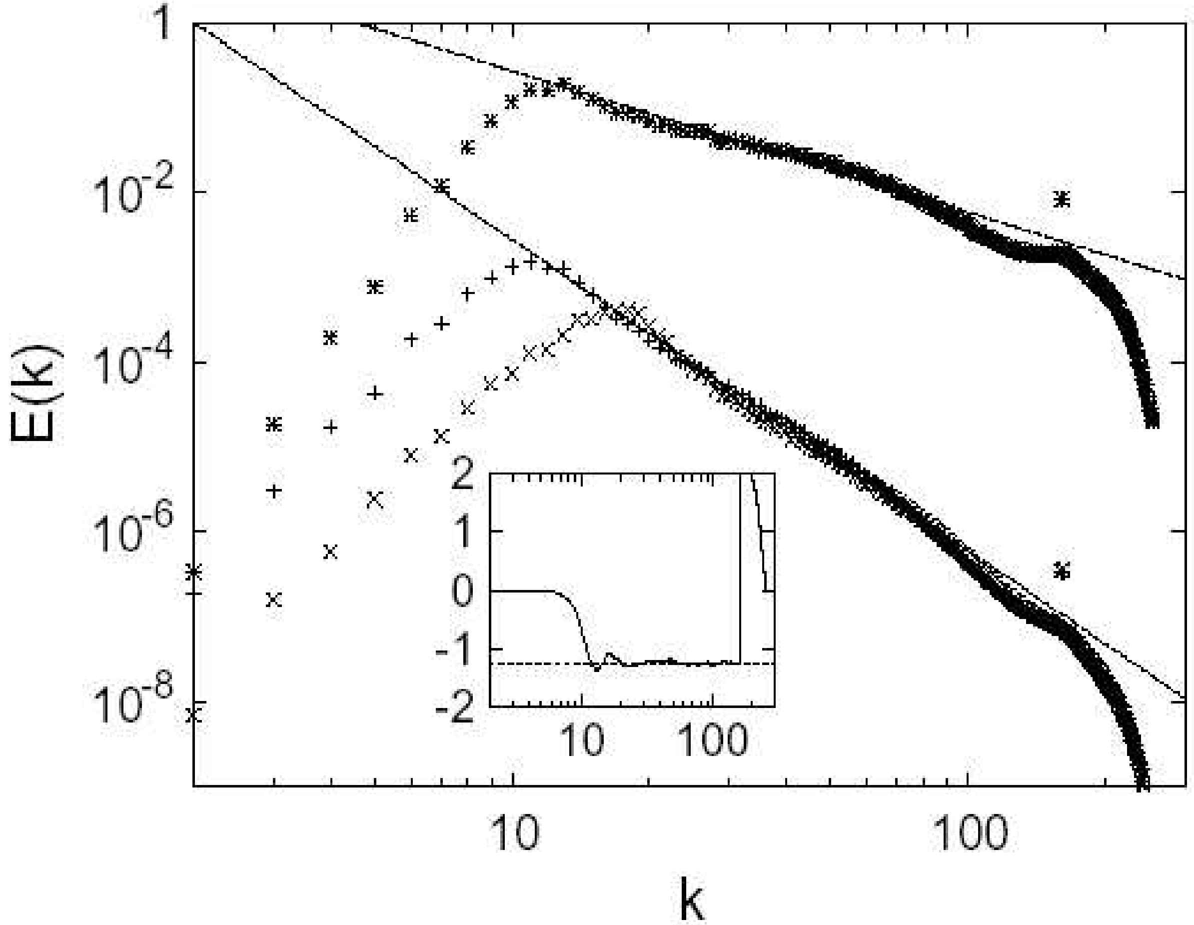


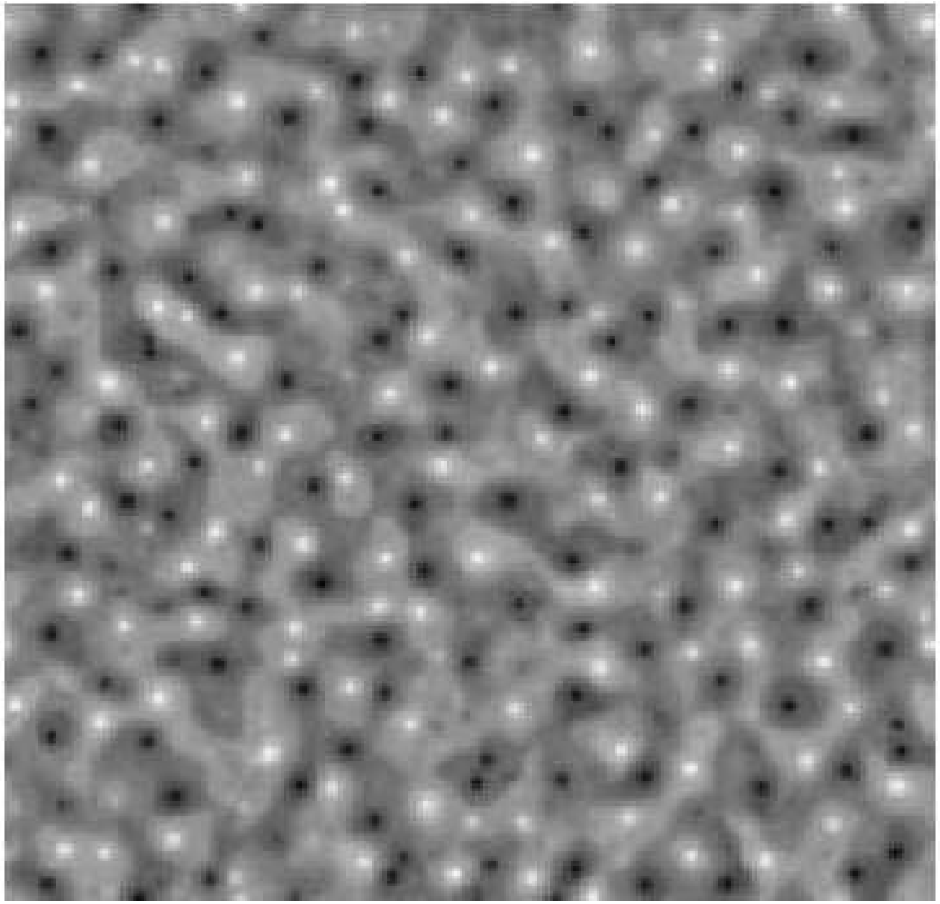
FIG. 2. Moments of potential vorticity increments $S_2(r)$ (dotted), $S_4(r)$ (dashed), and $S_8(r)$ (solid) normalized to their maximum values and averaged over time $N_\lambda \in [0, 400]$ for (a) $\lambda = 0$ (NS); (b) $\lambda = 20$, $\rho_s \simeq 0.32$; and (c) $\lambda = 40$, $\rho_s \simeq 0.16$.

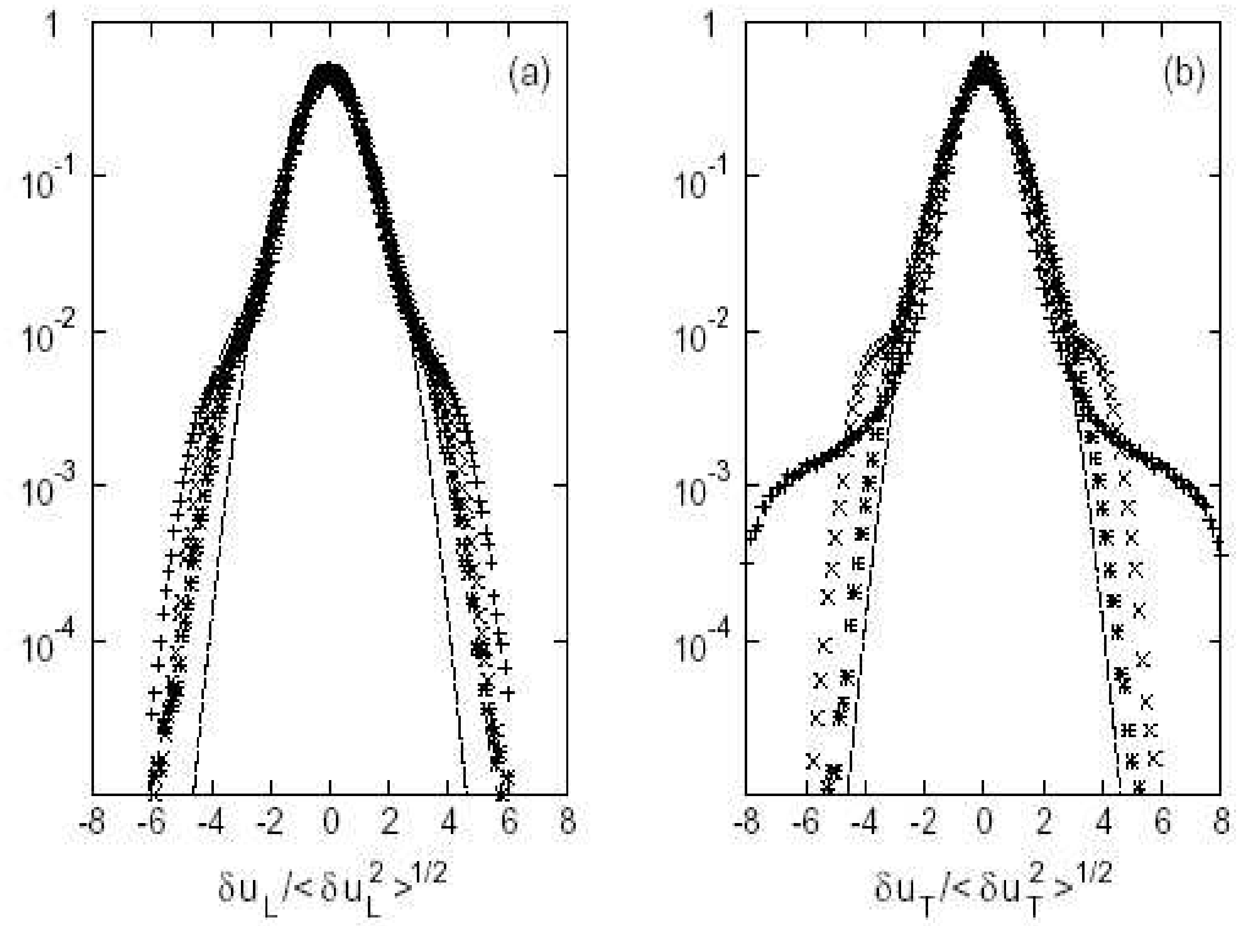
Boffeta et al (Europhys lett 2002) have shown that even when the HM equation is driven in the k << 1 regime and there are no physical boundaries like the box size leading to condensation effects, interesting quenched vortex like structures form at the characteristic size ~ size of eddy with maximum energy at that time.

This leads to a turbulent glass like state with very non Gaussian pdf's for velocity increments and has many feaures of intermittency like phenomenon.



E(k)
k
1
10-2
10-4
10-6
10-8
10
100
2
1
0
-1
-2
10
100





1
10-1
10-2
10-3
10-4
(a)
(b)
-8 -6 -4 -2 0 2 4 6 8
δu_L/<δu_L^2>^1/2
δu_T/<δu_T^2>^1/2

Two dimensional Navier Stokes turbulence has inverse energy cascade and displays no intermittency effects. On the other hand two dimensional MHD and EMHD equations display forward cascade of energy and give intermittency effect. The origin of intermittency is in the patchiness and burstiness of energy dissipation in localized regions when a steady flux of energy is moving through the system.

Quasi two dimensional HM equation with natural scales have inverse cascades of energy like 2d Navier-Stokes but because of natural boundaries at characteristic scales and properties of nonlinear terms do lead to coherent structures and non Gaussian pdf's even when accumulation of energy is not due to artificial box size limitations. Should these intermittency like phenomena in plasmas be classified as genuine intermittency ?

Nonlinear wave-wave interactions and impact on intermittency

Plasmas abound in dispersive waves and their nonlinear interactions can lead to novel phenomena.

We illustrate this with the drift wave-zonal flow coupling problem (Diamond et al, Zakharov et al) which has a profound effect on drift wave saturation physics.

Zonal flows are radially structured *k*❻ = 0 modes, which are not contained in the HM description of drift wave turbulence.

Zonal flows can be excited by nonlinear coupling of drift waves and can in turn react back on the drift waves, saturating them.

This coupling essentially corresponds to disparate scale interactions and hence emphasizes nonlocal interactions in k space.

Kinetic Wave Equation for drift waves

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \cdot \frac{\partial N_k}{\partial \vec{r}} - \frac{\partial \omega_k}{\partial \vec{r}} \cdot \frac{\partial N_k}{\partial \vec{k}} = \gamma_k N_k - \Delta\omega_k N_k^2$$

$$N_k \equiv \left| \frac{e\phi_k}{T} \right|^2 \left(1 + k_\perp^2 \rho_s^2\right)^2$$ is the action density for drift waves

$$\omega_k = \omega_* / \left(1 + k_\perp^2 \rho_s^2\right)$$ γ_k is the linear growth rate

The fast space and time variations have been Fourier transformed and the symbols r and t here refer to slow space and time variables.

$\Delta\omega_k$ term is a nonlinear interaction term which determines the mixing length saturation amplitude e.g. for ~ homog plasmas

$N_k = \gamma_k / \Delta\omega_k = \left(1 / k_r L_n\right)^2 \left(1 + k_\perp^2 \rho_s^2\right)^2$

Description of zonal flows

The equation for the potential ψ associated with zonal flows is given by the equation describing charging of surfaces by nonlinear polarization currents:

$$\left(\frac{\partial}{\partial t} + \nu - \mu \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2}{\partial x^2} \left[\sum_k \frac{\rho_s c_s k_x k_y}{\left(1 + k^2 \rho_s^2\right)^2} N_k \right]$$

Here ν is the neoclassical damping rate and μ is the viscous effect and the term on the right side describes the zonal flow drive by the Reynold stresses $-\rho_s^3 c_s \langle [\phi, \nabla^2 \phi] \rangle$ due to drift waves.

The zonal flow reacts back on the kinetic wave equation of drift waves through the doppler shift term $\delta\omega = -\rho_s c_s \left(1 + \rho_i^2 \nabla^2\right)\left(\nabla \psi \times \vec{k}\right) \cdot \hat{z}$ where the ρ_i^2 term gives the finite ion larmor radius correction.

Modulational instability analysis

The coupled drift wave zonal flow equations show that a homogeneous background of drift wave turbulence is unstable to modulational instabilities leading to the generation of zonal flows. Linear analysis indicates a growth rate given by

$$\gamma_q \cong -\nu + i\rho_s^2 c_s^2 q^2 \int \frac{k_\theta^2}{\left(1+k^2\rho_s^2\right)^2} \frac{k_r \partial N_{k0} / \partial k_r}{\left(\Omega - q \cdot V_g\right)} d^2k$$

Instability is like the bump on tail instability in plasmas and arises from the resonant denominator when $\partial N_{k0}/\partial k_r < 0$. It is a manifestation of inverse cascade and shows that energy gets pumped into the long scale zonal flows.

Quasilinear analysis

Like the bump on tail instability a random collection of zonal flow regions reacts back on the drift waves giving them a diffusive spreading in k_r:

$$\frac{\partial N_k}{\partial t} - \frac{\partial}{\partial k_r} D \frac{\partial N_k}{\partial k_r} = \gamma_k N_k - \Delta\omega_k N_k^2$$

where D is the diffusion coefficient due to zonal flow induced shearing of drift wave turbulence:

$$D = \sum_q k_\theta^2 q^2 \left| \frac{\partial \psi_q}{\partial r} \right|^2 R(\Omega, q)$$

$$R(\Omega, q) = 1/(\Omega - \boldsymbol{q} \cdot \boldsymbol{v}_g + i\gamma_k)$$

RESONANCE OVERLAP AND DIFFUSION

When the zonal flows are random we get a diffusion of k_r namely

$$\delta k_r^2 \sim Dt$$

which indicates a direct cascade of drift wave energy to shorter scales through zonal flow interactions.

Zero dimensional models

Saturated states may be understood from a simple zero dim model. (Diamond et al 1998)

$$\frac{dN}{dt} = \gamma N - \alpha VN - \Delta N^2$$

Linear growth (γN); Diffusive damping (αVN); Nonlinear damping (ΔN^2)

$$\frac{dV}{dt} = \alpha VN - \nu V + (noise)$$

Modulational instability growth (αVN); Neoclassical damping (νV)

Saturated States:
$N=\nu/\alpha$; $V=(\gamma-\Delta\nu/\alpha)/\alpha$

Hence in the presence of zonal flow, saturated drift wave turbulence amplitude determined by zonal flow damping

$N/N_{mix\ length} \sim \nu L/c_s$

In the presence of noise complex nonlinear relaxation osillations may occur.

Drift wave-Zonal flow System

Nonlinear relaxation oscillations give intermittent patchy turbulence and bursty transport!! Observed in large scale gyrokinetic simulations.

Thus inclusion of zonal flow interaction modifies HM turbulence of drift waves profoundly and leads to novel intermittency effects.

Strong Plasma turbulence regime

As $\nu \rightarrow 0$; $N \rightarrow 0$

This is because with reduced damping the zonal flow becomes large and quenches the turbulence. In reality the plasma enters into a strongly turbulent state with the zonal flow modulations trapping drift waves in a manner analogous to particle trapping in large amplitude waves. Basically a spectrum of resonantly excited waves produces diffusion when autocorrelation time τ_{ac} is less than the bounce time τ_b and produces trapping effects when $\tau_b < \tau_{ac}$.

We now look for novel nonlinear stationary states with wave trapping effects. Large amplitude BGK wave type solutions !!

Nonlinear BGK Mode equations

$$(v_{gx} - U)\frac{\partial N_k}{\partial x} + k_y \frac{\partial}{\partial x}\left(v + \tau \frac{\partial^2 v}{\partial x^2}\right)\frac{\partial N_k}{\partial k_x} = 0$$

$$\left(\mu \frac{\partial^2}{\partial x^2} - \nu\right) v + U\frac{\partial v}{\partial x} = -\frac{\partial}{\partial x}\sum_k \frac{k_x k_y}{(1+k^2)^2} N_k$$

Characteristics of Kinetic equation are:

$$\frac{\mathrm{d}k_x}{\mathrm{d}x} = \frac{k_y(\mathrm{d}/\mathrm{d}x)(v + \tau(\mathrm{d}^2 v/\mathrm{d}x^2))}{v_{gx} - U}, \qquad \frac{\mathrm{d}k_y}{\mathrm{d}x} = 0$$

Giving a constant of motion:

$$W = v + \tau v'' - (1+k^2)^{-1} + \frac{k_x}{k_y} U, \qquad k_y = k_{y0}$$

And the solution:

$$N_k(k_x, k_y, x) \equiv N_k(W(k_x, x), k_y)$$

Wave trapping equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{-2k_x k_y}{(1+k^2)^2}$$

$$\frac{\mathrm{d}k_x}{\mathrm{d}t} = k_y \left[\frac{\mathrm{d}v}{\mathrm{d}x} + \tau \frac{\mathrm{d}^3 v}{\mathrm{d}x^3}\right]$$

$$\frac{\mathrm{d}k_y}{\mathrm{d}t} = 0$$

Near a minimum of zonal flow

$$\left(\frac{\mathrm{d}^2 k_x}{\mathrm{d}t^2}\right) + \frac{2k_y^2 v''_{\mathrm{m}}}{(1+k^2)^2} k_x = 0$$

A nonlinear oscillator with characteristic bounce frequencies of order $\sqrt{2k_y^2 v''_{\mathrm{m}}}$

Depending on the value of *W* (“energy”), the drift wave quasi - particles may either be trapped by zonal flow modulations or stay untrapped !

Velocity shear structure equation

For the nonlinear stationary states we make appropriate choices for the trapped and untrapped drift wave distributions (Bohm-Gross and power law) and arrive at the nonlinear structure equation for the zonal flow.

$$V'' + \bar{\mu} V' - K^2 V + \beta V^{3/2} = 0$$

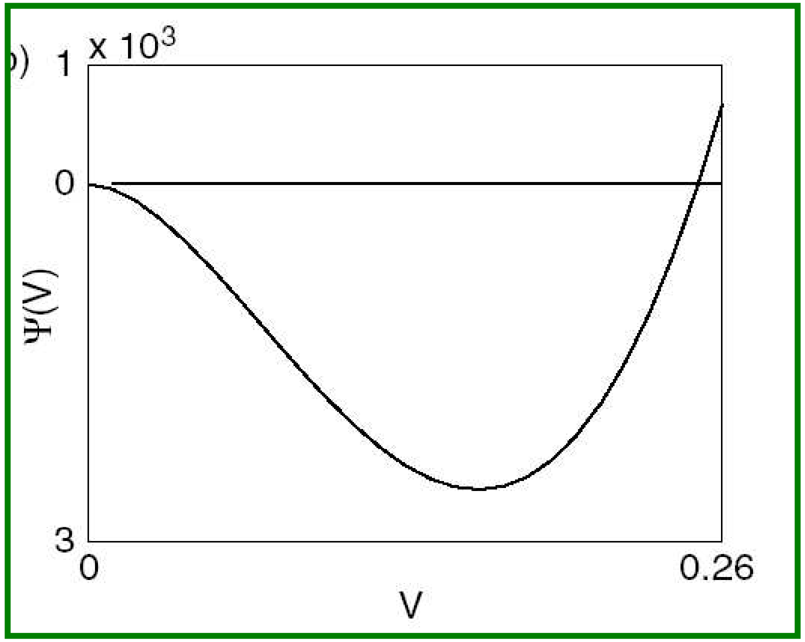
where the last term comes from the trapped drift wave trajectories. It must be appreciated that saturation of modulation instability in this theory arises because the resonant waves responsible for growth of the instability are now trapped in the zonal flow modulations and exchange energy back and forth with them. Thus typical saturation amplitudes are likely to be of order $\gamma\tau_b \sim 1$.

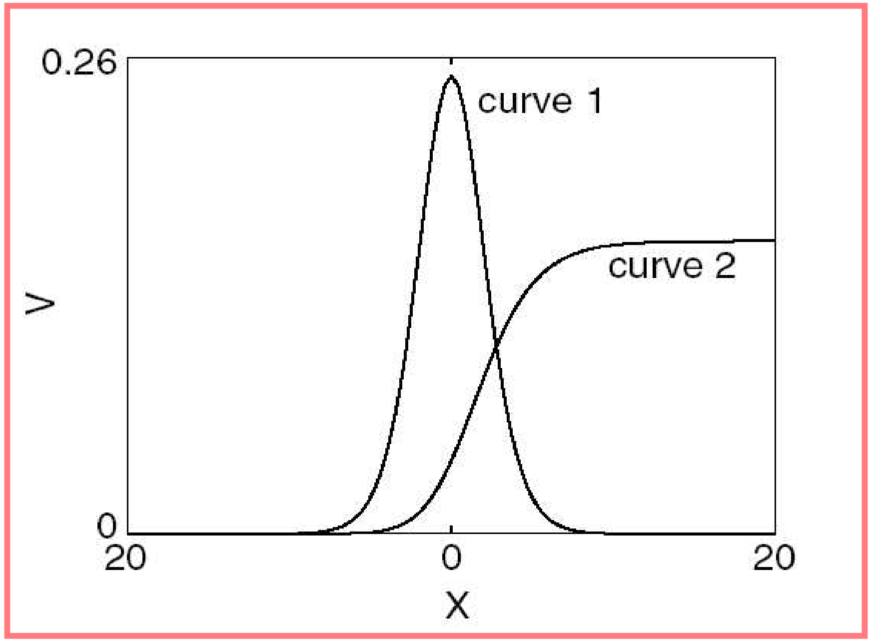
Pseudopotential for Nonlinear Zonal Flow Waves

Nonlinear solutions of this equation for various cases may be found by the Sagdeev pseudo-potential method. In the absence of viscosity μ the pseudopotential is given by

$$\Psi(V) = -K^2(V^2/2) + \frac{2}{5}\beta V^{5/2}$$

Novel Saturated states





Shocks and Solitons

SOLITONS (when viscosity is absent):

$$V = (25/16)(K^4/\beta^2)\operatorname{sech}^4[(K/4)(x - Ut)]$$

SHOCKS (when viscosity dominates):

$$V = \frac{K^4}{\beta^2}\left\{\frac{\exp(K^2(x - Ut)/\bar{\mu})}{[1 + \exp(K^2(x - Ut)/2\bar{\mu})]^2}\right\}.$$

Implications for Intermittency

Nonlinear entities in coupled zonal flow-drift wave turbulence may consist of such radially propagating solitons or shocks which are coherent zonal flow structures sustained by a sea of trapped and untrapped drift waves.

They can take energy in and out of unstable regions in inhomogeneous plasmas and form a novel class of dissipative structures in a driven system.

Implications……

We have investigated such nonlinear strctures in a variety of situations: plasma wave-ion acoustic wave coupling (Kaw et al 1975), coupled whistler wave- magnetosonic wave systems (Das, Singh, Kaw POP 2002), Alfven wave –ion acoustic wave system (Singh et al 2005).

How these structures contribute to intermittency in the strongly turbulent plasma needs to be quantitatively investigated. Some progress in obtaining PDF's for Zonal flow shear can be made using the instanton and mapping closure methods on the velocity structure equation (Kaw, Das and Singh '05) .

Summary and Discussion

- Plasma Intermittency in fluid models with no natural scales is well defined as a phenomenon leading to deviations from self similarity in the inertial range. New effects are due to wave propagation/interaction effects (Exs MHD, EMHD in $kd_e << 1$ or $kd_e >> 1$ regime etc).

- In fluid models with direct energy cascade and with natural scales in them new intermittency like effects may arise because of energy accumulation/structure formation at the natural scales (2D EMHD at $kd_e \sim 1$, 2D and 3D MHD/ Hall MHD at $kd_i \sim 1$ etc)

Summary …. Contd.

- In systems with natural scales and inverse cascade (like Hasegawa-Mima equation at Larmor radius scale) coherent structure formation and non-gaussian pdf's may arise but without significant impact on dissipative structures and scale similarity; these may have to be interpreted as fake intermittency effects.

- Genuine and novel intermittency effects may arise because of wave propagation and nonlinear structure formation by wave wave interaction effects which may produce patchiness and relaxation oscillations and take energy in an inhomogeneous plasma in and out of the unstable (energy injection) regions in physical space.

Summary …. Contd.

- Dissipative nonlinear plasma structures like shocks, current filaments and sheets, magnetic island chains, rotational and tangential discontinuities, waves with trapped particles etc may naturally form in plasmas and bring in dissipation on fractal objects giving novel scaling effects which then lead to intermittency like phenomena

Summary and Discussion

- The observed plasma intermittency phenomena in experiments and simulations need to be appropriately classified and distinguished from each other and the rich physics needs to be isolated with care .

Indirect evidences of intermittency

- PDF of velocity fields show highly non gaussian features due to such quasistationary structures. (Bofetta et al.)
- Single point PDF studies. Kurtosis of ϕ close to gaussian but $\nabla^2 \phi$ deviates strongly from gaussian value. Also seen from mapping closure studies (Das and Kaw POP 1995).
- Structure function scalings studies remain open for investigation.

Aspects which were not covered

- Intermittency as non local interaction in ‘k’ space.
- Modulation instabilities, shear flow generation. Impact on transport.
- PDF determination by mapping closure ~ emphasis on dissipation. Kraichnan, She et al. in fluids; Das and Kaw in Plasma.

Weak Turbulence theories

- Wave aspect given prominence.
- Prediction of the scaling index possible.
- Very successful in the context of gravity waves.
- Context of drift waves and other the scaling index does not agree with the simulations.
- Approach towards strong turbulence by broadening the resonance wave matching conditions.

Finally …

- Plasma turbulence fertile area.
- Determination of exact results! (has not been pursued so far)
- Standard intermittency studies have been limited. Most conclusions derived on the basis of indirect evidences.
- Presence of natural scales and quasi-structures: S_p scaling need not be a power law $\sim [f(r)]^{\zeta p}$ instead .. 'Intermittency like'!

Extra slides

In 2d

$$\left\langle \left(\delta v_r\right)^2 \right\rangle = \frac{\int \left(\delta v_r\right)^2 d^2 x}{\int d^2 x} = \int \left|v_k\right|^2 k dk = \int E\left(k\right) dk$$

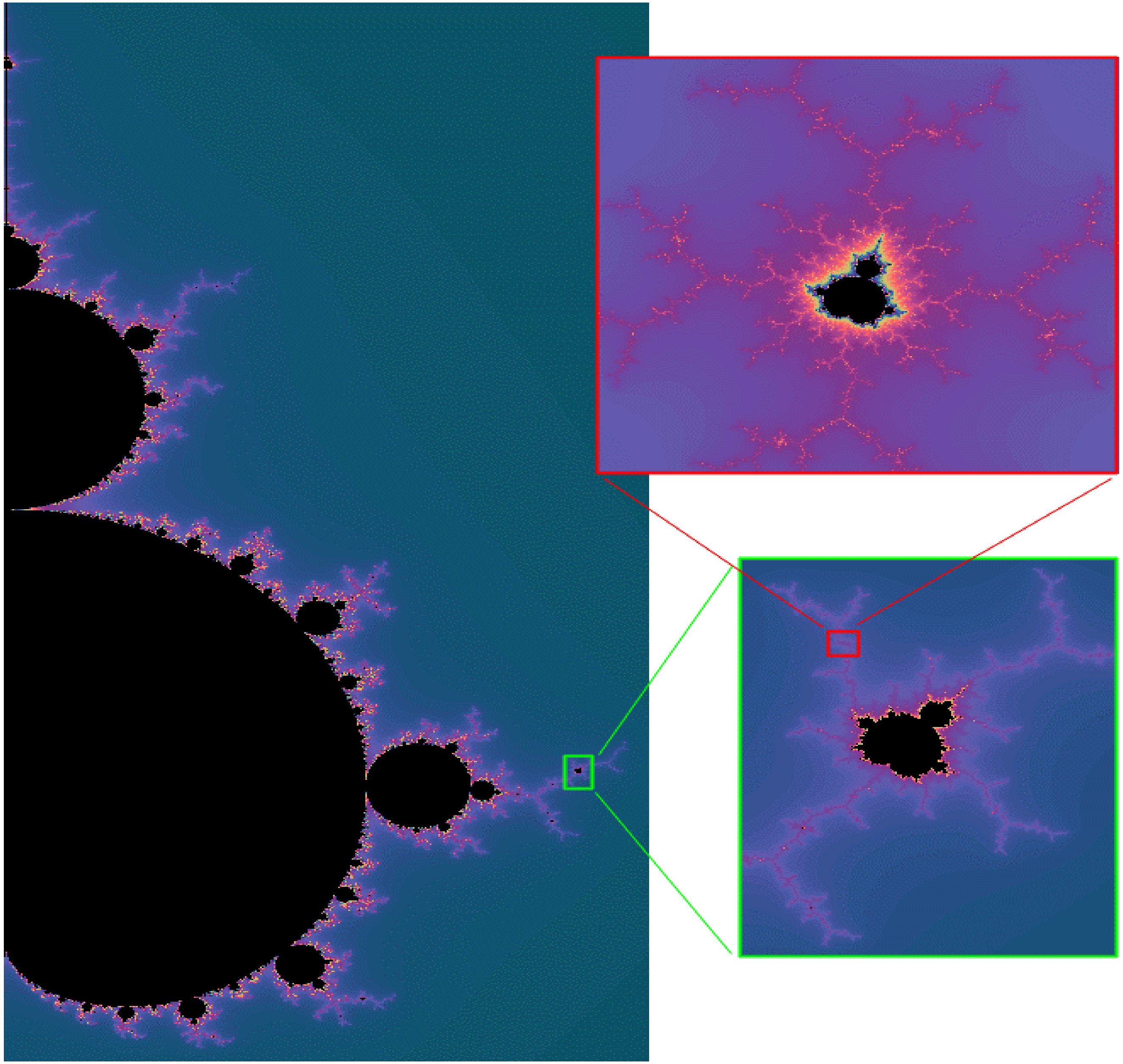
Turbulence: a necessary evil

Desirable consequences:

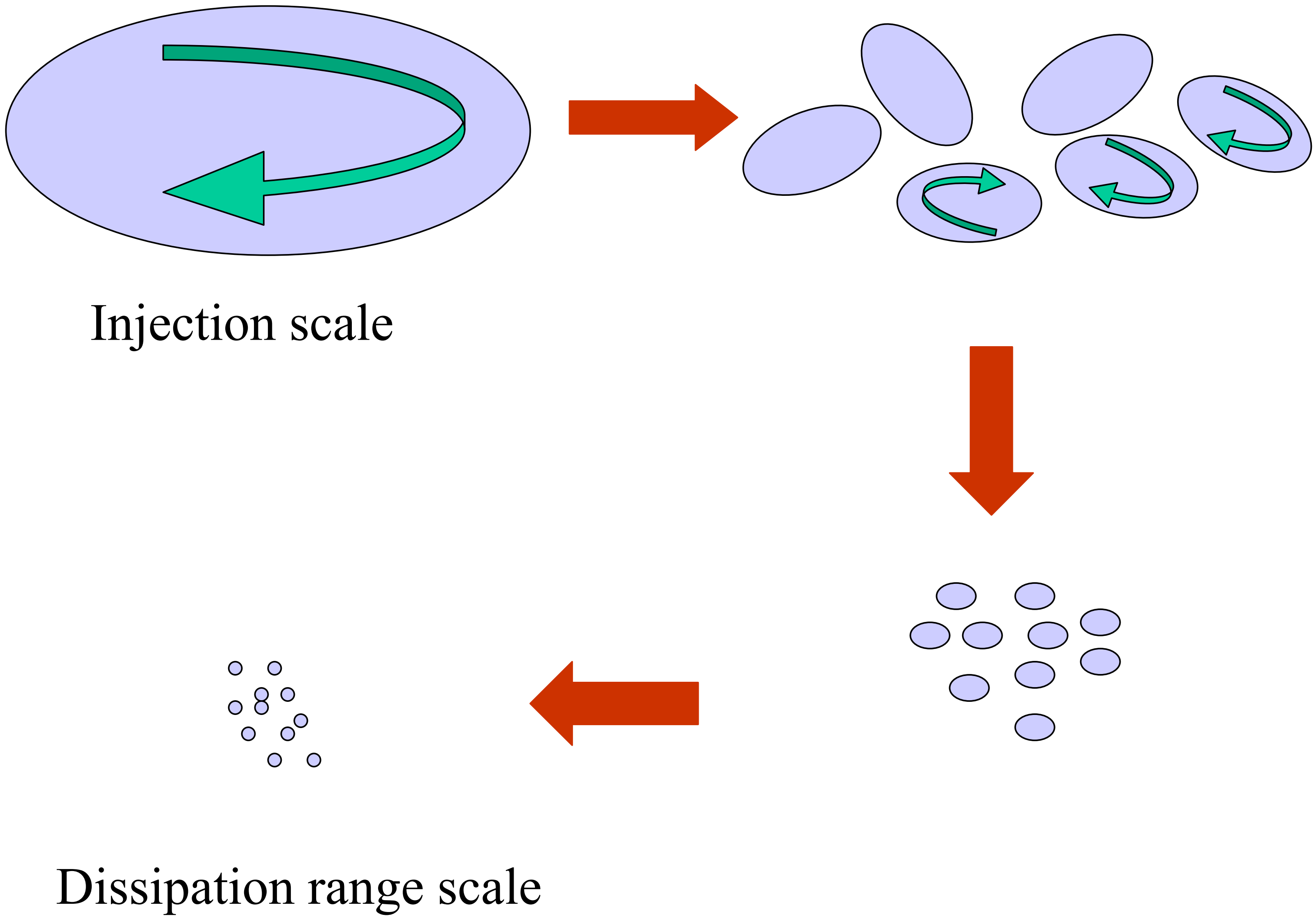
- Mixing of air and fuel in automobiles on useful time scales.
- Transport and dispersion of heat, pollutants etc.
- Fusion: removal of He ash.

Undesirable features:

- Enhances consumption of energy in automobiles, aircrafts etc.
- Air travel safety in question.
- Fusion: Confinement degradation.



Energy Cascade



$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{\mathrm{g}} \cdot \nabla - \frac{\partial}{\partial \boldsymbol{r}}\delta\omega \cdot \frac{\partial}{\partial \boldsymbol{k}}\right) N_k = 2\gamma_k N_k - \Delta\omega N_k^2$$

Numerical results

Extensive numerical simulations (Biskamp, Mueller 2000,2003) have shown that for isotropic 3D MHD the energy spectra are given by the Kolmogorov $k^{-5/3}$ scaling whereas for anisotropic case with significant mean magnetic field B_0, the perpendicular energy spectra are given by Iroshnikov-Kraichnan scaling $k^{-3/2}$. This is consistent with the ideas put forward by Goldreich – Sridhar (1995) that wave interaction effects are crucial for anisotropic case.

Intermittency effects have likewise shown significant influence of mean magnetic fields. For the isotropic case with no mean magnetic fields one finds intermittency consistent with a generalized She Levegue formula (Biskamp and Mueller 2000)