Comparisons of gyrokinetic PIC and CIP codes

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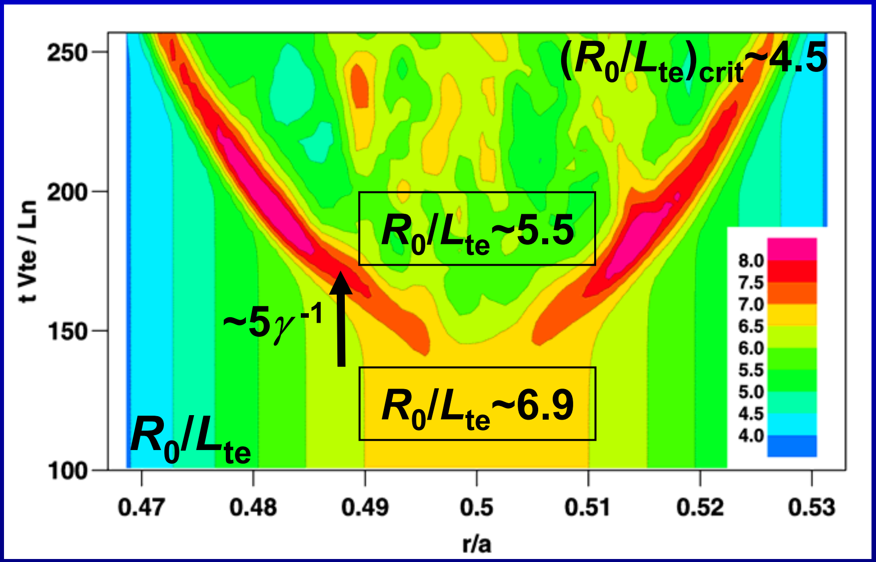
Outline

- Introduction
- Gyrokinetic Vlasov CIP code
- Comparisons of ITG simulations between PIC and CIP
- Summary

Motivation to develop gyrokinetic Vlasov code

- In toroidal simulation, strong profile relaxation is often observed
  - difficult to get quasi-steady χ
  - → In reality, χ may be defined for quasi-steady profile balanced with heat source/sink

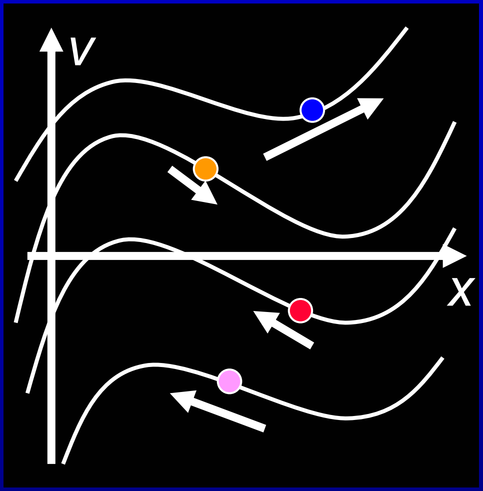
ETG turbulence in PS tokamaks



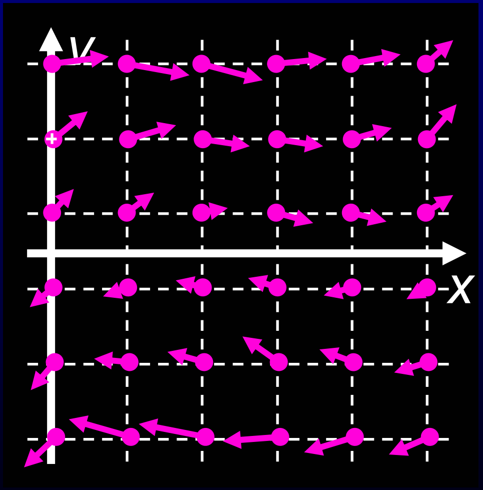
- Issues in realistic long time simulation of tokamak micro-turbulence
  - Heat/particle-source/sink
    - → determine transport level balanced with heat source/sink
    - → simulate profile formation, modulation experiment
  - Collision
    - → collisional zonal flow damping, neoclassical effects
    - → eliminate fine structures in phase space

Main features of PIC and Vlasov simulations

- Particle-In-Cell (PIC) simulation
  - nonlinear δf PIC method (Parker 1993)
    → $DF/Dt=0$, $DG/Dt=0$ are assumed
  - difficult to implement non-conservative effects
  - limited for turbulent time scale simulation
  - relatively small memory usage
  - full torus global calculation is possible



- Vlasov simulation
  - CFD scheme in 5D phase space
    → difficult to find stable CFD scheme
  - huge memory usage
  - limited for local flux tube model
  - non-conservative effects can be implemented
  - long time simulation is possible

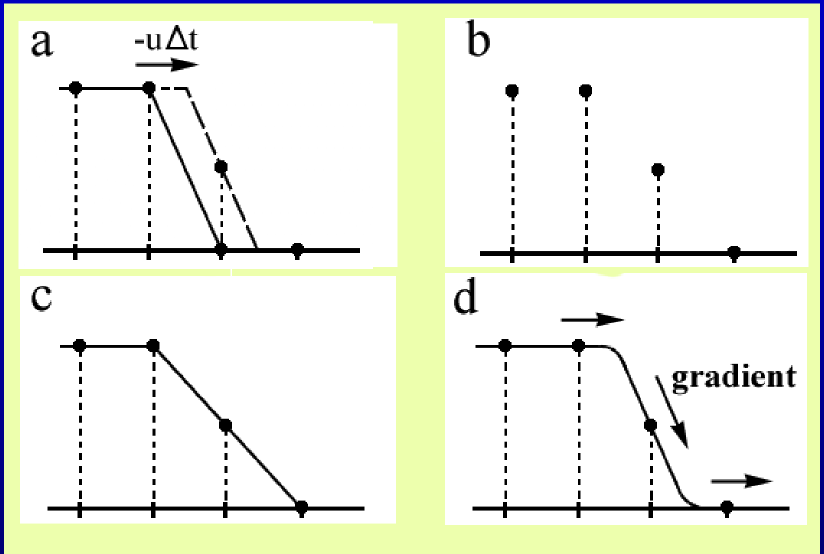


Main concept of CIP method (Yabe 1991)

CIP: Constrained Interpolation Profile method

- Let us consider a simple advection equation

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = 0$$



→linear interpolation causes numerical diffusion

→higher order spline causes numerical oscillations

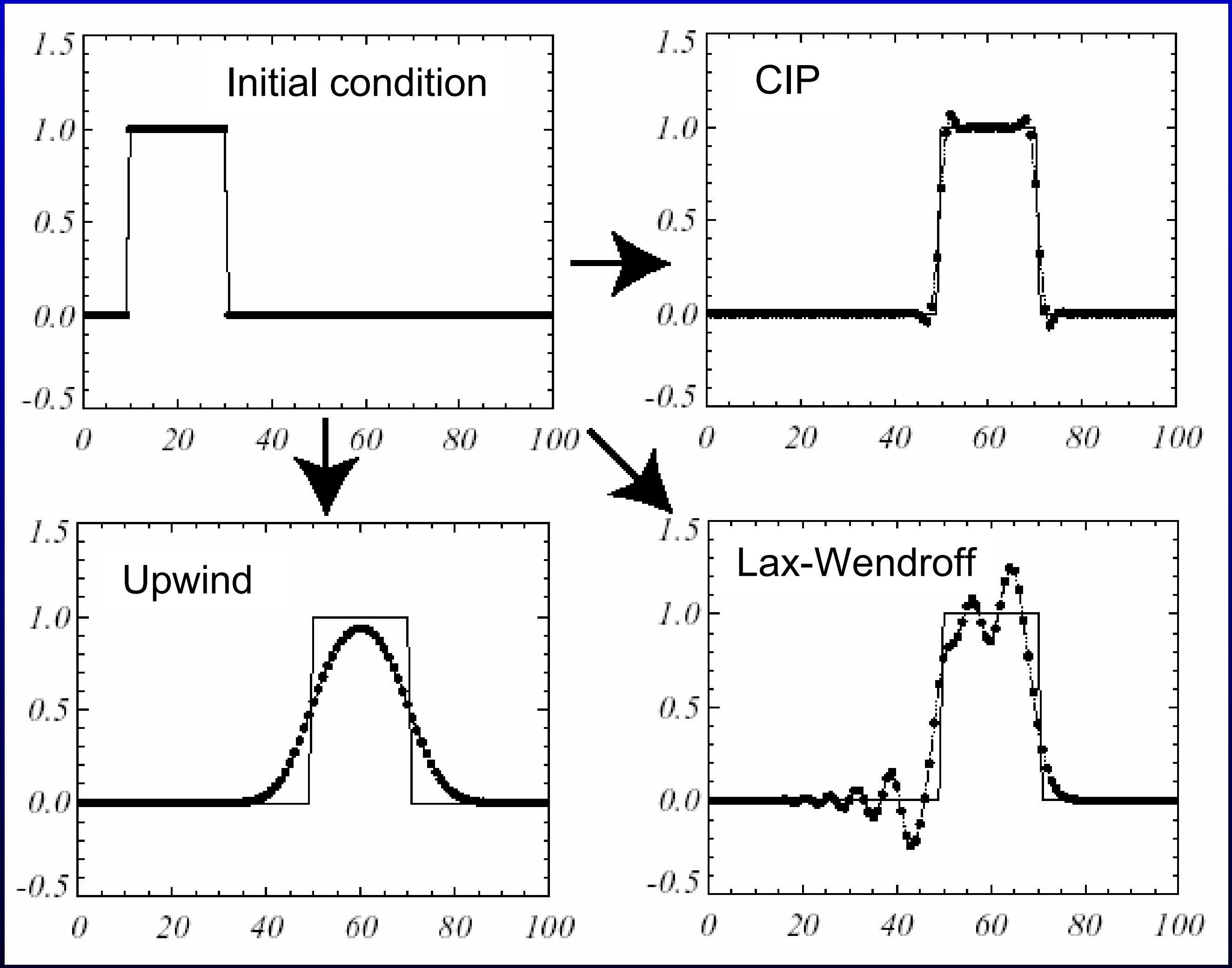
- Keep information between grids by solving $g = \frac{\partial f}{\partial x}$

$$\frac{\partial g}{\partial t} + u\frac{\partial g}{\partial x} = -\frac{\partial u}{\partial x}g$$

→ Hermite interpolation

Comparison among CIP and other methods

- Propagation of square wave (after 200 time steps) (Kudoh 2002)



ITG turbulence in 4D drift-kinetic system

- 4D drift-kinetic-gyrokinetic-Poisson system

$$\frac{\partial f}{\partial t} - \frac{c}{B_0}\frac{\partial \phi}{\partial y}\frac{\partial f}{\partial x} + \frac{c}{B_0}\frac{\partial \phi}{\partial x}\frac{\partial f}{\partial y} + v\frac{\partial f}{\partial z} - \frac{e}{m}\frac{\partial \phi}{\partial z}\frac{\partial f}{\partial v} = 0$$

$$-\left(\nabla^2 + \nabla_\perp \cdot \frac{\rho_{ti}^2}{\lambda_{Di}^2}\nabla_\perp\right)\phi + \frac{1}{\lambda_{De}^2}\left(\phi - \langle\phi\rangle\right) = 4\pi e\left(\int f dv - n_0\right)$$

- Numerical model
  - Time integration using directional splitting (Cheng 1976)

    2D CIP(x-y) and 1D CIP(z,v)

    leap-frog like splitting rule → $xy/2$-$z/2$-v-$z/2$-$xy/2$
  - Field solver using FFT
  - Fourier filter to emulate 2D-FEM(x-y) and 1D-FSP(z) in PIC

$$S_m(k_x,k_y,k_z) = \left(\frac{\sin\,\Delta x k_x/2}{\Delta x k_x/2}\right)^{m+1}\left(\frac{\sin\,\Delta y k_y/2}{\Delta y k_y/2}\right)^{m+1}\exp\left[-\frac{1}{2}\left(\frac{k_z}{k_0}\right)^2\right]$$

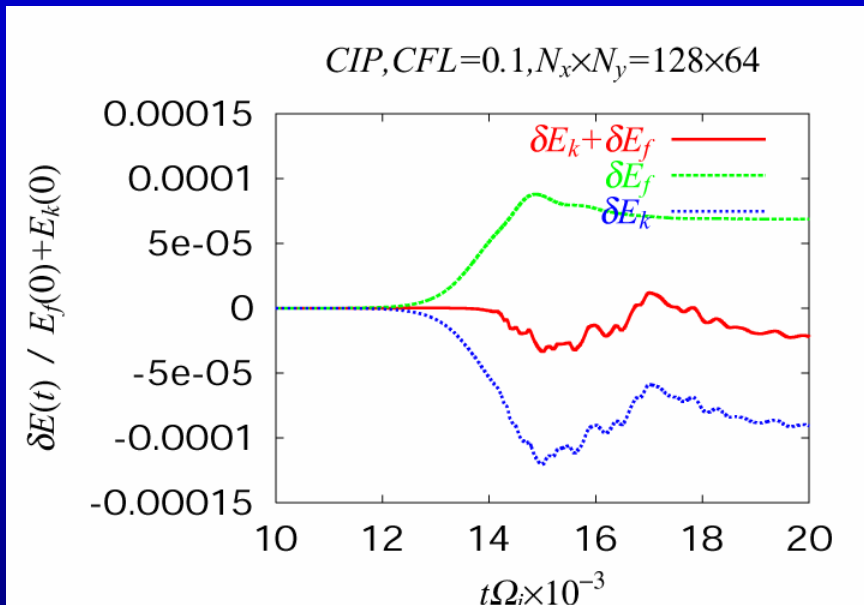
m : order of spline function, k_0 : width of gaussian FSP in k_z

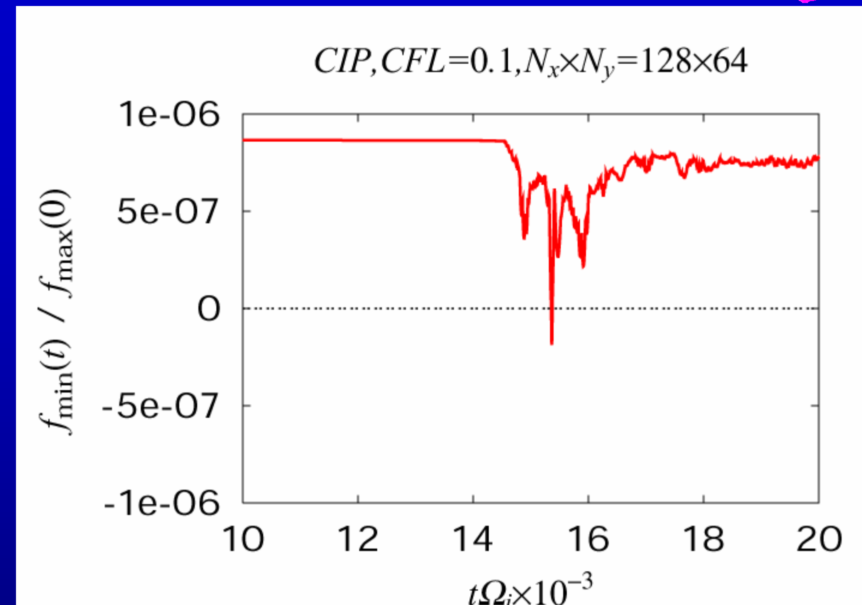
Benchmark parameter

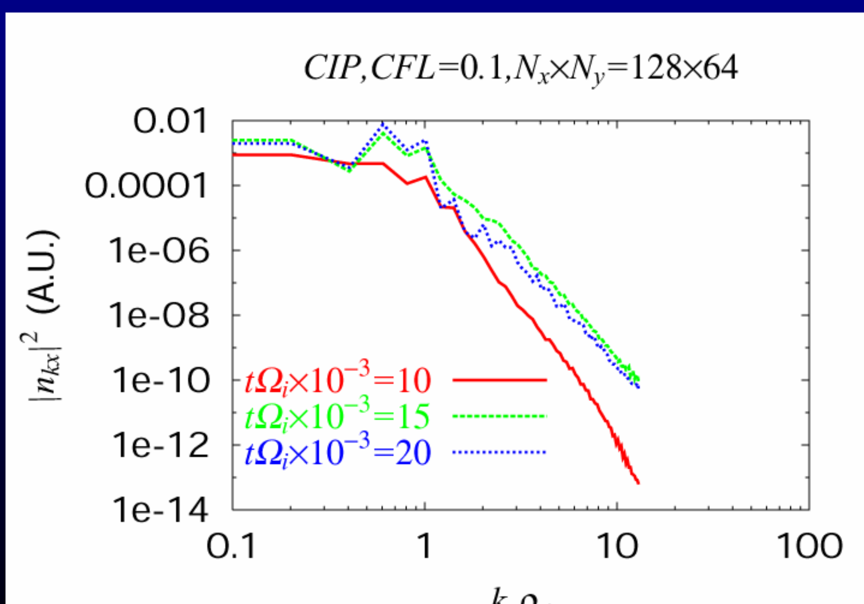
Ion temperature gradient driven (ITG) turbulence is simulated

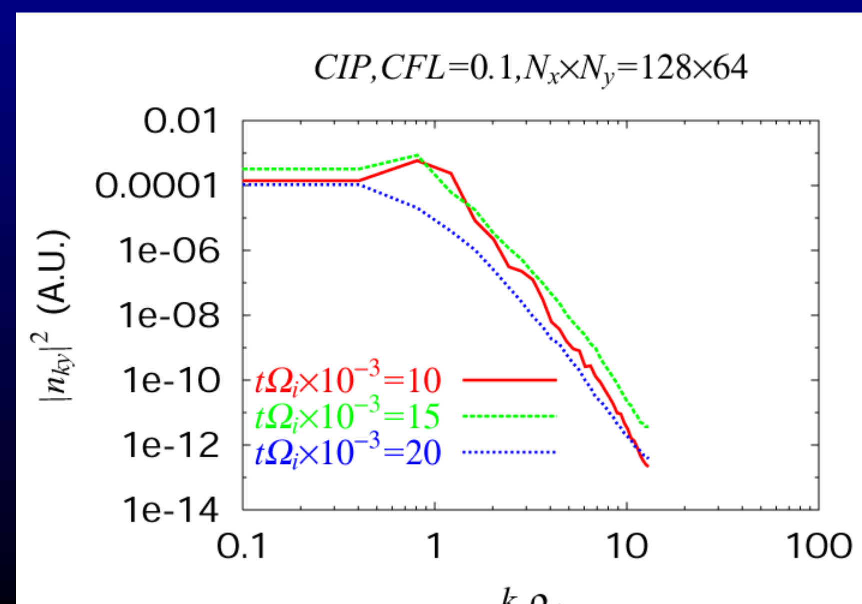
- Calculation model
  - slab geometry (*x*,*y*,*z*), periodic in *x*, *y*, *z* directions
  - fixed boundary in *v* direction $f(v_{\max}) = f(v_{\min}) = 0$
  - uniform **B** in *z* direction, no magnetic shear
  - flat *n*, T_e profiles
  - T_i profile $T_i = c_0\left[1 + c_2 \sin(\pi x / L_x)^2\right]$
- Benchmark parameters
  - m_i=1836m_e, B_0=2.5T, T_{i0}=T_e=5keV, L_{ti}=0.3×128ρ_{ti}
  - L_x=2L_y=32ρ_{ti},L_z=8000ρ_{ti},L_v=±5v_{ti}
- Standard case
  - CFL=0.1, $N_x \times N_y \times N_z \times N_v$=128×64×16×64

Numerical properties of GK Vlasov CIP code









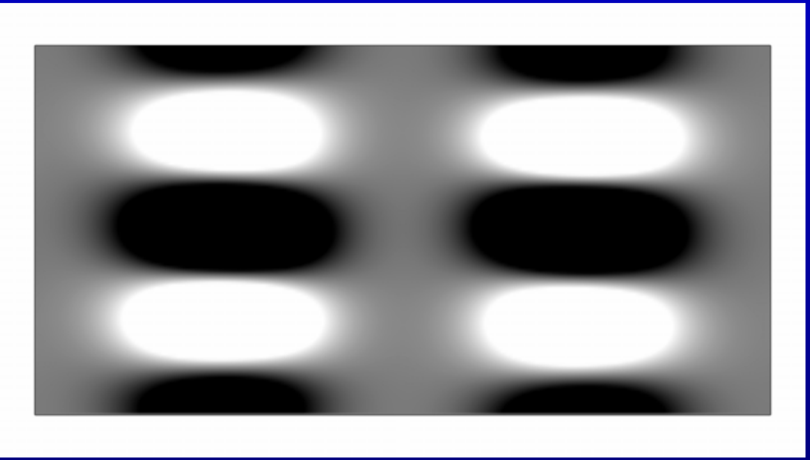
Gyrokinetic slab PIC code G3D

- Numerical model
  - finite element δ f PIC method
  - 2D FEM(x-y) + Fourier mode expansion (z)
  - 4^{th} Runge-Kutta method
- Calculation model
  - slab geometry (x,y,z), periodic in x, y, z directions
  - uniform **B** in z direction, no magnetic shear
  - flat n, T_e profiles
  - T_i profile $T_i = c_0\left[1 + c_2 \sin(\pi x / L_x)^2\right]$
- Benchmark parameters
  - m_i=1836m_e, B_0=2.5T, T_{i0}=T_e=5keV, L_{ti}=0.3×128ρ_{ti}
  - L_x=2L_y=32ρ_{ti},L_z=8000ρ_{ti}
- Standard case
  - Δt=20Ω_i^{-1}, $N_x \times N_y$=16×16, k_z=0~6/(2πL_z)

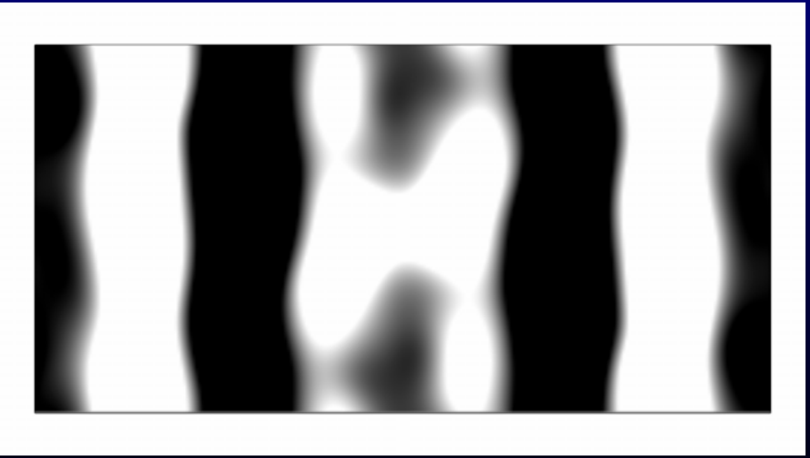
Linear eigenfunction and zonal flows

- CIP

linear phase

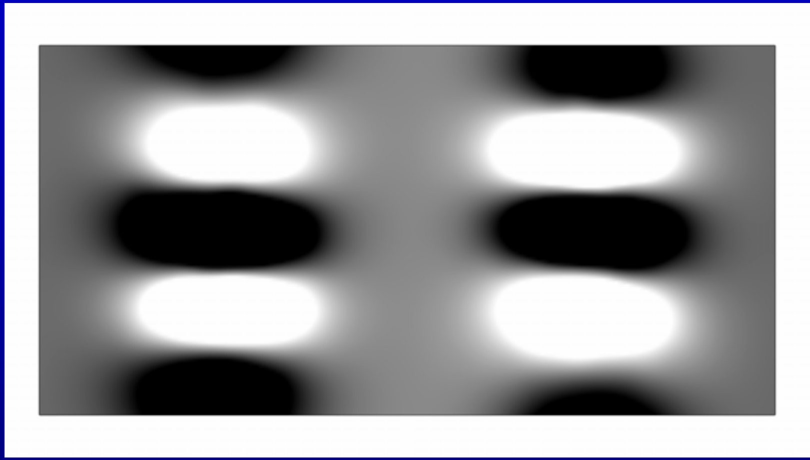


nonlinear phase

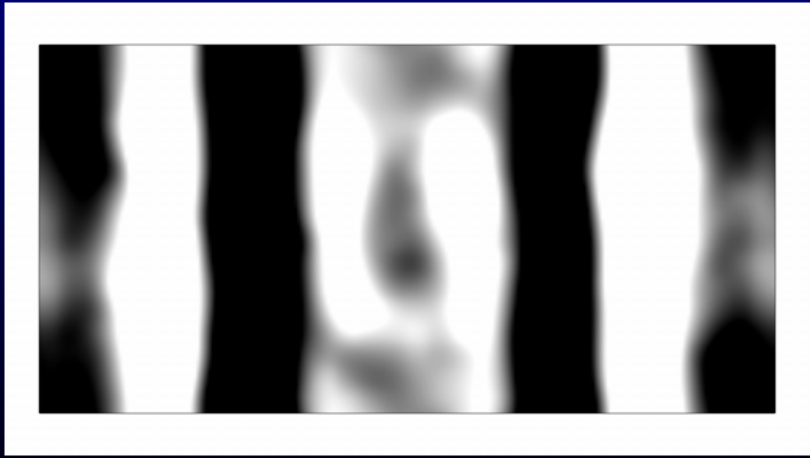


- PIC

linear phase

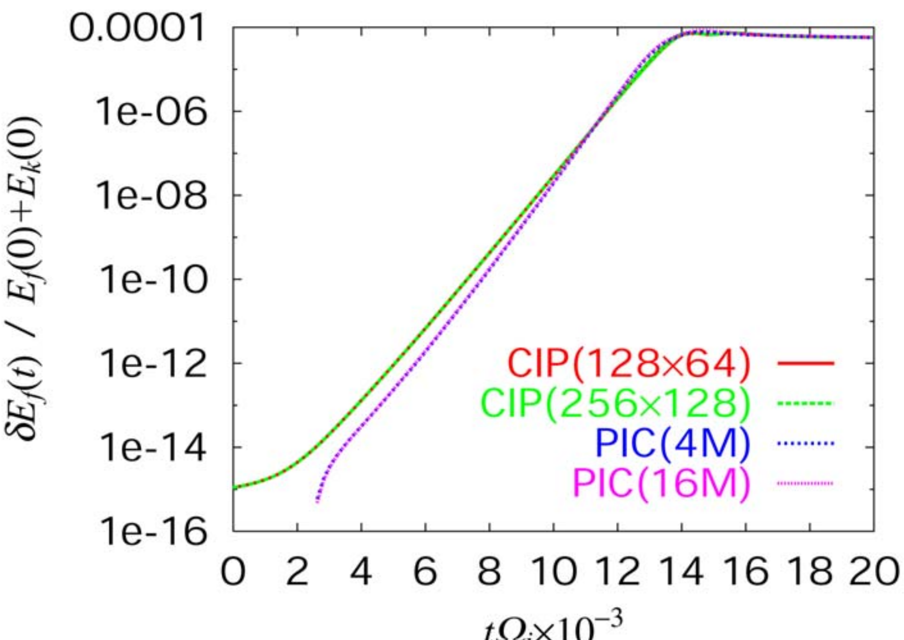


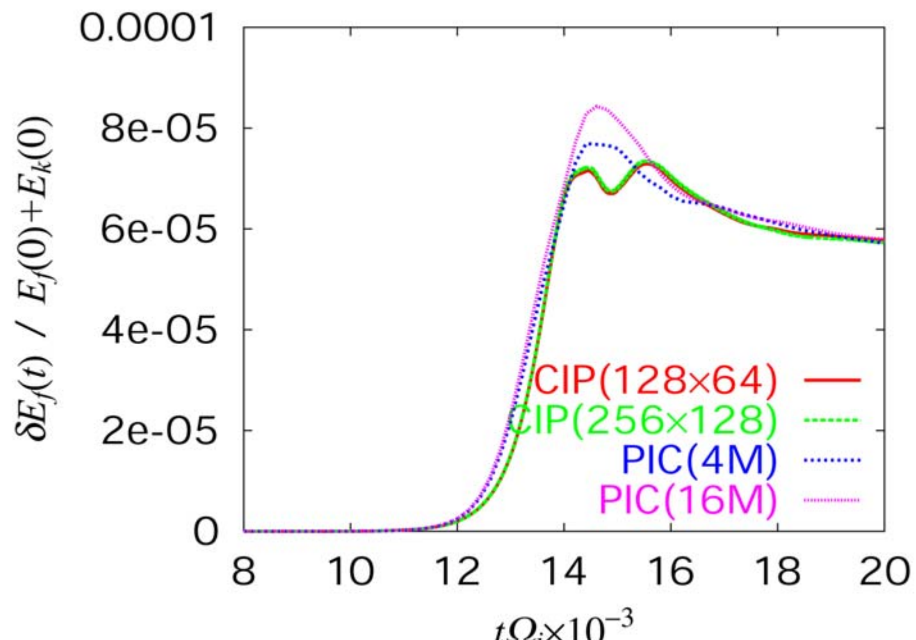
nonlinear phase



Linear growth rates and saturation amplitude

- Linear growth rates
- Saturation amplitude

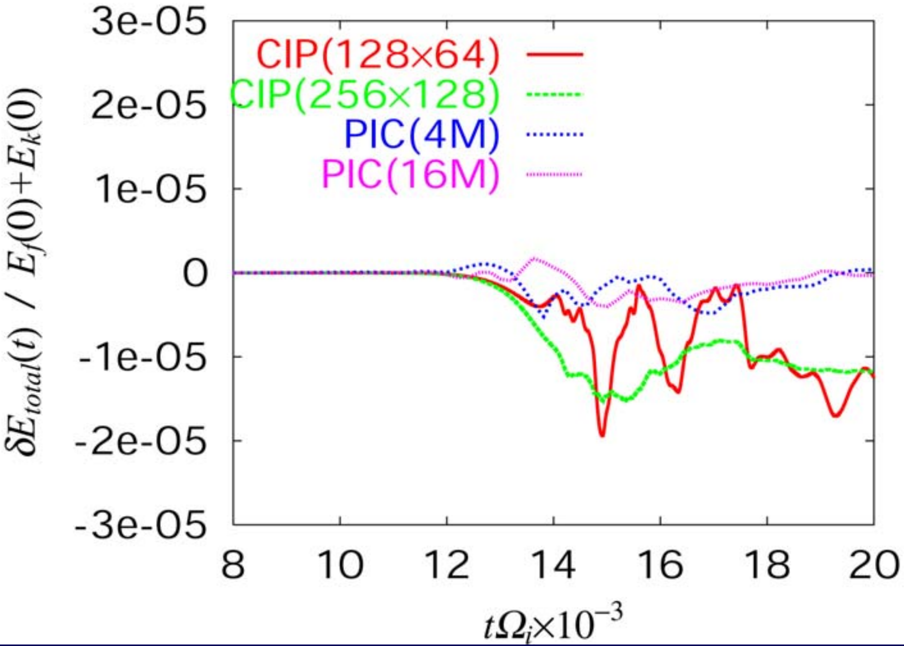


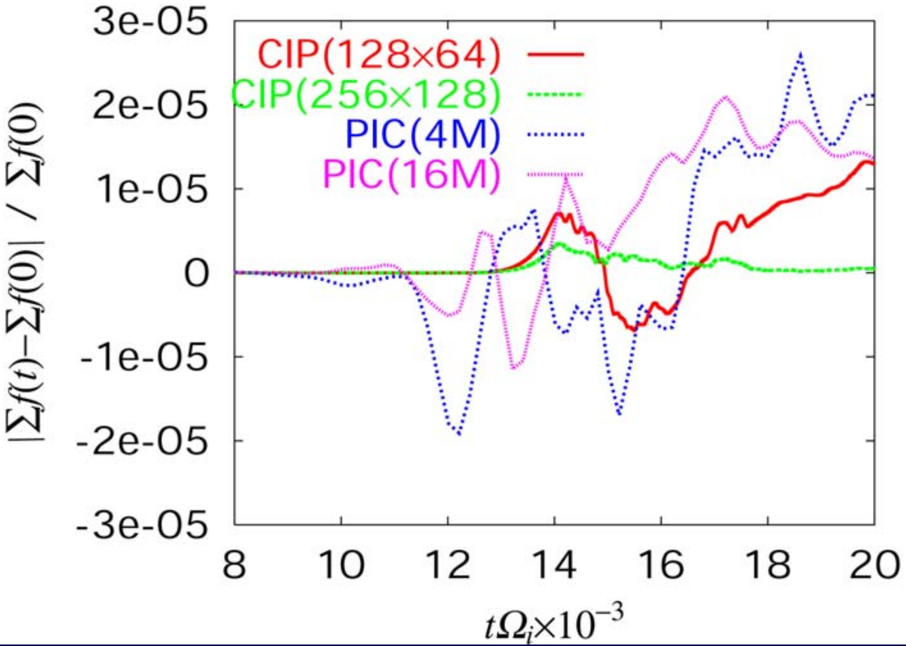


- Results are converged against mesh/particle number
- Linear growth rates in CIP and PIC codes differ by ~7%
- Saturation levels coincide with each other

Energy and particle conservation

- Energy conservation
- Particle conservation





- Both codes show reasonably good energy and particle conservations $<2 \times 10^{-5}$
- PIC (CIP) code gives better energy (particle) conservation

Summary

- 4D drift-kinetic-GK-Poisson system is solved using CIP method
  - Code is stable (positivity is satisfied, converged spectrum)
  - Relative errors of particle and energy conservations are $<2\times10^{-5}$
  - ITG growth rate and saturation level agree well with PIC code

  → Results obtained are almost equivalent to PIC code
- Computational cost on JAERI Origin3800 system
  - CIP ($N_x \times N_y$=128×64) ~1.7GB,120Gflops･h (32PE 3.8h)
  - PIC (4M particles) ~ 27GB, 35Gflops･h (64PE 0.5h)

  → Vlasov code is possible solution to study non-conservative effects
- Future works
  - development of 5D toroidal code
  - benchmark against gyrokinetic toroidal PIC code GT3D
  - development of heat source, collisions etc…