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Turbulent fluxes and entropy production rate

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Motivation

- Particle and heat fluxes versus density and temperature gradients?
- Turbulence EquiPartition Theory (fluid) Isichenko and Yankov 97

$$\Gamma = -D_{turb}\left(\partial_r n_{eq} + \frac{2}{L_{\nabla B}} n_{eq}\right)$$

$$Q = -\frac{1}{\varsigma - 1} D_{turb}\left(\partial_r T_{eq} + (\varsigma - 1)\frac{2}{L_{\nabla B}} T_{eq}\right) \quad ; \quad \varsigma = \frac{5}{3}$$

- Traditional formulation

$$\begin{pmatrix} \dfrac{\Gamma}{n_{eq}} \\ \dfrac{Q}{n_{eq} T_{eq}} \end{pmatrix} = -\begin{bmatrix} D_{nn} & D_{nT} \\ D_{Tn} & D_{TT} \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial_r n_{eq}}{n_{eq}} \\ \dfrac{\partial_r T_{eq}}{T_{eq}} \end{pmatrix}$$

Thermodiffusion (D_{nT})

B. Coppi & Spight 78,

Tang et al. 86, Terry 89,

Waltz & Dominguez 89

Nordman, Weiland and Jarmèn 90

Heat pinch (D_{Tn})

Weiland & Nordman 93

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Motivation (cont.)

- TEP and thermodiffusion terms are additive in particle flux Baker and Rosenbluth 00

- Raises several questions:

  - what is the thermodynamical meaning of TEP terms?
  - does Onsager symmetry principle apply?
  - implications for the heat flux
  - are pinches consistent with the second principle?

$\rightarrow$ calculate entropy production rate.

Outline

- Experimental evidence of off-diagonal terms in particle flux.
- Introduction to pinch mechanisms: Turbulence Equipartition Theory. Extension to interchange turbulence: curvature and thermodiffusion.
- Entropy production rate: fluid and kinetic theories.
- Examples: particle (e/D) and impurity transport.

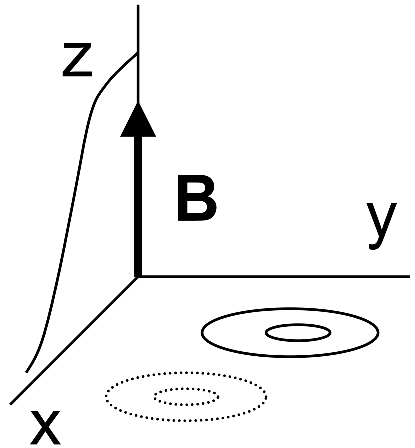
Turbulence EquiPartition Theory

- ExB convection in an inhomogeneous magnetic field

Isichenko and Yankov 97, Naulin 98.

$$\partial_t n + \nabla\cdot(n\mathbf{v_E}) = 0 \quad ; \quad \mathbf{v_E} = \frac{\mathbf{B}}{B^2}\times\nabla\phi$$

- Equivalent to the advection of n/B



$$(\partial_t + \mathbf{v_E}\cdot\nabla)\left(\frac{n}{B}\right) = 0$$

- Quasi-linear theory

$$\partial_t n_{eq} = \partial_x D_{turb} B \partial_x\left(\frac{n_{eq}}{B}\right)$$

- Relaxes toward canonical profile $n_{eq} = n_{can} = B(x)$

A step further : 2D interchange turbulence

- Equations for density and temperature Braginskii 65

$$\frac{d_t n}{n} = -\boldsymbol{\kappa}.\left(\nabla\phi + \frac{T}{e}\frac{\nabla n}{n} + \frac{\nabla T}{e}\right)$$

$$\frac{d_t T}{T} = -\boldsymbol{\kappa}.\left((\varsigma - 1)\nabla\phi + (\varsigma - 1)\frac{T}{e}\frac{\nabla n}{n} + (2\varsigma - 1)\frac{\nabla T}{e}\right)$$

$$d_t = \partial_t + \mathbf{v_E}\cdot\nabla \quad ; \quad \boldsymbol{\kappa} = \lambda\frac{2}{B}\left(\frac{\mathbf{B}}{B}\times\frac{\nabla B}{B}\right) \quad ; \quad \varsigma = \frac{5}{3}$$

- Curvature terms come from compressibility. Yancov $\lambda=1/2$, Weiland model $\lambda=1$, trapped electrons $\lambda=1/4+2s/3$ ($s=rdq/qdr$) $\rightarrow$ link between density and current See e.g. H. Weisen 04, GLF23: another function….
- Here λ and adiabatic index $\varsigma > 1$ are free parameters.

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Turbulent fluxes

- Expression of fluxes

$$\begin{pmatrix} \dfrac{\Gamma}{\overline{n}_{eq}} \\ \dfrac{Q}{\overline{n}_{eq}\overline{T}_{eq}} \end{pmatrix} = -\sum_{\mathbf{k}\omega} i \frac{\left|\tilde{v}_{E,\mathbf{k}\omega}\right|^2}{D_{\mathbf{k}\omega}} \begin{bmatrix} \omega-(2\varsigma-1)\omega_d & \omega_d \\ \omega_d & \dfrac{\omega-\omega_d}{\varsigma-1} \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial_r \overline{n}_{eq}}{\overline{n}_{eq}} \\ \dfrac{\partial_r \overline{T}_{eq}}{\overline{T}_{eq}} \end{pmatrix}$$

$$\overline{n} = \frac{n}{n_{can}} \qquad \overline{T} = \frac{T}{T_{can}}$$

$$n_{can} = n_0 \exp\left\{-\frac{2}{R_0}\int_0^r \lambda dr\right\} \qquad T_{can} \propto n_{can}^{\varsigma-1}$$

- Onsager symmetry is found. Does not guarantee 2nd principle is respected → entropy production rate.

TEP limit

- When $\omega \gg \omega_D$

$$d_t \bar{n} = 0 \quad ; \quad d_t \bar{T} = 0 \quad ; \quad d_t = \partial_t + \mathbf{v_E} \cdot \nabla$$

- Leads to turbulent diffusion equation

$$\begin{pmatrix} \Gamma \\ Q \end{pmatrix} = -\begin{bmatrix} D_{turb} & 0 \\ 0 & \frac{1}{\varsigma - 1} D_{turb} \end{bmatrix} \begin{pmatrix} \partial_r \bar{n}_{eq} \\ \bar{n}_{eq} \partial_r \bar{T}_{eq} \end{pmatrix}$$

$$D_{turb} = \sum_{\mathbf{k}} \frac{\left| \tilde{v}_{E\mathbf{k}} \right|^2}{\Delta\omega_{\mathbf{k}}}$$

- Diffusion applies on profiles normalised to canonical values: thermodynamical forces.

Equivalence with pinch formulation

- Equivalent to particle curvature pinch

$$\Gamma = -D_{turb}\partial_r \overline{n}_{eq} = -D_{turb}\left(\partial_r n_{eq} + \frac{2\lambda}{R_0} n_{eq}\right)$$

- Heat pinch also predicted

$$Q = -\frac{1}{\varsigma - 1} D_{turb} n_{eq}\left(\partial_r T_{eq} + \frac{2\lambda}{R_0}(\varsigma - 1) T_{eq}\right)$$

- Quasi-linear theory not mandatory. Works also for fractional kinetics.

Entropy production rate

- Total entropy $S = \int d^3\mathbf{x} n \mathrm{Log}\left(T n^{1-\varsigma}\right)/(\varsigma - 1)$

  $\partial_t S = 0$ → does not tell much.

e.g. Hazeltine and Meiss 2003

- Entropy for profiles $S_{eq} = \frac{1}{\varsigma - 1}\int dV n_{eq} \mathrm{Log}\left(T_{eq} n_{eq}^{1-\varsigma}\right)$

$$\partial_t S_{eq} = -\int dV n_{eq} \left\{ \frac{\Gamma}{\overline{n}_{eq}} \frac{\partial_r \overline{n}_{eq}}{\overline{n}_{eq}} + \frac{Q}{\overline{n}_{eq}\overline{T}_{eq}} \frac{\partial_r \overline{T}_{eq}}{\overline{T}_{eq}} \right\}$$

- $\partial_t S_{eq} > 0$ whatever the spectrum of fluctuations

- Consistent with definition of thermodynamical fluxes and forces.

General form of fluxes

- Practical expressions of fluxes

$$\begin{pmatrix} \dfrac{\Gamma}{\overline{n}_{eq}} \\ \dfrac{Q}{\overline{n}_{eq}\overline{T}_{eq}} \end{pmatrix} = -\begin{bmatrix} D_{nn} & D_{nT} \\ D_{nT} & D_{TT} \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial_r \overline{n}_{eq}}{\overline{n}_{eq}} \\ \dfrac{\partial_r \overline{T}_{eq}}{\overline{T}_{eq}} \end{pmatrix}$$

$$D_{nn} \approx D_{turb} \quad ; \quad D_{TT} \approx \frac{1}{\varsigma - 1} D_{turb}$$

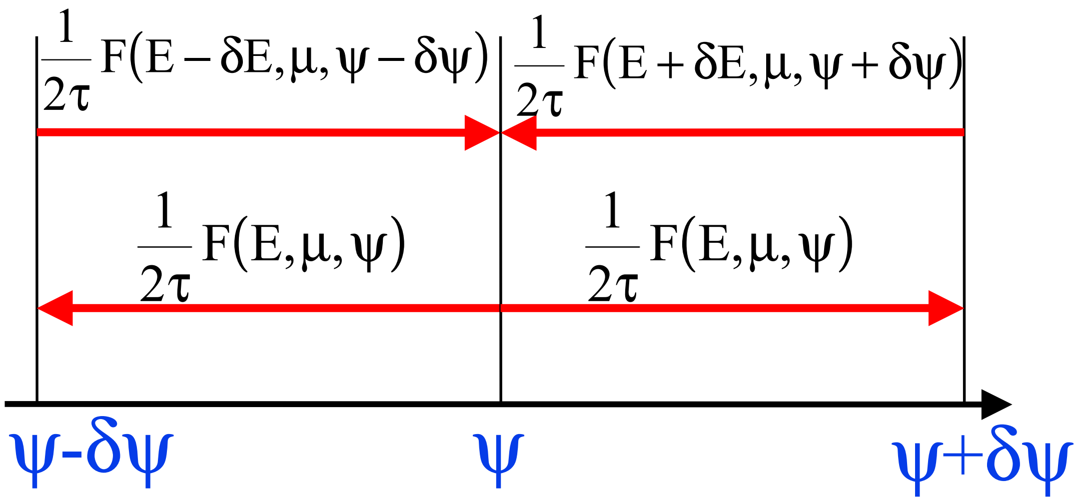
$$D_{nT} \approx \left\langle \frac{\omega}{\omega_d} - \varsigma \right\rangle = \sum_{\mathbf{k}} \frac{\left|\tilde{v}_{E\mathbf{k}}\right|^2}{\Delta\omega_{\mathbf{k}}} \frac{2(\omega_{\mathbf{k}} - \varsigma\omega_d)\omega_d}{\Delta\omega_{\mathbf{k}}^2}$$

- D_{nT} sign depends on fluctuation spectrum. Electron pinch velocity is inward for phase velocities in the ion diamagnetic direction.

Turbulence EquiPartition - Kinetics

Isichenko, Gruzinov & Diamond 96 - Baker & Rosenbluth 98

- Trapped particles : adiabatic invariant $J_{//}(E,\mu,\psi)=\oint mv_{//}d\ell$
- Energy variation $(\partial_E J_{//})\delta E+(\partial_\psi J_{//})\delta\psi=0 \rightarrow \delta E=\Omega_d\delta\psi$



- Transport equation $\partial_t n+\partial_\psi \Gamma=0$

$$\Gamma=-D\int\frac{dEd\mu}{\Omega_b}\left(\partial_\psi F+\Omega_d\partial_E F\right) \quad ; \quad D=\frac{\delta\psi^2}{2\tau}$$

General QL expression for the kinetic entropy production rate

- If a tokamak, the entropy production rate is always positive when using a quasi-linear kinetic theory

$$\partial_t S_{eq} = \pi \int d^3\mathbf{J}\,(2\pi)^3 F_{eq} \sum_{\mathbf{n}\omega} \left| h_{\mathbf{n}\omega} \right|^2 \delta(\omega - \mathbf{n}.\boldsymbol{\Omega}) \left[\mathbf{n} \cdot \partial_{\mathbf{J}} \mathrm{Log}\left(F_{eq}\right) \right]^2$$

Action variables → $d^3\mathbf{J}$

Hamiltonian components → $h_{\mathbf{n}\omega}$

Resonances → $\delta(\omega - \mathbf{n}.\boldsymbol{\Omega})$

Equilibrium distribution function → F_{eq}

Samain 70

Entropy production rate also involves canonical profiles

- Trapped particles

$$\partial_t S_{eq} = \pi \int dV n_{eq} \sum_{\mathbf{k}\omega} \left| v_{E,\mathbf{k}\omega} \right|^2 \left\langle \delta(\omega - \omega_d) \left\{ \frac{\partial_r \overline{n}_{eq}}{\overline{n}_{eq}} + \left(E - \frac{3}{2} \right) \frac{\partial_r \overline{T}_{eq}}{\overline{T}_{eq}} \right\}^2 \right\rangle_E$$

- In the limit $\omega >> \omega_D$, gives the same result as fluid equations, including reversal of thermodiffusion sign.

Passing particles

- Final expression:

$$\partial_t S_{eq} = \pi \int dV n_{eq} \sum_{\mathbf{k}\omega} \left| v_{E,\mathbf{k}\omega} \right|^2$$

$$\left\langle \delta\left(\omega - \omega_d - k_{//} v_{T_{//}} \zeta\right) \left\{ \frac{\partial_r \overline{n}_{eq}}{\overline{n}_{eq}} + \frac{1}{2}\left(\zeta^2 - 1\right) \frac{\partial_r \overline{T}_{//eq}}{\overline{T}_{//eq}} \right.\right.$$

$$\left.\left. + (h-1) \frac{\partial_r \overline{T}_{\perp eq}}{\overline{T}_{\perp eq}} + \zeta \left(\frac{\partial_r V_{//eq}}{v_{T_{//}}} - \frac{k_{//}}{k_\theta \rho_s} \right) \right\}^2 \right\rangle_{\zeta,h}$$

Turbulent spin-up Dominguez & Staebler 93, Diamond et al 94, XG 01, Coppi 02

Discussion

- Transport matrix satisfies $D_{nn}D_{TT} - D_{nT}^2 > 0$ (2nd principle).
- Thermodiffusion D_{nT} changes sign with average fluctuation phase velocity $<\omega/\omega_D - \varsigma>$. Inward particle transport for ITG dominated turbulence.
- Minimum of entropy production corresponds to finite density and temperature gradients: generalisation of stiffness concept.
- However a state with zero fluxes such that

$$n_{eq} = n_0 e^{-\frac{2}{R_0}\int_0^r \lambda dr} \qquad T_{eq} \propto n_{eq}^{\varsigma-1}$$

is not attainable because it is linearly stable Angioni 05.

Discussion (cont.)

- Other pinch mechanisms:

- inward thermodiffusion of passing electrons due to parallel dynamics - order $o(\omega/k_{//}v_{Te})$ Jenko 00, Hallatschek 01

- fractional kinetics Castillo del Negrete 05 or CTRW Van Milligen 04

$$\frac{\partial n(x,t)}{\partial t} = S(x) - \frac{n(x,t)}{\tau_D(x)} + \int_0^1 dx' p(x-x',x';t)\frac{n(x',t)}{\tau_D(x')}$$

$$p\left(x-x';\frac{dn}{dx}(x';t)\right) = \zeta(x',t)P_{sym}(x-x',1,\sigma_1) + [1-\zeta(x',t)]P_{sym}(x-x',2,\sigma_2).$$

$$\zeta(x',t) = \Theta\left[\left|\frac{dn}{dx}(x',t)\right| - \left(\frac{dn}{dx}\right)_{crit}\right].$$

- coupling between passive scalar and vorticity transport $d_t(n-\nabla^2\phi)=0$ Priego 05 $\rightarrow$ clustering of particles in vortical structures.

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Testing theory with the TRB code

- TRB solve 3D fluid equations .
- Same as simplified 2D model + polarization drift and parallel dynamics.
- Neoclassical transport is not included.
- Covers Ion Temperature Gradient / Trapped Electron Mode turbulence.

Equations solved by the TRB code

$$d_t n_e = -V_{dte} \cdot [\nabla p_e - n_e \nabla \phi] + S_n$$

$$d_t p_e = -\varsigma V_{dte} \cdot [\nabla (T_e p_e) - p_e \nabla \phi] + S_{pe}$$

$$d_t \Omega = -n_i \nabla_{//} V_{//i} + \left[p_i, \nabla^2 \phi\right] + (1 - f_t)[n_e, \phi] + f_t V_{dte} \cdot [\nabla p_e - n_e \nabla \phi] - V_{di} \cdot [\nabla p_i - n_i \nabla \phi]$$

$$d_t V_{//i} = -n_i \nabla_{//} \phi - \nabla_{//} p_i + S_{v//i}$$

$$d_t p_i = -\varsigma V_{di} \cdot [\nabla (T_i p_i) + p_i \nabla \phi] - \varsigma p_i \nabla_{//} V_{//i} + S_{pi}$$

$$d_t = \partial_t - \left(\frac{\mathbf{B}}{B^2} \times \nabla \phi \right) \cdot \nabla \quad ; \quad \Omega = n_{e,eq} \left[\frac{1 - f_t}{T_{e,eq}} (\phi - \langle \phi \rangle) - \nabla^2 \phi \right] \quad ; \quad \varsigma = \frac{5}{3}$$

TRB calculations agrees with QL theory for particle transport

Two additive contributions:

- Curvature pinch, depends on magnetic shear

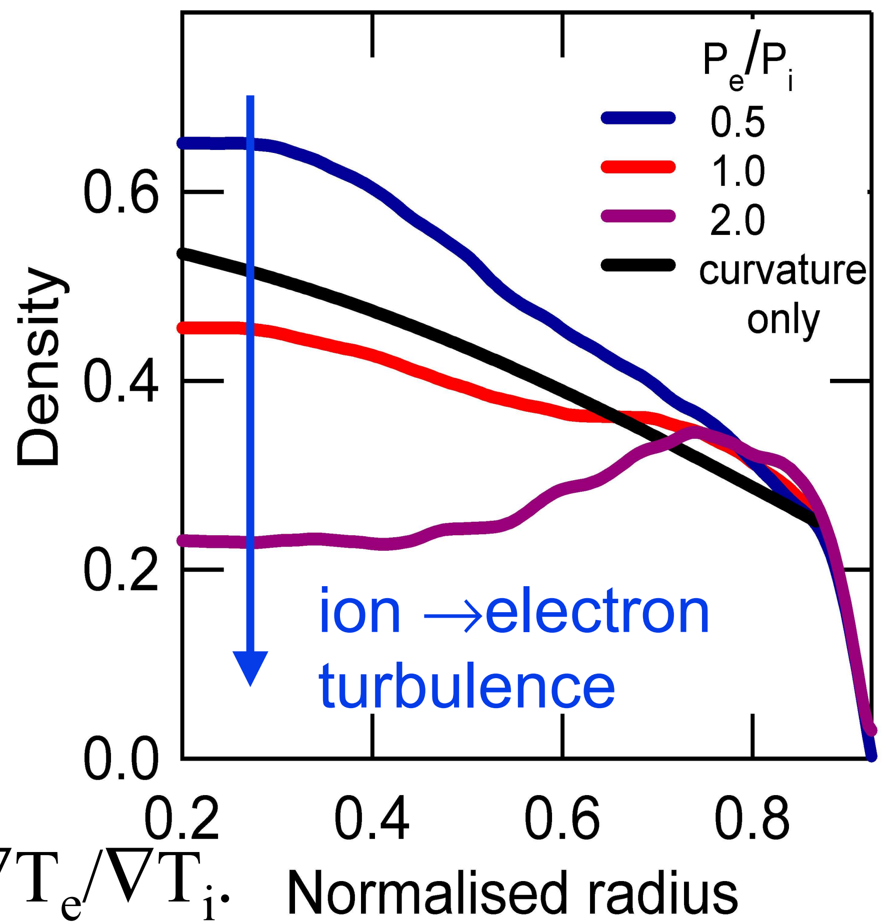
$$\frac{V}{D} \propto s \qquad s = \frac{r}{q}\frac{dq}{dr}$$

- Thermo-diffusion

$$\frac{V}{D} \propto \frac{\nabla T}{T}$$

changes sign when increasing $\nabla T_e / \nabla T_i$.

TRB simulations



Impurity transport

- Issue: is the impurity pinch velocity turbulent or neoclassical?
- Prediction of quasi-linear theory

$$\Gamma_{Zturb} = -D_Z \left\{ \frac{dn_Z}{dr} + C_q(s)\frac{2}{R}n_Z - C_T(\omega)\frac{dT_Z}{T_Z dr}n_Z \right\}$$

↑ Diffusion

↑ Curvature pinch inward

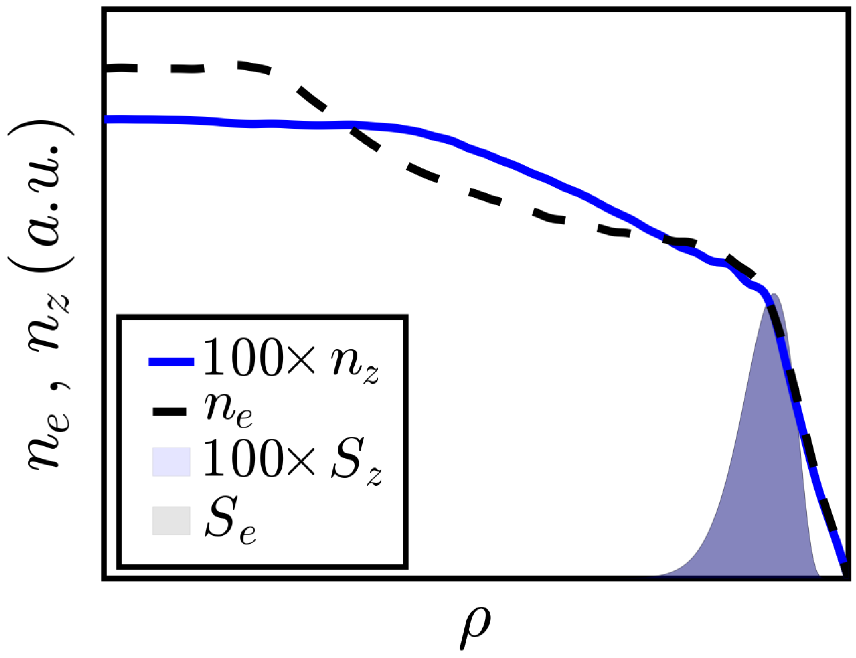
↑ Thermodiffusion ~1/Z, outward for ion turb.

TRB has been modified to add an impurity Dubuit 05

Evidence of an inward impurity pinch

- Both Carbon and electron sources are peripheral.
- Peaked electron and impurity densities → inward pinch. Similar to Estrada-Mila 05
- Profiles are different.

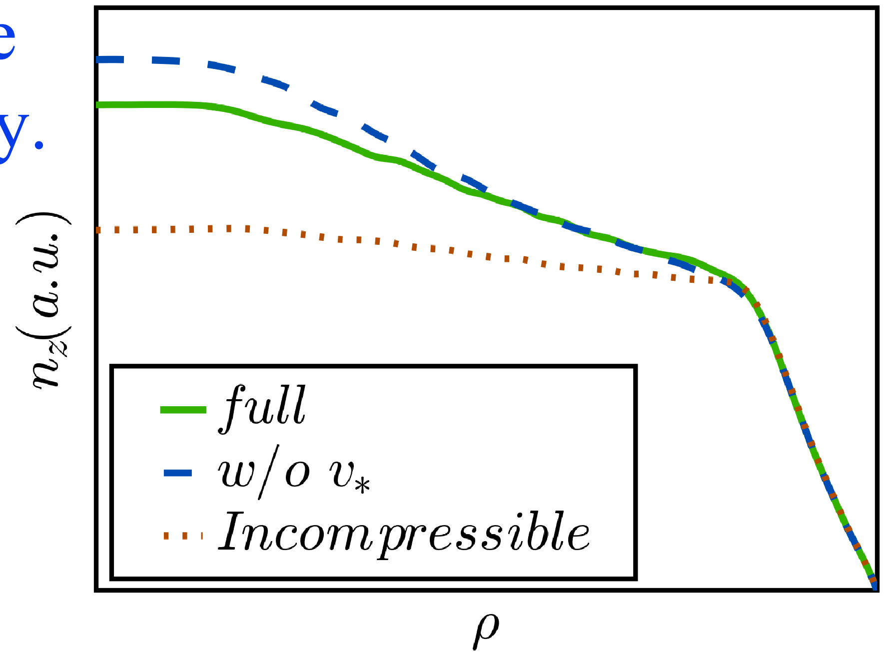
Dubuit 05



Curvature seems to be the dominant mechanism

- Thermodiffusion and curvature pinches can be switched-off numerically.
- Curvature pinch is dominant.
- Thermodiffusion is directed outward and is small.

Dubuit 05



- Unexplained small residual contribution

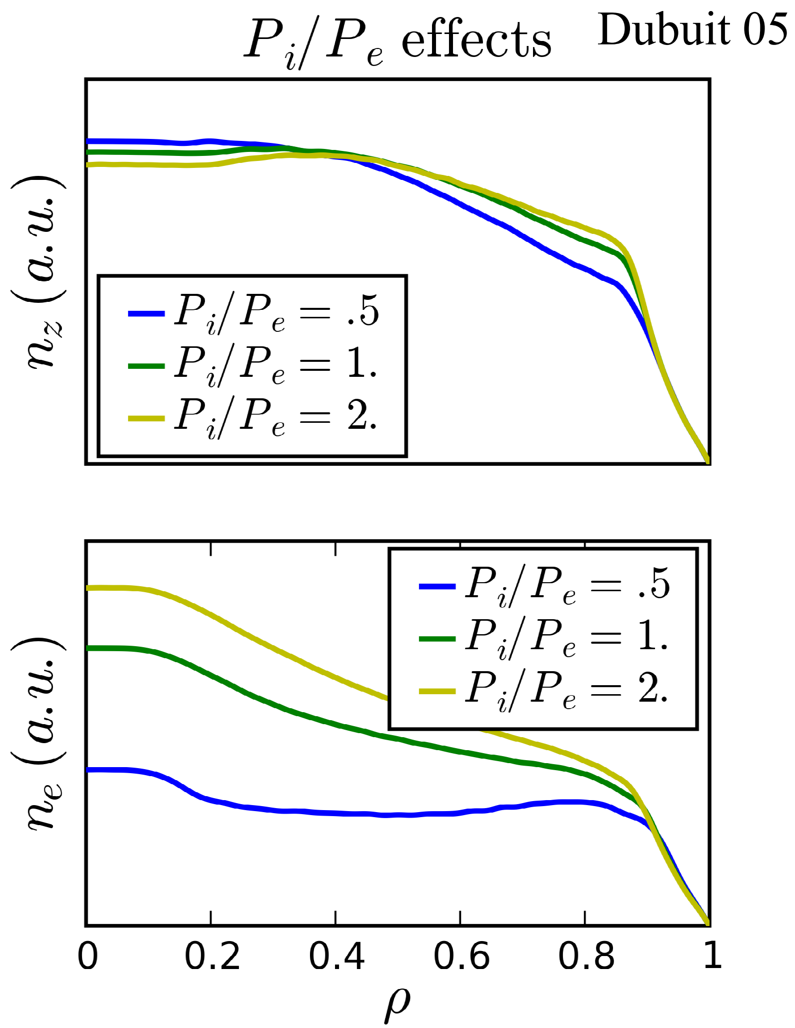
Effect of P_e/P_i

- Increasing the ratio of ion to electron heating sources:

- electron density profile is more peaked

- impurity profile is less peaked.

- Consistent with quasi-linear prediction. Estrada-Mila 05, Ongoing comparison with GS2 Angioni 05



Conclusions

- Curvature pinch is not a thermodynamical force: geometrical effect that leads to new definitions of density and temperature forces

$$\overline{n} = n / n_{can} \qquad \overline{T} = T / T_{can}$$

$$n_{can} = n_0 I^{-1} \quad ; \quad T_{can} = T_0 I^{-(\varsigma-1)} \quad ; \quad I = \exp\left(\frac{2}{R_0}\int_0^r \lambda dr\right)$$

- Onsager symmetry is found once this normalisation is done.
- Entropy production rate is always positive, within the frame of quasi-linear theory.

Conclusions (cont.)

- Some of these predictions have been tested with the TRB code:

- Particle pinch velocity is the sum of curvature and thermodiffusion pinches. Somewhat formal however,except fot trace impurities.

- Impurity pinch velocity is dominated by curvature pinch.

- For ion turbulence: particle thermodiffusion pinch velocity is inward and impurity pinch velocity is outward.

- Collisonality is an issue.

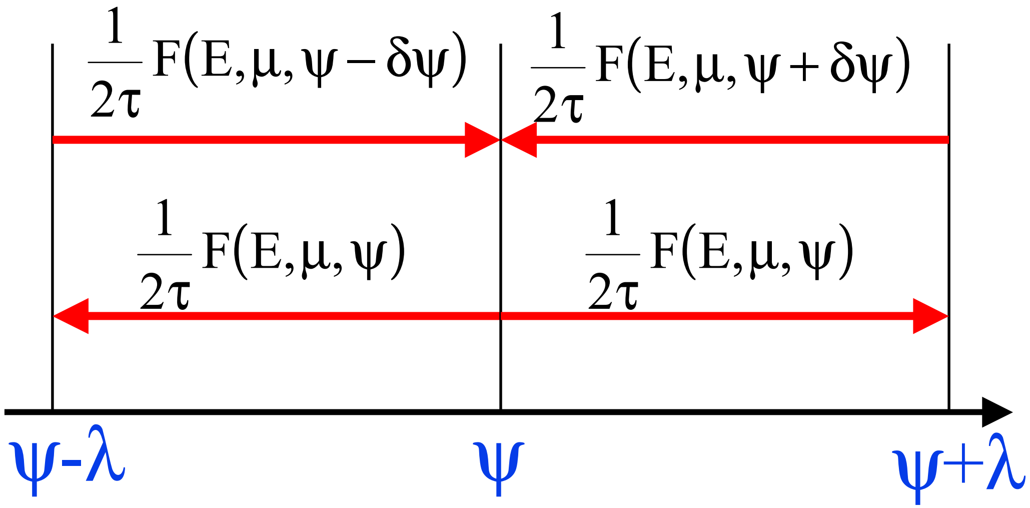
Transport equations

$$\partial_t \overline{n}_{eq} + \frac{1}{r}\partial_r\left(r\Gamma\right) = 0$$

$$\frac{1}{\varsigma - 1}\overline{n}_{eq}\partial_t \overline{T}_{eq} + \frac{1}{r}\partial_r\left(rQ\right) = -\frac{1}{\varsigma - 1}\Gamma\partial_r \overline{T}_{eq}$$

Diffusion : conventional model

- Random walk: step size $\delta\psi$, time step τ , distribution function $F(E,\mu,\psi,t)$

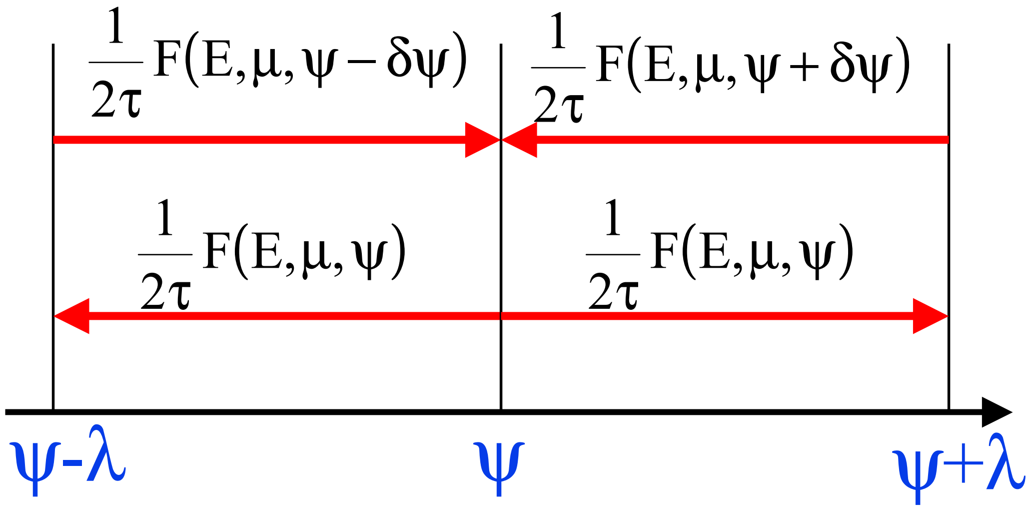


- Particle balance $\rightarrow$ diffusion equation $D = \delta\psi^2/2\tau$

$$\partial_t F = D\partial_{\psi\psi} F \quad \rightarrow \quad \partial_t n = D\partial_{\psi\psi} n$$

Thermodiffusion

- Step size $\delta\psi$ depends on energy E $\rightarrow$ D(E), e.g. $D=D_0+D_1(E/T-3/2)$



- Temperature gradient appears in particle flux

$$\partial_t F = D\partial_{\psi\psi} F \quad \rightarrow \quad \partial_t n = \partial_\psi \left(D_0 \partial_\psi n + D_1 n \frac{\partial_\psi T}{T} \right)$$

Diffusion and pinch velocity

- Particle flux

$$\Gamma_e = -D\frac{dn_e}{dr} + Vn_e$$

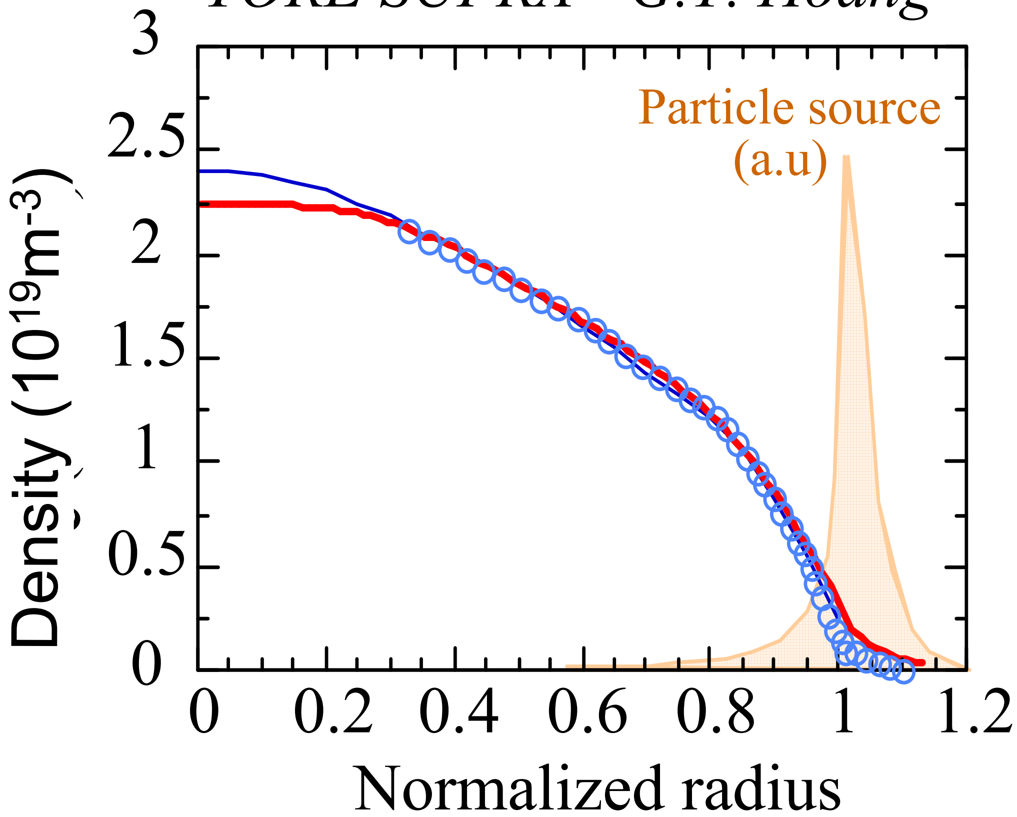
- Diffusion is turbulent

$$D = D_{turb}$$

- pinch velocity= collisions + turbulence

$$V = V_{coll} + V_{turb}$$

TORE SUPRA - G.T. Hoang



- In a reactor:

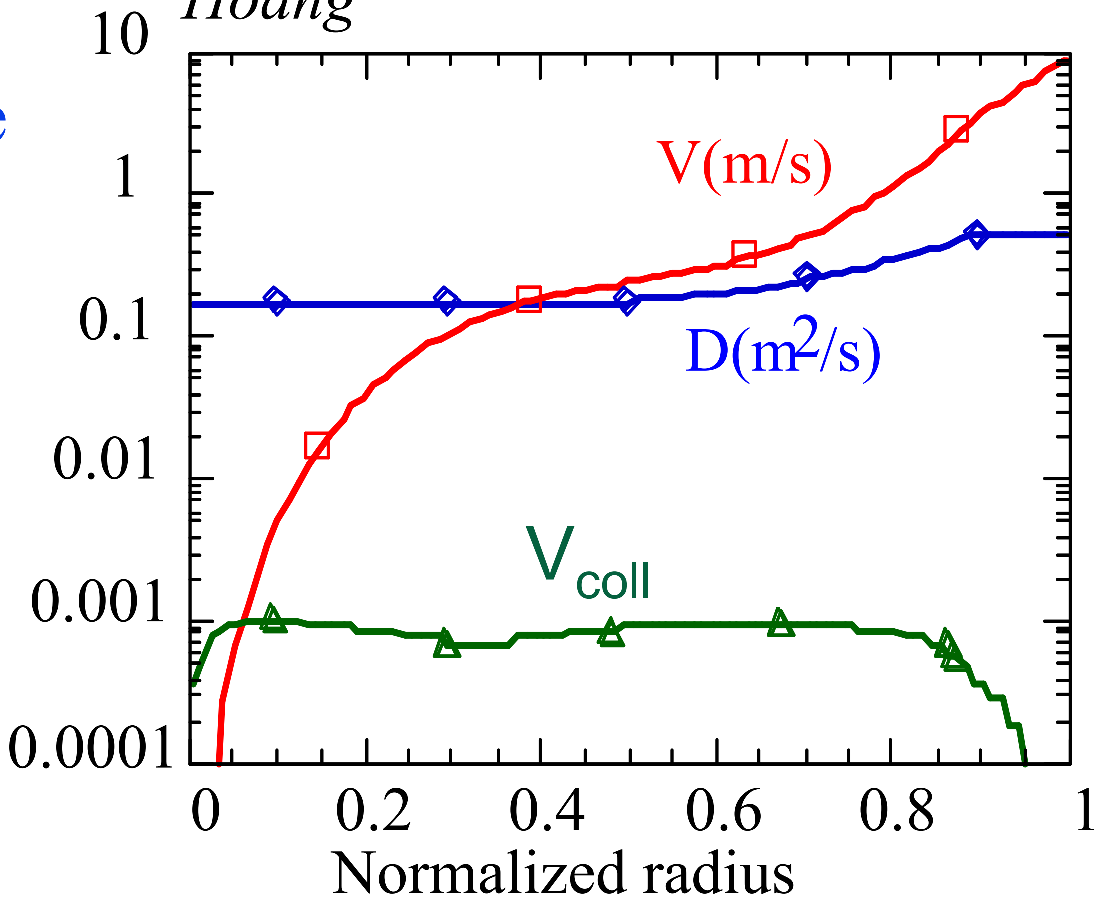
- ionisation source localised in the edge → Γ_e=0

-V_{coll}~V_{Ware}=0. Turbulent pinch V_{turb} → density peaking?

Pinch is turbulent in L-mode

- In TORE SUPRA, and TCV: no Ware pinch, $V_{Ware}=0$.
- Peaked density profile, ionisation source localised in the edge → turbulent pinch velocity.

Diffusion and pinch velocity vs radius - TORE SUPRA - G.T. Hoang



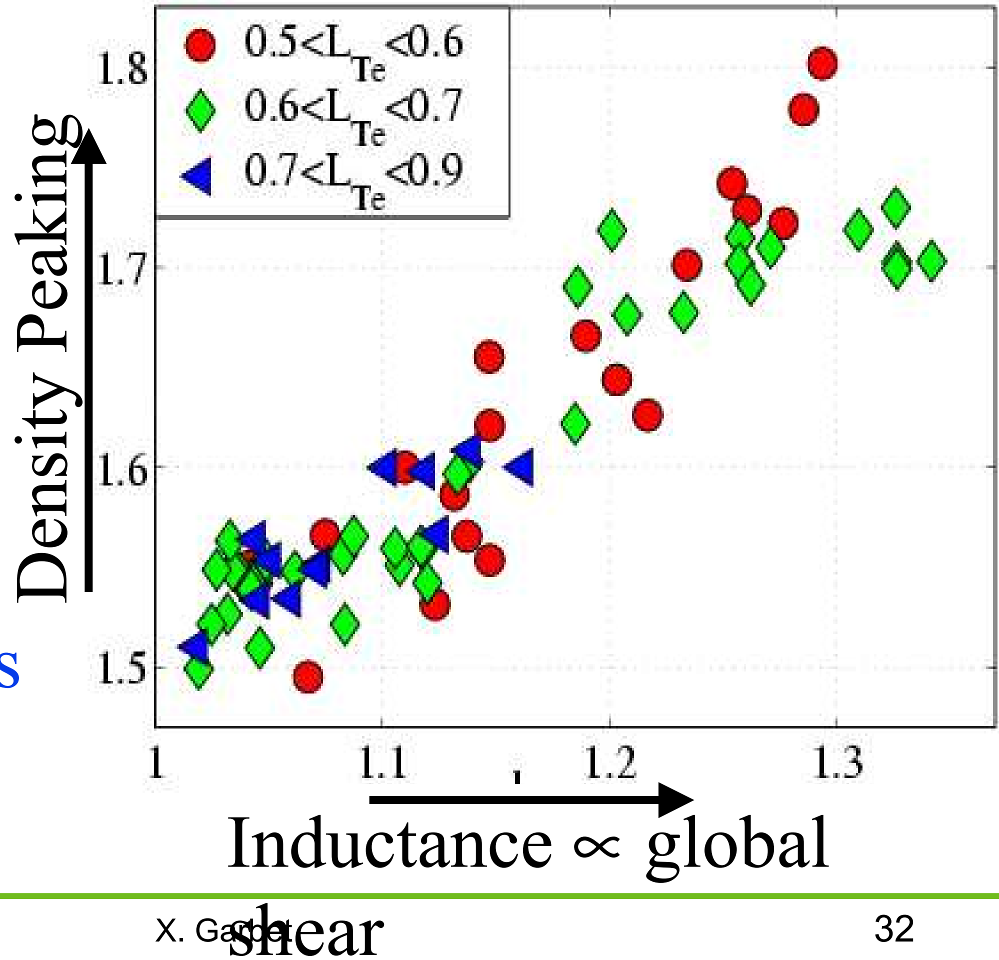
Density and safety factor profiles are correlated

- Combined heating and current drive :
- consistent with curvature pinch
- no indication of thermodiffusion:

e/ion-mode transition?

→ density and q profiles are correlated in JET, DIIID, TCV, TS…

JET- H. Weisen, A.Zabolotsky



Density profiles are peaked at low collisionality

H. Weisen, A.Zabolotsky

- V/D decreases with collisionality.
- Consistent with observations on AUG and JET.
- At large collisionality, $V_{turb} \approx V_{ware}$

(*J. Stober, M.Valovic*)