Wave-particle interaction

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Summary

Motivation
Wave-particle dynamics
Simple limits of this interaction
- One wave number
  - Hydrodynamic case
  - Kinetic case
- Several wavenumbers
  - Quasilinear spreading
    - Stochastic/chaotic regime
    - Prescribed/self-consistent field

Less simple (but important) limits

Objectives

Tutorial
Caveats
- Phase space resolution in kinetic codes
- Measurement of numerical transport
- Folklore about Landau damping

Plasma physics as statistical physics
Plasma physics as chaos theory

Why bothering with this ?

Universality of the wave-particle problem
- Space plasmas
  - Particle acceleration
  - Anomalous dissipations
- Magnetic fusion
  - Heating by waves
  - Kinctic instabilities

Universality of the 1D wave-particle interaction paradigm
- Classical Langmuir wave problem
- Heating waves acting through $\vec{E}$
- Transit time magnetic pumping
- Interaction of fast particles with low frequency MHD modes: transport in r, $v_{//}$ unaffected
- Drift waves and zonal flows

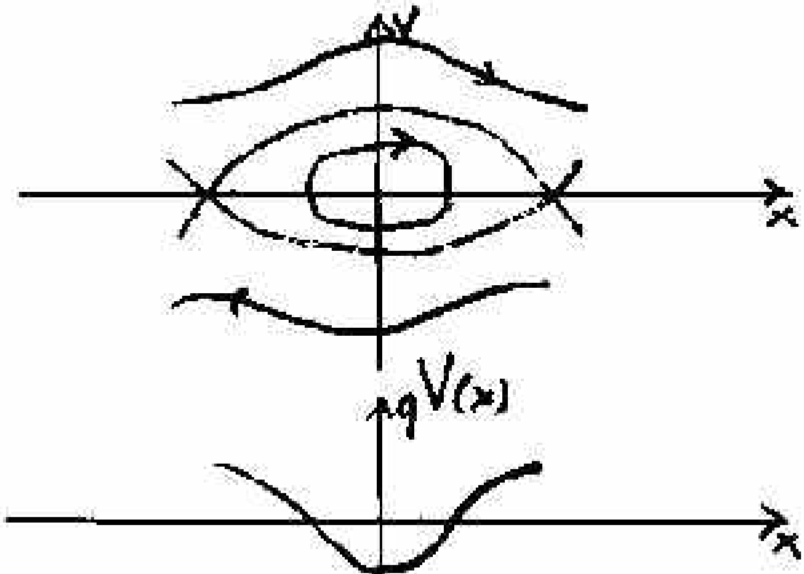
Wave-particle interaction:
The simplest case corresponds to one electron in the field of a Langmuir wave

Langmuir waves correspond to vibrations of the electrons with respect to the ions (along the magnetic field if present); essentially 1 dimensional.

In the presence of a Langmuir wave one electron may be trapped or passing; a separatrix separates these two types of motion.

Trapping is a strong resonance:

Average electron velocity = wave phase velocity



One electron has a negligible action on the wave. What about many ?

Need for a symmetrical description of waves and particles: Now derived by successive steps.

Wave-particle interaction for heating waves acting through $\vec{E}$

A particle at r sees a field ~ $\exp i\left[k_{\parallel} r_{\parallel} + \vec{k}_{\perp}\cdot\vec{r}_{\perp} - \omega t\right]$

If $\mathbf{r}_{\perp}(t)$ modulated with one or several periods, Fourier analysis yields components of the type

$$\exp i\left[k_{\parallel} r_{\parallel} + \left(-\omega + \sum_i n_i \Omega_i\right) t\right]$$

frequencies in $\mathbf{r}_{\perp}(t)$

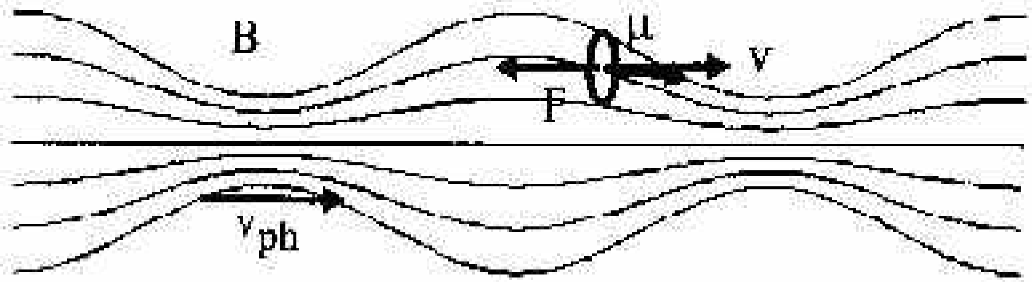
This brings the resonance condition

$$\omega = k_{\parallel} v_{\parallel} + \sum_i n_i \Omega_i$$

Transit Time Magnetic Pumping

Magnetic moment approximately conserved:

$$\frac{d}{dt}\mu \approx 0 \quad \text{if} \left(\frac{dB}{dt}/B\right)/\Omega \ll 1$$



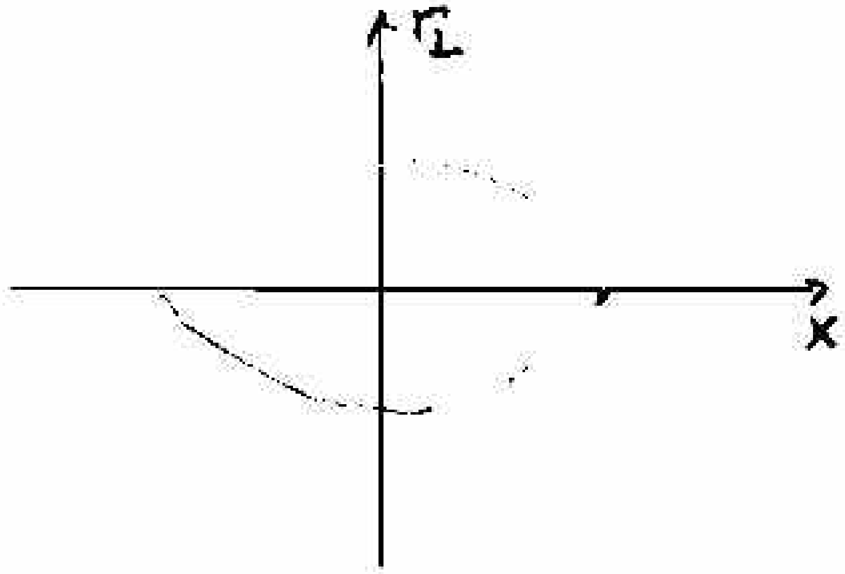
Force on magnetic moment: $F = -\mu \nabla B$

similar to Landau damping with substitution:

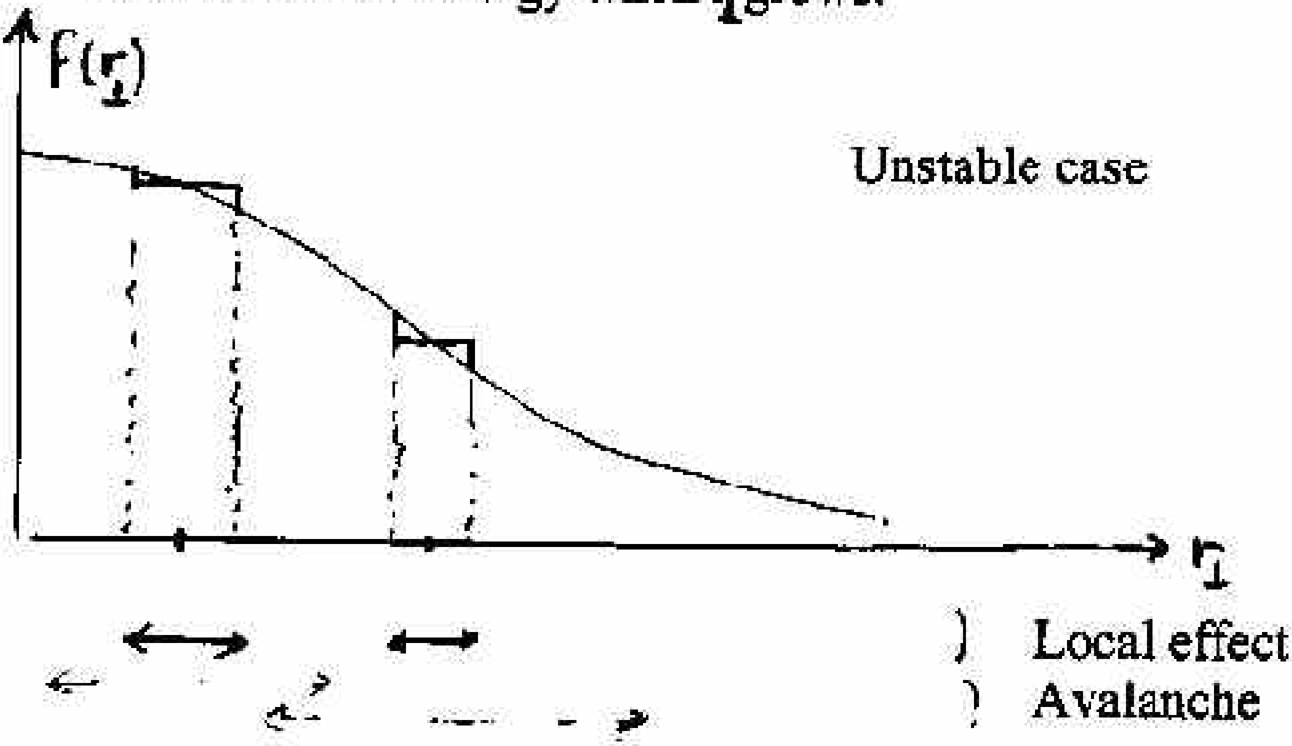
$$\mu \rightarrow q$$
$$\nabla B \rightarrow E$$

For low frequency waves in a tokamak, wave-particle resonance translates into an eye of cat in $(x, r_\perp)$

$x = n\varphi - m\theta - \omega t$



Particle looses energy when $r_\perp$ grows.



Trapping into banana orbits also possible

Drift waves and zonal flows behave like particles and waves

Drift waves may be described by a wave kinetic equation which is similar to a Vlasov equation where v becomes k

The zonal flow behaves like a wave exchanging energy with the pseudo-particles

Trapping possible!

The dynamics is defined by a self-consistent Hamiltonian

$$H_{sc} = \sum_{l=1}^{N} \frac{p_l^2}{2m} + \sum_{j=1}^{M} \omega_j I_j - \sum_{l=1}^{N} \sum_{j=1}^{M} \nu_j \sqrt{I_j} \cos(k_j x_l - \theta_j),$$

$$\nu_j = \frac{4\omega_p}{k_j} \sqrt{\frac{n_{tail}}{N n_{bulk}}} \left(\frac{\partial \epsilon}{\partial \omega}(k_j, \omega_j) \right)^{1/2},$$

$$\dot{\theta}_j = \omega_j + \nu_j \ldots,$$

H is made up of free particle terms, of harmonic oscillator terms, and of coupling terms.

The coupling terms take on a natural structure for the potential of a Langmuir wave:

- it is sinusoidal in space
- also in time in the limit of a small coupling: θ_j evolves like $\omega_j t + \phi_j$
- its amplitude scales like the square root of the wave energy.

The structure of the coupling implies the constant of the motion:

$$P = \sum_{l=1}^{N} p_l + \sum_{j=1}^{M} k_j I_j$$

which is the total wave-particle momentum.

The growth or decay of a wave impacts on particles.

The self-consistent dynamics was originally derived by mixing a Vlasovian and a granular description of the plasma

H is the generalisation to M>1 waves of the self-consistent dynamics introduced with M=1 for describing the saturation of the cold beam-plasma instability

Onischenko et al., 1970
O'Neil et al., 1971
Mynick and Kaufman, 1978
Tennyson, Meiss, and Morrison, 1994

Found to be a general model for electrostatic instabilities
del-Castillo-Negrete, 1998
Crawford and Jayaraman, 1999

Describes vorticity dynamics in marginally stable shear flows: del-Castillo-Negrete, 2000

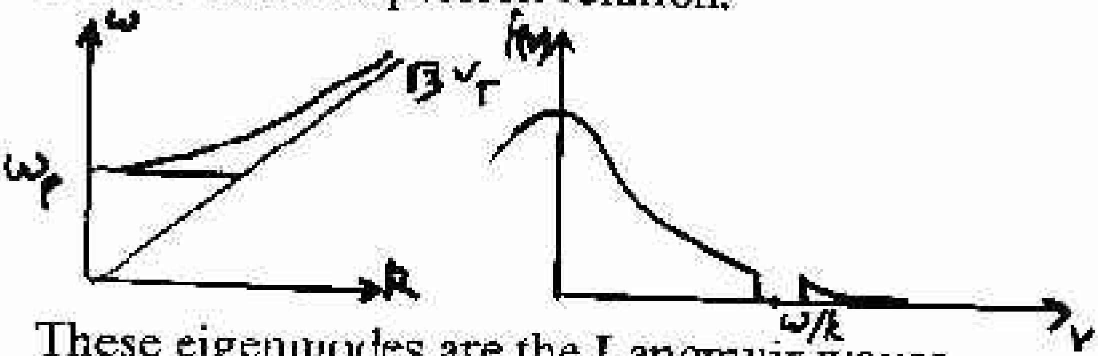
Generalised to the case with source and sink
Berk, Breizman, and Pekker, 1995

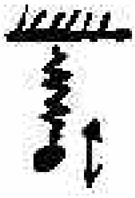
Derivation from a description of the plasma as a N-body system
Antoni, Elskens, DFE, 1998

Langmuir waves can be described as harmonic oscillators

For describing Langmuir waves, the plasma can be considered as a set of N* electrostatically coupled particles in a 1D system with spatial period L.

Looking for the small collective oscillations of this mechanical system with a wave-number k leads to the Bohm-Gross dispersion relation.





These eigenmodes are the Langmuir waves.

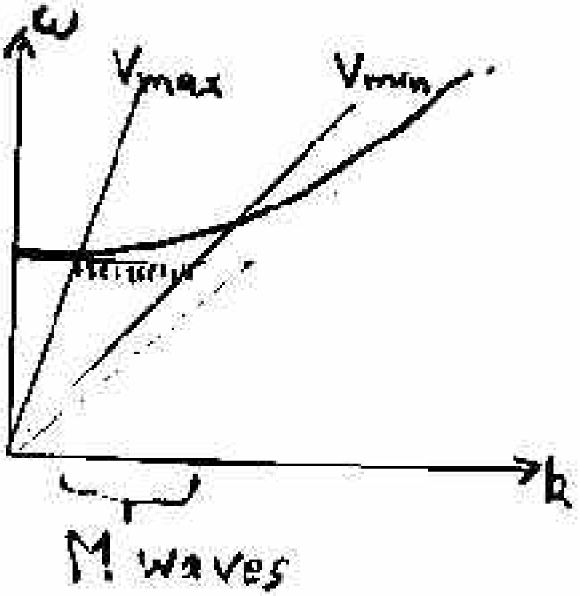
In the linear regime their frequency is amplitude independent: these vibrations are harmonic oscillators.

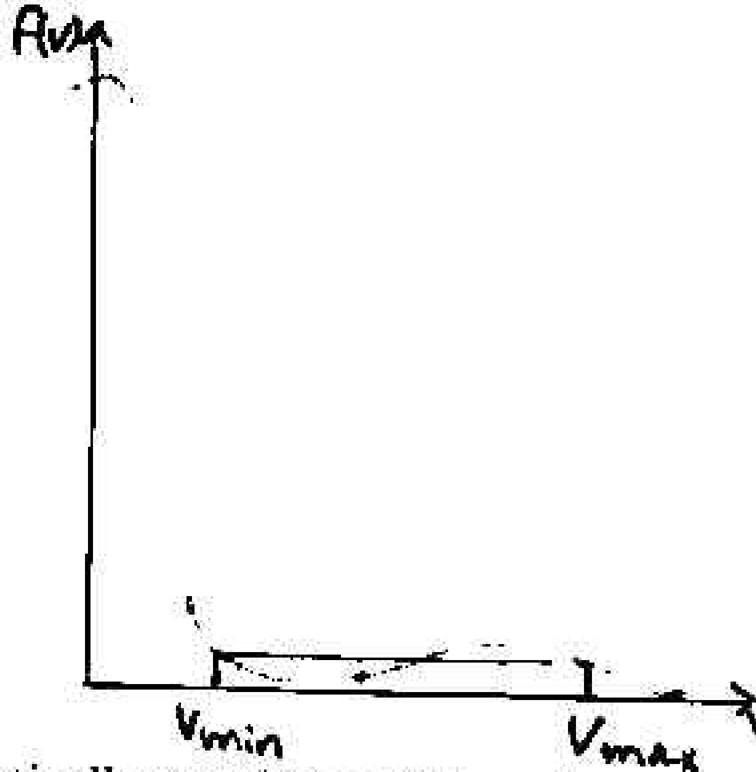
Caveat: particles may not be resonant or close to resonance with the wave (Bohm and Gross, 1949).

The harmonic oscillator model is the analogue for waves of the free particle model for particles.

Need for a description of the case where there are resonant particles.

Wave-particle interaction is cast into an explicit mechanical system





We separate the N* electrostatically coupled particles of a plasma with spatial period L into N tail particles and N*-N bulk particles.

A rigorous classical mechanics calculation enables to go from the original $N^{(*)}$-body problem to a field particle interaction problem:

- N resonant particles
- M harmonic oscillators defining the field:

  M Langmuir waves due to the collective vibrations of the bulk.

$N + M \ll N^*$

The mechanical approach enables considering one mechanical realisation of the plasma

An equilibrium is found if particles are set on monokinetic beams, and if each beam is an array of particles (destructive interference of spontaneous emissions).

Partly reminiscent of Dawson multi-beam approach (1960).

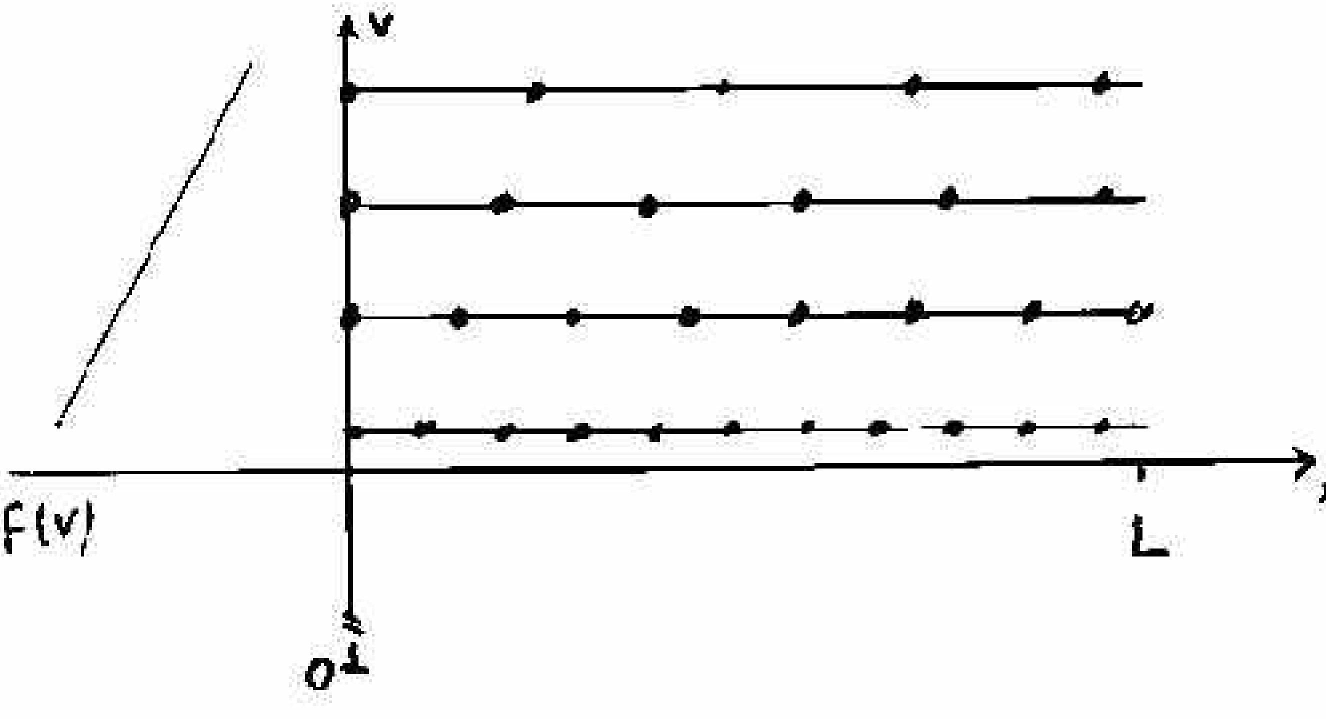
Perturbing this equilibrium leads to a Floquet problem at 2(N+M) dimensions which is explicitly solvable!

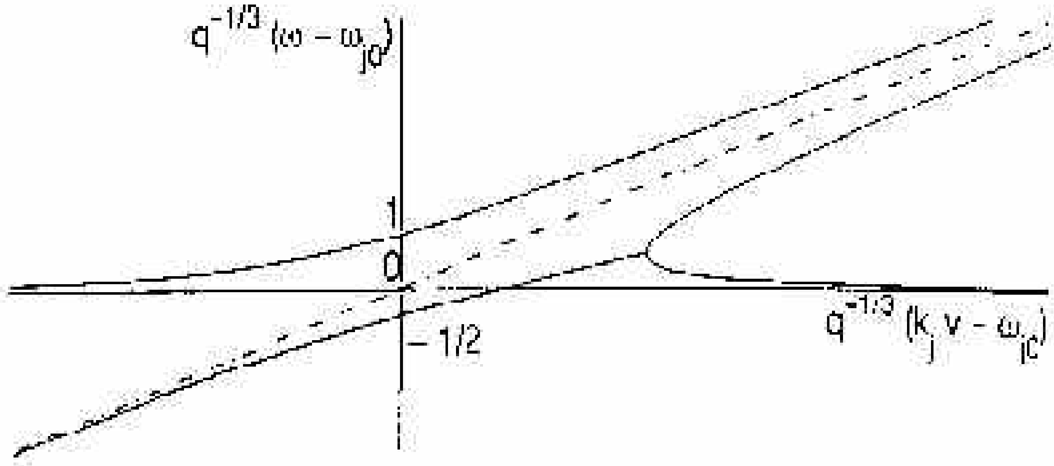
One finds the (natural) result that a given wave-number corresponds to 2 eigenmodes per beam.

One wave is the sum of many eigenmodes.

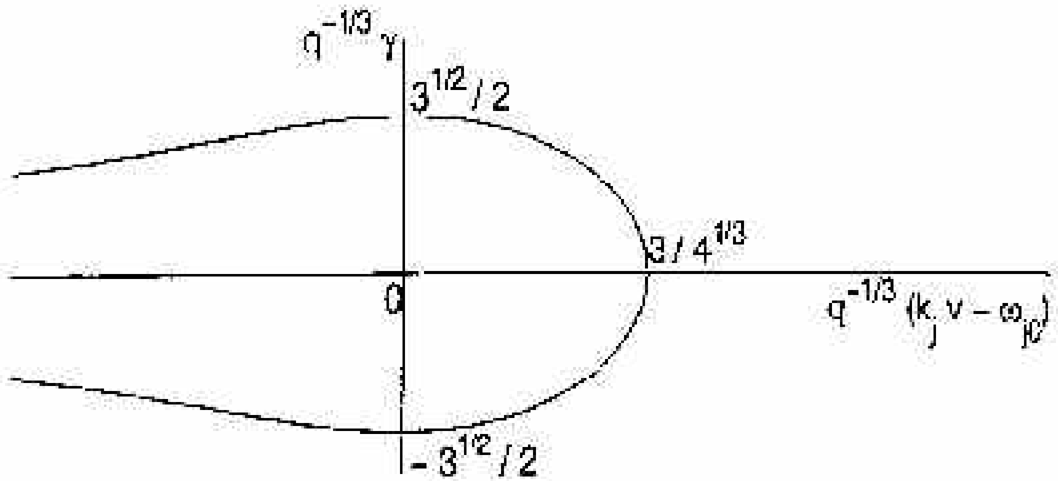
Only one if no resonant particle (Bohm-Gross case)

Only finite Fourier sums are used in this calculation. The Vlasovian approach involves the Fourier and Laplace transforms, pole hunting, and analytical continuation.

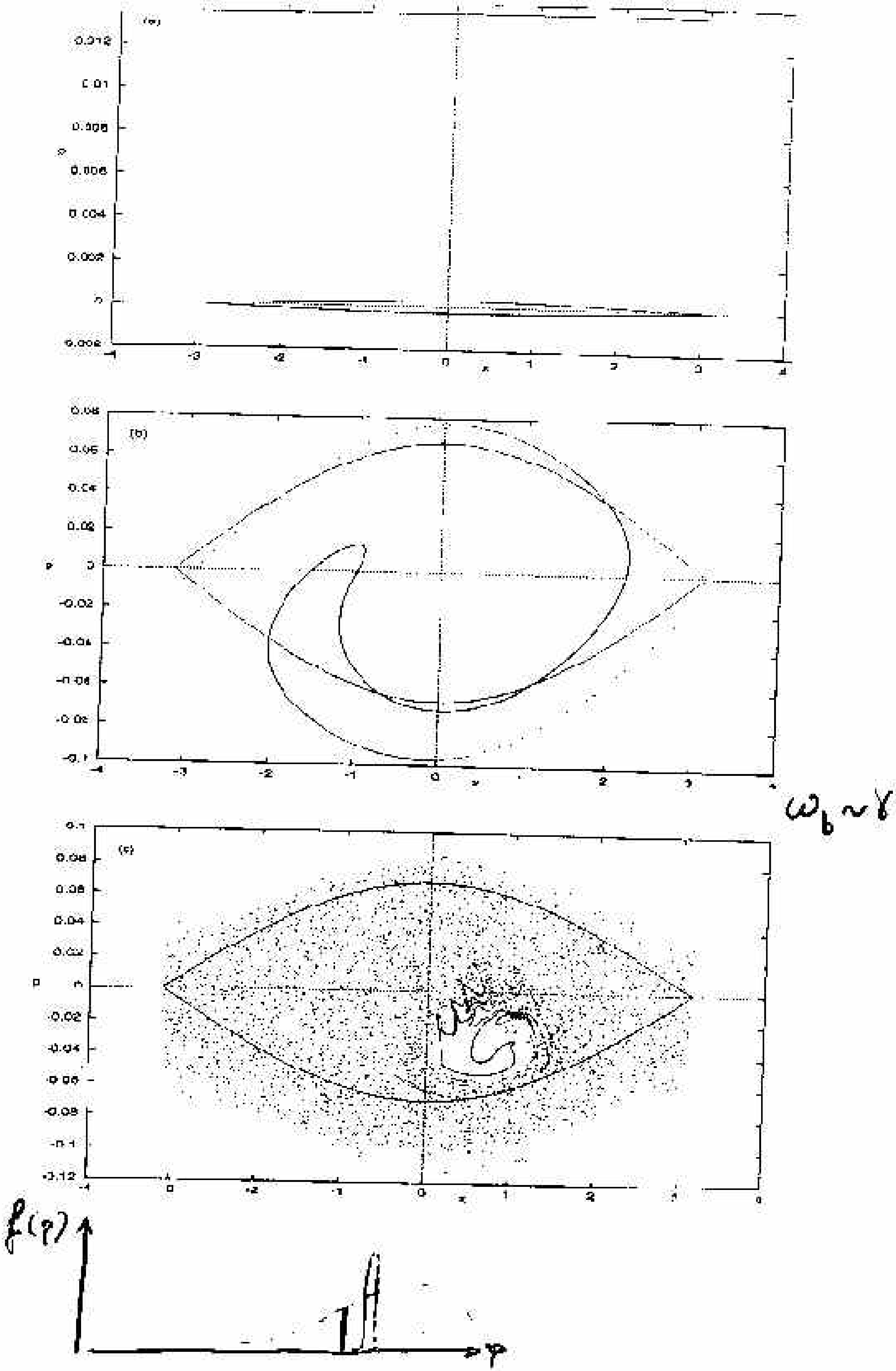


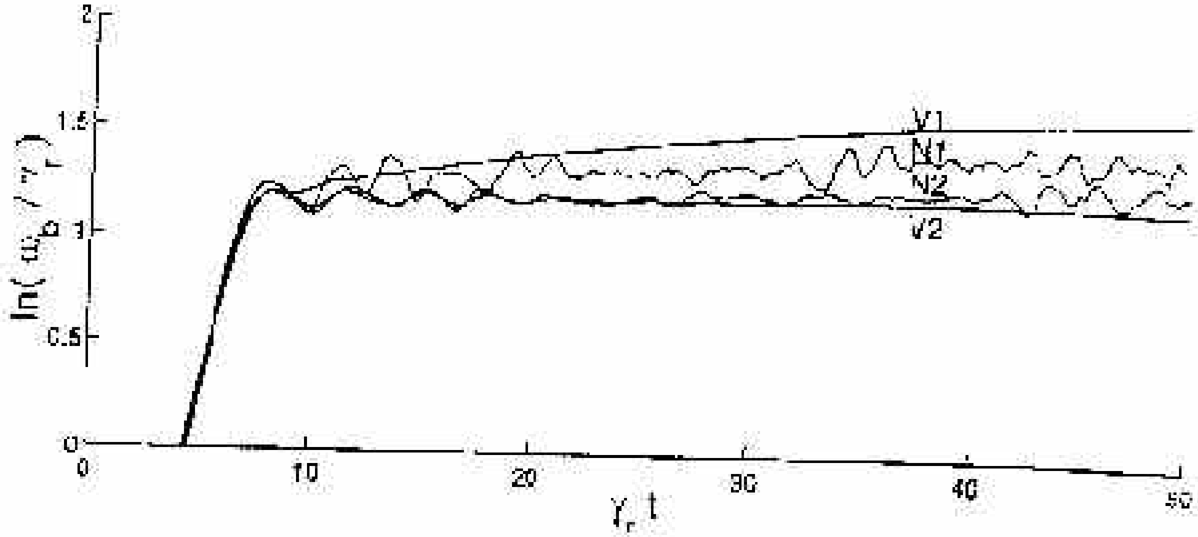


$q^{-1/3}(\omega - \omega_{j0})$
1
0
- 1/2
$q^{-1/3}(k_j v - \omega_{j0})$

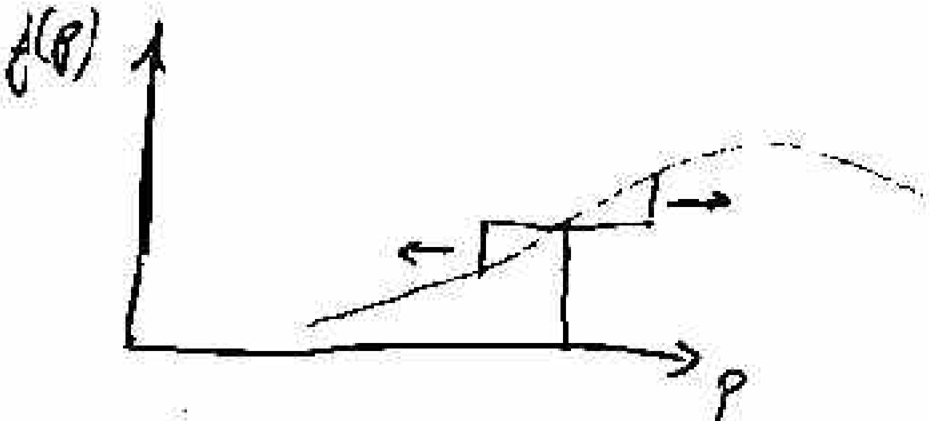


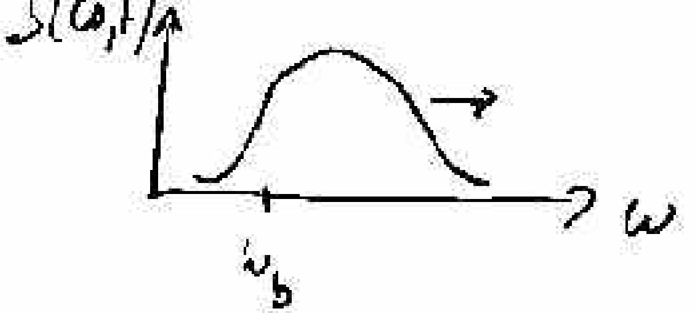
$q^{-1/3}\gamma$
$3^{1/2}/2$
$3/4^{1/3}$
0
$q^{-1/3}(k_j v - \omega_{j0})$
$-3^{1/2}/2$


(a)
(b)
(c)
p
x
$\omega_b \sim \gamma$
$f(p)$
p



Doveil et al. (2001)
Firpo et al. (2001)



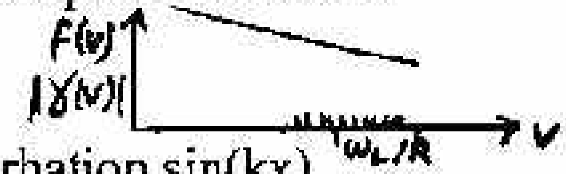


The classical Landau-van Kampen theory is recovered by using mathematical tools not more intricate than the finite Fourier sum

Equilibrium: - vanishing wave amplitudes
- particles set on monokinetic beams which are arrays of particles.

Perturbing this equilibrium leads to a 2(N−M) dimensions Floquet problem which is explicitly solvable!

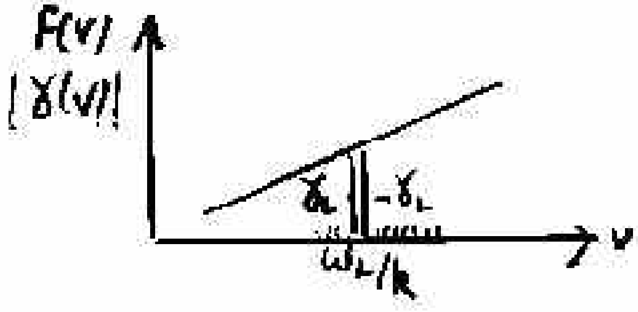
In the limit where N and the number of beams are large (Vlasovian limit), the beam modes correspond to the van Kampen-Case modes.



If the slope is negative, an initial perturbation sin(kx) damps with time like $\exp(\gamma_L |t|)$; $\gamma_L < 0$ is the Landau damping rate. Landau damping = phase mixing.

If the slope is positive, an initial perturbation sin(kx) grows with time like $\exp(\gamma_L t) + \exp(-\gamma_L t) - \exp(-\gamma_L |t|)$
Result found by Y. Elskens (2002) as a result of a remark by A. Samain. Vlasovian limit highly singular!

Absolute values reflect the reversibility of Hamiltonian dynamics.
Caveat: model recovering Landau damping with one damped wave!



Landau damping cannot correspond to an eigenmode

The reversibility of Hamiltonian dynamics implies that eigenvalues occur in conjugate pairs.

A Landau unstable mode occurs with a damped mode which cannot hide the unstable one.

If Landau damping was a damped eigenmode, its unstable companion would dominate!

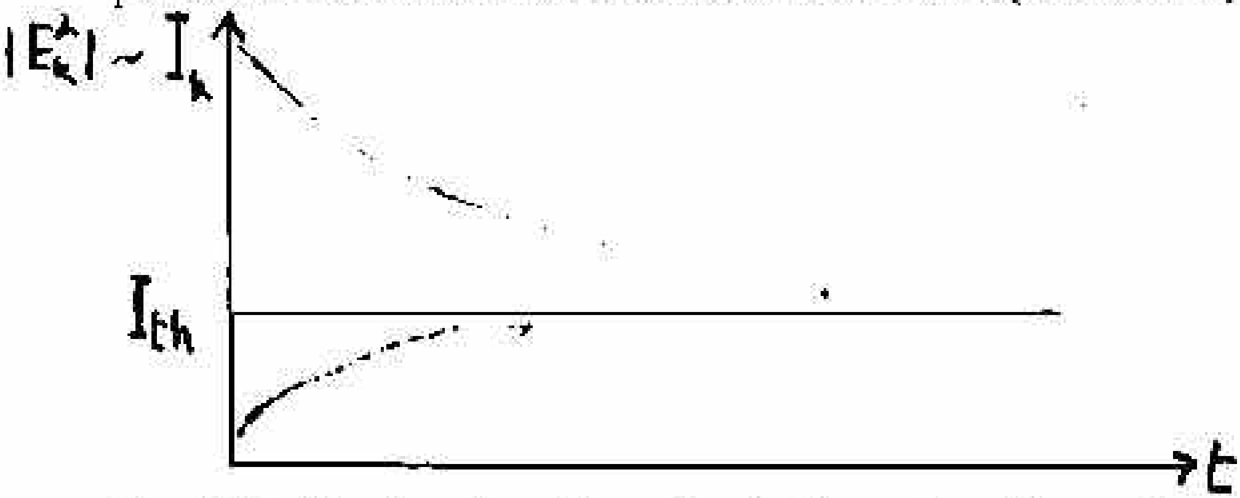
This explains why in the classical Landau calculation the instability corresponds to an eigenmode, but not the damping.

Landau damping is the exponential relaxation toward the thermal level

Considering random initial positions and phases, perturbation theory at second order in the coupling parameter v yields

$$\frac{\mathrm{d}\langle |E_k^2| \rangle}{\mathrm{d}t} = 2 \underbrace{\gamma_L(k,t)}_{\sim \partial_v f(\omega_k/k,t)} \langle |E_k^2| \rangle + \underbrace{S_k}_{\sim \frac{1}{N} f(\omega_k/k,t)}$$

S corresponds to the spontaneous emission of waves by particles. Goes to 0 in the Vlasovian limit (N infinite).



For N finite, Landau damping is the relaxation of Langmuir waves toward their thermal level.

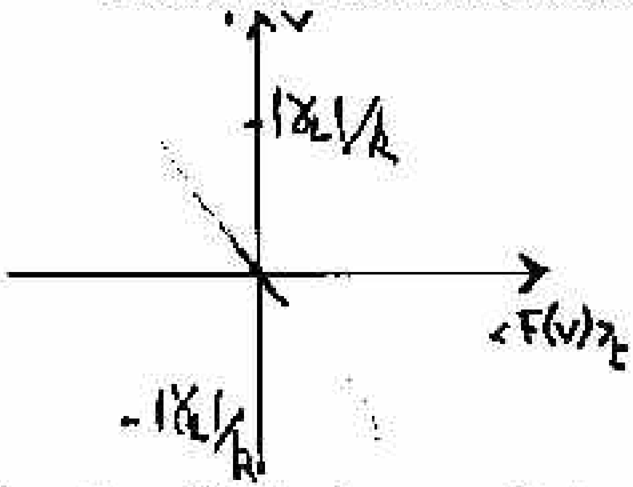
The Vlasovian limit is highly singular.

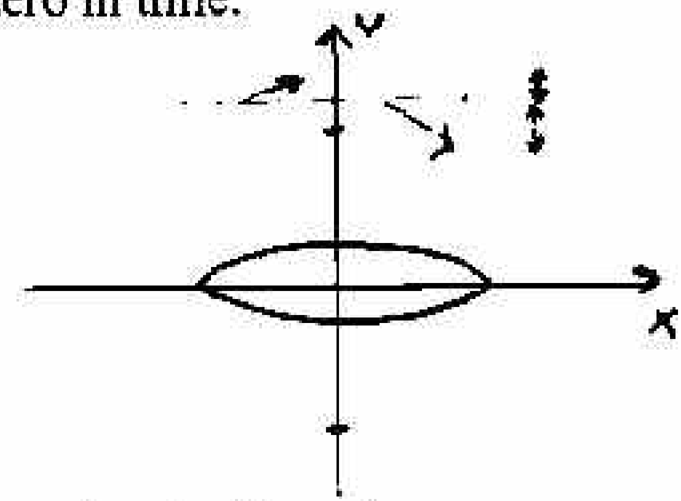
This straightforward calculation works for both instability and damping.

Synchronisation or diffusion are the signatures of the Landau effect on particles

A similar calculation yields the average force acting on a particle for a single Langmuir wave with $\gamma_L < 0$ or $\gamma_L > 0$

The force fluctuates about zero in time.





Landau effect corresponds to a synchronisation of particles with a single wave. Experiment. Doveil, Macor, DFE PRL 2005

Trapping has no role in the Landau effect.

It is a weakly resonant effect.

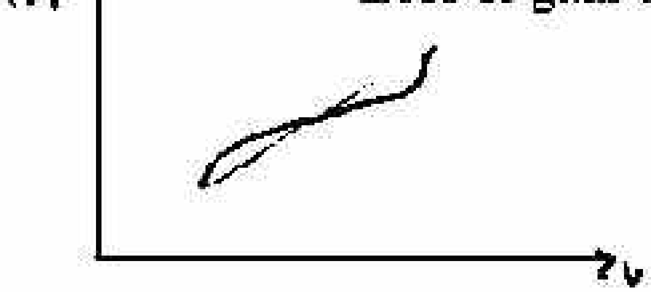
Caveat: the surfer model!

In the case of a broad spectrum of Langmuir waves the previous second order calculation leads to the QL Fokker-Planck equation for the evolution of the particle distribution function

Diffusion tends to diminish the slope of the distribution function: Gain or loss of particle momentum

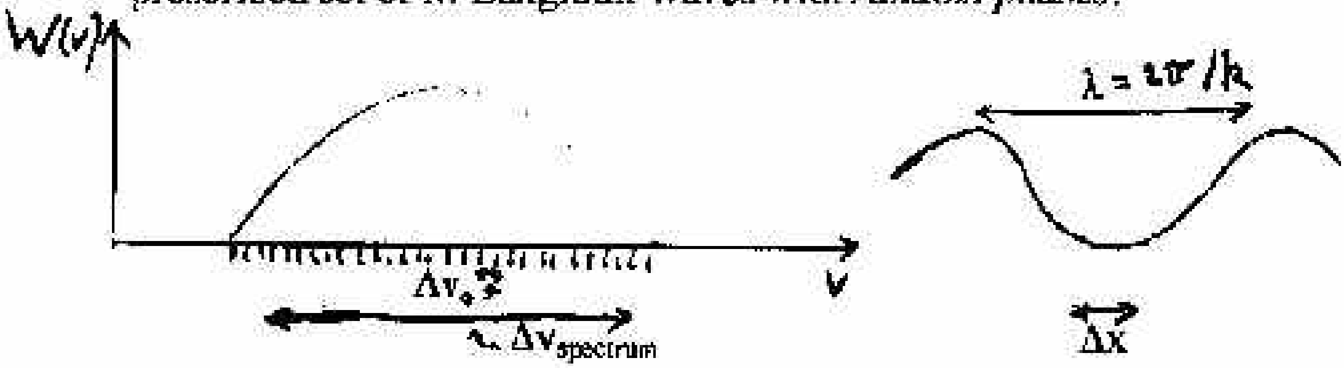
Loss or gain of wave amplitude.

f(v)



Traditional quasilinear theory takes advantage of the stochastic character of the waves, not of chaos in the particle motion

To understand particle diffusion it is useful to first consider the case of a prescribed set of M Langmuir waves with *random phases*.



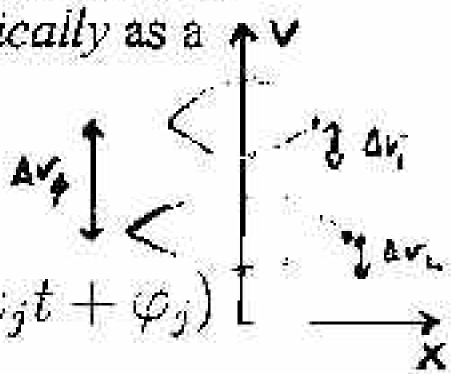
If the wave spectrum is broad, classical perturbation theory predicts
$<\Delta v^2> = 2 D_{QL} t$, $<k^2 \Delta x^2> = 2 k^2 D_{QL} t^3 / 3$
for $\tau_c << t << \tau_{max} = \mathrm{Min}(\tau_{spread}, \tau_{discr})$
$\tau_{spread} = (k^2 D_{QL})^{-1/2}$, $\tau_c = (k \Delta v_{spectrum})^{-1}$, $\tau_{discr} = (k \Delta v_\phi)^{-1}$.
Averages over the random phases.

D_{QL} is defined over a time where the spreading of orbits is negligible (chaos unimportant).
The diffusion is due to the randomness of the field, not to chaos in the motion: it is stochastic, not chaotic.

τ_{spread} bounds the time over which the dependence of x(t) over M phases is small.

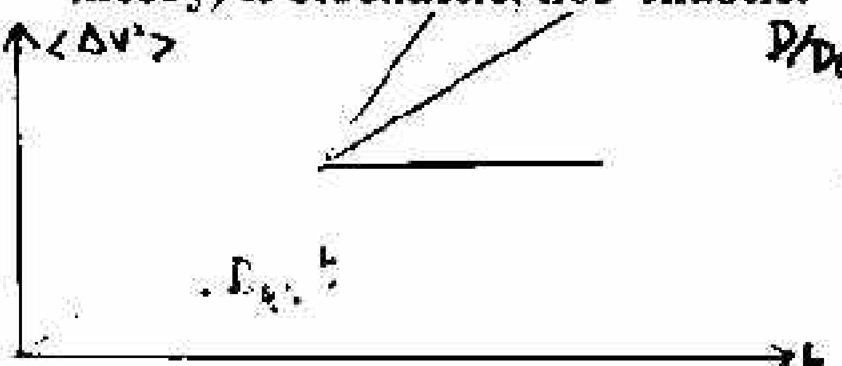
Diffusion may be stochastic or chaotic

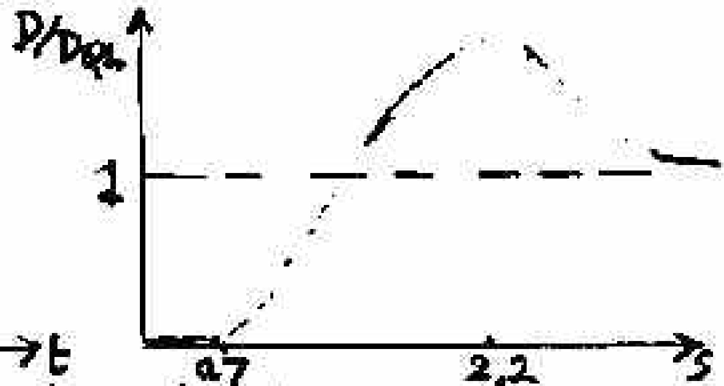
For a prescribed set of M Langmuir waves with random phases several scenarios are *observed numerically* as a function of the resonance overlap parameter $s = (\Delta v_1 + \Delta v_2)/ \Delta v_\phi$.



$$H_{nsc} = \frac{p^2}{2m} - \sum_{j=1}^{M} V_j \cos(k_j x - \omega_j t + \varphi_j)$$

A small s means no chaos; the dynamics feels the discreteness of the wave spectrum for $t > \tau_{discr} = (k\Delta v_\phi)^{-1}$; the dynamics is quasiperiodic and $<\Delta v^2>$ saturates.
The initial QL diffusion (predicted by perturbation theory) is stochastic, not chaotic.





A large s means strong chaos; the dynamics stays diffusive and QL for long times: chaotic diffusion.

For intermediate values of s with global chaos, D is initially QL (stochastic), then becomes superquasilinear (chaotic); Cary, DFE & Verga, 1990.
$D = D_{QL}$ for s large is by no means trivial!

Locality of wave-particle interaction and the lack of confinement in velocity due to chaos imply diffusion

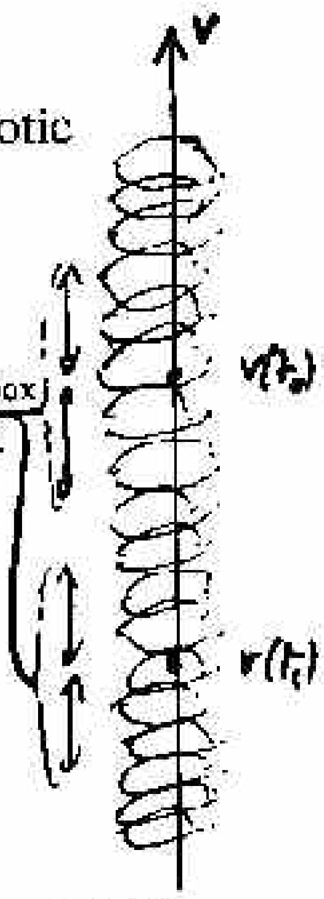
The origin of the chaotic diffusion can be understood with the concept of *resonance box*.

The larger $|v-\omega/k|$, the smaller the influence of the wave on the particle:

Even in the chaotic regime, waves strongly non resonant may be treated through perturbation theory:

There is locality in the interaction.

For strong resonance overlap, only waves with $|v-\omega/k| < \Delta v_{box} \sim (D_{QL}/k)^{1/3}$ contribute to the chaotic transport (Bénisti & DFE, 1997).



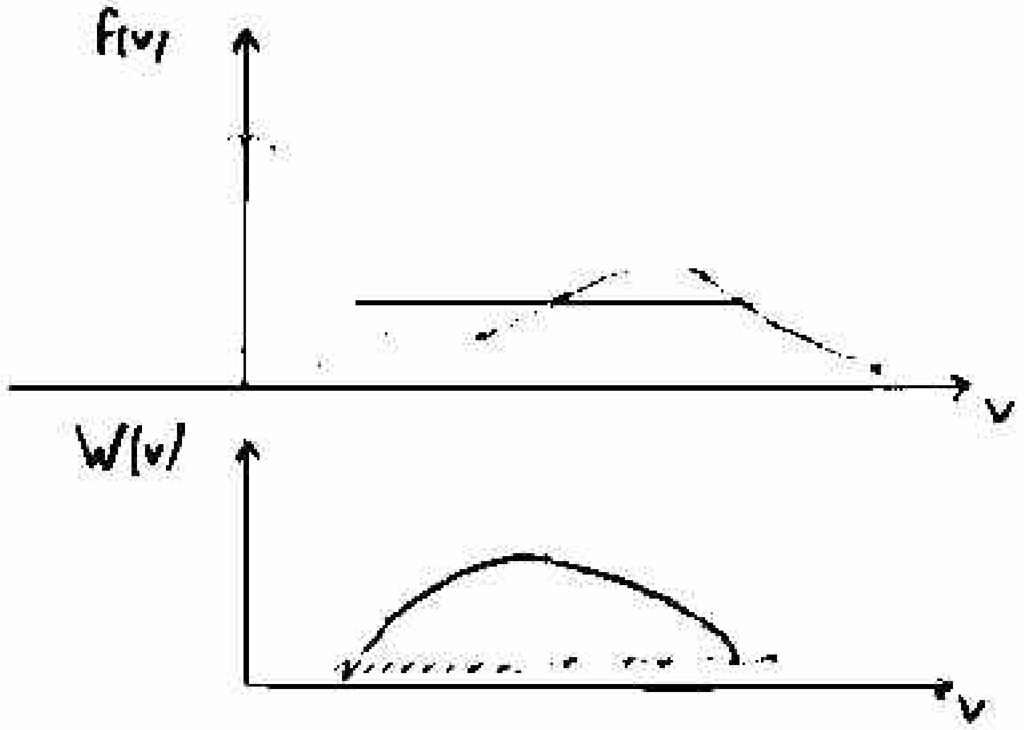
Since chaos makes the orbit unconfined in v, it Visits a series of resonance boxes of width $2\Delta v_{box}$ Where the wave random phases are independent.

The velocity undergoes a series of independent Increments. Yields a diffusion according to the central limit theorem.

A random initial particle position only does not yield a Gaussian statistics to the velocity:

Randomness of the phases is important.

Quasilinear theory was developed to describe the weak warm beam-plasma instability



Vedenov, Velikhov & Sagdeev, 1962

Drummond & Pines, 1962

Quasi linear: mode couplings are neglected except for their effect on the space averaged distribution function

$$\partial_t f = \partial_v \Big(D_{QL}(v) \partial_v f \Big), \qquad D_{QL}(v) = \frac{4\pi^2 e^2}{m^2 L} |E^2_{k(v)}|$$

$$\frac{d|E_k^2|}{dt} = \gamma_L(k,t)|E_k^2| \qquad \gamma_L(k,t) = \frac{\pi \omega_p^3}{2nk^2} \partial_v f(v(k),t)$$

Quasilinear assumption is wrong in the saturation regime

Velocity diffuses, but positions spread faster

$<\Delta v^2> = 2 D_{QL} t$ $\quad <k^2 \Delta x^2> = 2 k^2 D_{QL} t^3 / 3$

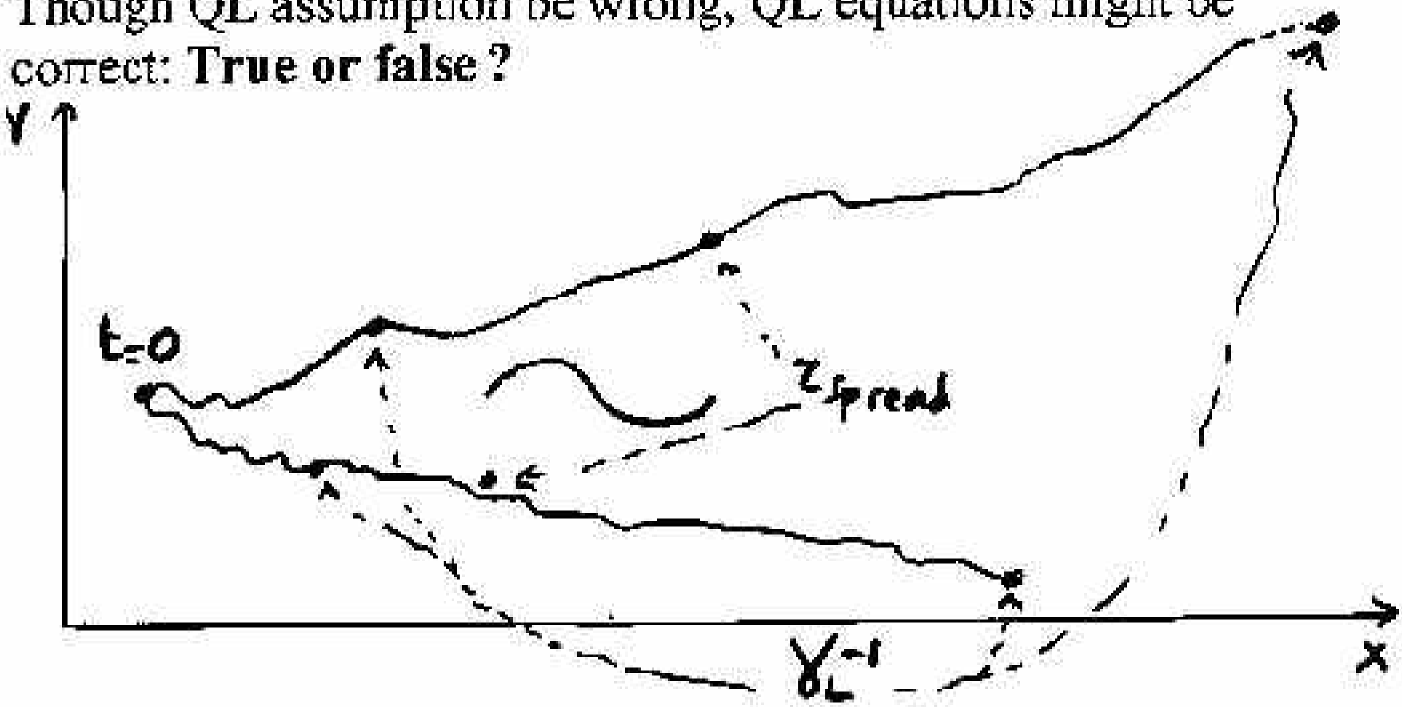
$<k^2 \Delta x^2> \sim 1$ for $t = \tau_{spread} = (k^2 D_{QL})^{-1/3}$

QL assumption is correct for the weakly nonlinear initial regime of the instability

Close to ballistic motion: $\gamma_L^{-1} << \tau_{spread}$

However saturation involves $\gamma_L^{-1} >> \tau_{spread}$ and a strong mode coupling is seen numerically and experimentally.

Though QL assumption be wrong, QL equations might be correct: **True or false ?**

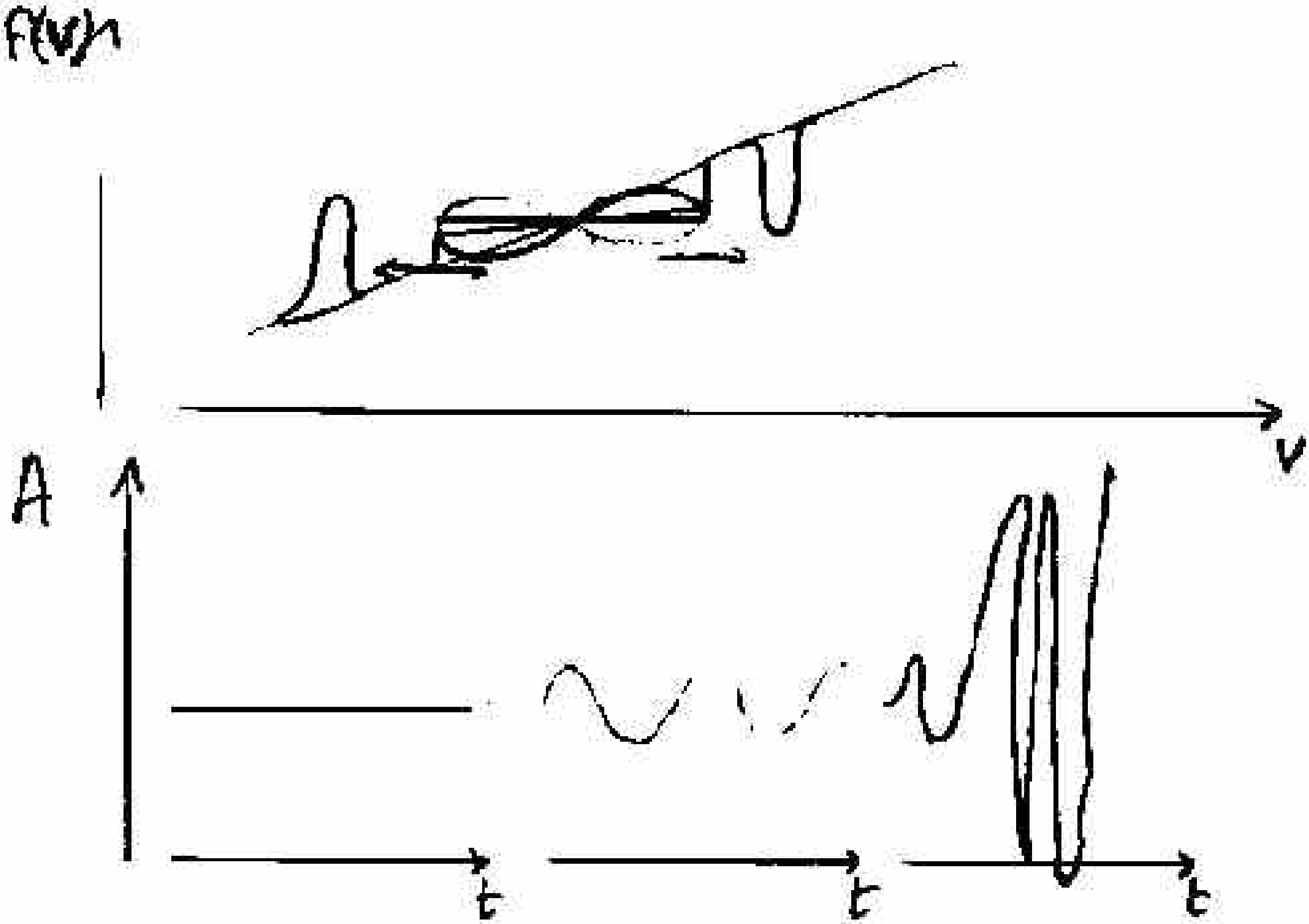


Kinetic instability with wave and damping makes possible time dependent NL effects

$$\frac{df}{dt} = \nu (f_0 - f)$$

γ_d Damping of E due to collisions, etc... $\gamma = \gamma_L - \gamma_d$

Important parameter: ν/γ



MHD spectroscopy, Berk, Breizman et al. '95-'98

q (t), mode characteristics

For strong resonance overlap there is a cross-over between the initial non chaotic QL regime and the final chaotic one

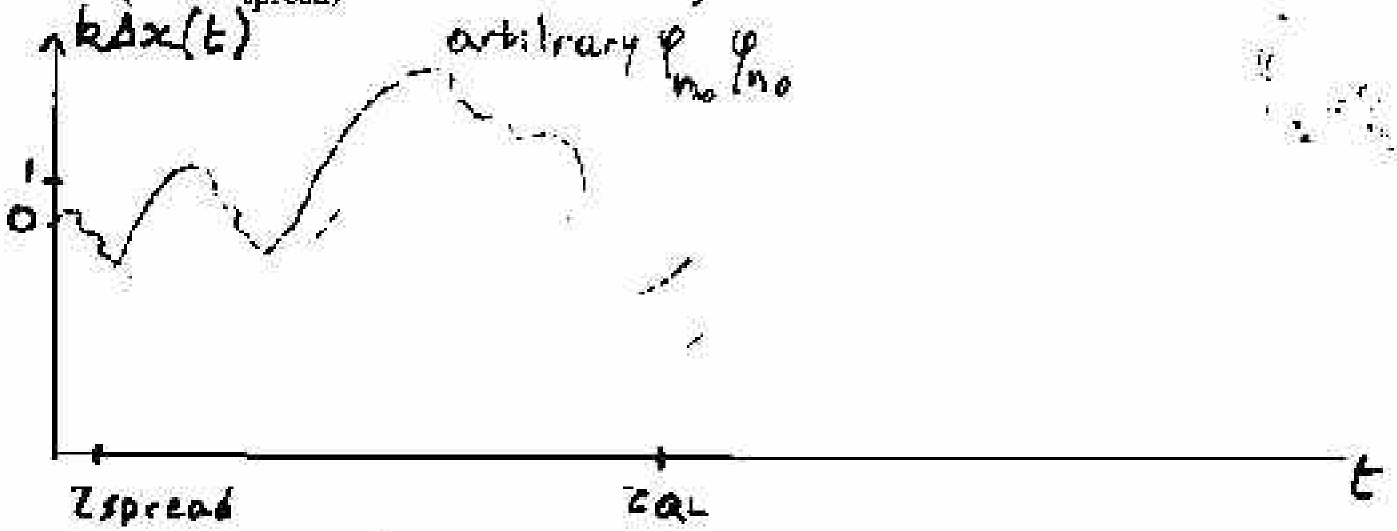
The traditional requirement $< k^2 \Delta x^2(t) > \ll 1$ for the initial QL estimate forces x(t) to have a small dependence over all M phases simultaneously.

Let $\Delta v(t) = v(t) - v(0)$. The formal integration of the equation of evolution of v reveals $<\Delta v^2(t)>$ to take its QL value if x(t) has a small dependence over any $2 \ll M$ phases: this extends the temporal validity of the initial QL estimate up to $\tau_{QL} \gg \tau_{spread}$ (Bénisti & DFE, 1997).

τ_{QL} is computed rigorously by linear theory, since the variation of x(t) is small when any two phases are varied (DFE & Elskens, 2002): $\tau_{QL} = \tau_{spread} |\ln(k\Delta v_\phi \tau_{spread})|$ for strong resonance overlap (e.g. continuous spectrum).

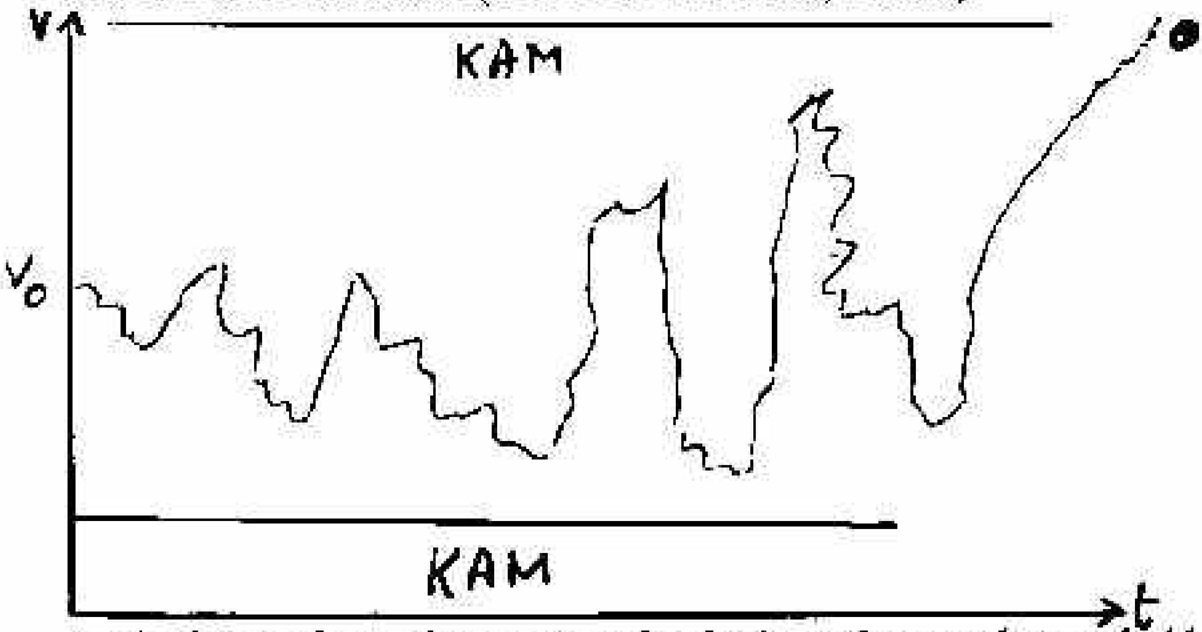
Therefore $D = D_{QL}$ for $t \ll \tau_{QL}$.

Spin off : rigorous calculation of the Lyapunov exponent ($\sim 1/\tau_{spread}$) of the chaotic dynamics.



Quasilinear diffusion is correct over the whole chaotic regime if resonance overlap is strong

The existence of a diffusion over a time $\tau_{QL} >> \tau_{spread}$ makes possible the rigorous extension of the QL estimate up to the time where orbits hit the (KAM) boundaries of the chaotic domain (DFE & Elskens, 2002).



An independent rigorous calculation shows that <dv/dt> = dD_{QL} (v)/dv: extension of the Landau formula (1937) for the regular regime.

On this basis the particles are shown to obey the QL Fokker-Planck equation (DFE & Elskens, 2002).

In fact the chaotic dynamics is truly Markovian only for an infinite resonance overlap: no loss of memory in Hamiltonian chaos after a time given by the inverse of the Lyapunov exponent ($t \sim \tau_{spread}$).

QL controversy: are QL equations correct in the saturation regime ?

Laval & Pesme (1983) predicted a renormalization of the growth rate and diffusion coefficient by 2.2.

Aimed at checking this prediction, a first experiment reached a weakly nonlinear regime where mode coupling was strong, but no renormalization was found (Tsunoda, Doveil, Malmberg, 1991).

This was felt as a *paradox*, and motivated the new mechanical approach.
This approach was encouraged by the progress in the understanding of low dimensional Hamiltonian chaos.

The Tsunoda et al. experiment initiated a controversy involving 20 analytical, numerical, and experimental papers over 2 decades.

Liang and Diamond (1993) and Shapiro and Sagdeev (1997) denied any renormalization.

Chaos and the weakness of interactions lead to QL equations in the saturation regime of the weak warm beam-plasma instability

During the saturation of the weak warm beam-plasma instability the dynamics enters the regime $\tau_{spread} << \gamma_L^{-1}$ where chaos is essential: strong mode coupling is present.

$<\Delta I_j>$, $<\Delta p_l>$, and $<\Delta p_l^2>$ related to $[t, t+\Delta t]$, t & $\Delta t >> \tau_{spread}$ can be computed in this regime by taking advantage of:

1) the large spreading of particle positions at $t >> \tau_{spread}$ Introduces chaotic random phases,
2) the absence of direct interaction between 2 particles or 2 waves: simpler than Boltzmann's problem,
3) the weak mutual influence between any particle and any wave: mean field limit.

QL equations turn out to be correct in the regime $\tau_{spread} << \gamma_L^{-1}$ (DFE & Elskens, 2002).

The derivation is made up of a few explicit steps which can be easily followed by a graduate student (6 pages in PoP, May 2003). *Not yet rigorous.*

This result is not a trivial extension of the initial QL regime: Non QL estimates hold for $\tau_{spread} \sim \gamma_L^{-1}$ Confirms a conclusion reached by Laval & Pesme (1983).

Conclusion

Wave-particle dynamics

- 2 types of coupled objects: waves and particles

Simple limits of this interaction

One wave number

Hydrodynamic case } { Trapping

Kinetic case } { $\omega_b \sim \gamma$

Several wavenumbers

Quasilinear spreading:•diffusion and penetration

Stochastic/chaotic regime $t \gg \tau_{spread}$

Prescribed/self-consistent field:

- Chaotic decorrelation

Less simple (but important) limits:

- Time dependent nonlinear state
- MHD spectroscopy, Pinches PPCF 2005

<u>Caveats</u>

Phase space resolution in kinetic codes

Measurement of numerical transport

Folklore about Landau damping

Plasma physics as statistical physics

From N body to irreversible evolution by chaotic mechanics

Plasma physics as chaos theory: resonance box concept

Elskens, DFE, *Microscopic physics of plasmas and chaos*, IOP 2003

<u>Future</u>

Self-consistency in fast particle-wave interaction

Non Valsovian effects