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Confinement of Fast Ions

(Or all you never thought you wanted to know about funny fast ion orbits)

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Outline

- Introduction
- Fast ion orbits in a tokamak
- Orbit averaged Fokker-Planck equation
- Effect of collisional slowing down on fast ions
- Transport of fast ions induced by magnetic field ripple
- On the transfer of torque from fast ions to the bulk plasma; exemplified on ICRF accelerated fast ions
- Fast ions and sawtooth redistribution
- On fast ions in plasmas with a current hole
- Conclusions

Fast ions in tokamaks

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- Fast ions have v >> v_{th}.
- The are several sources of fast ions, e.g. :
  - Alpha particles born in fusion reactions.
  - Neutral Beam Injected (NBI) ions for heating.
  - Acceleration of ions by waves used for plasma heating.
- The fast ions are weakly collisional, and the main collision process is slowing down on the electrons.

$$t_s = 6.27 \cdot 10^{14} \frac{A}{Z^2} \frac{(T_{e,eV})^{3/2}}{n_e \ln \Lambda}$$

, T_e=10 keV, n_e=4·10^{19} m^{-3}

$t_{s,\alpha}$~1s

- Orbit bounce time, τ_b, is typically ~ 10^{-6} to 10^{-4}s → $\tau_b/t_s<<1$

Toroidal and poloidal movement

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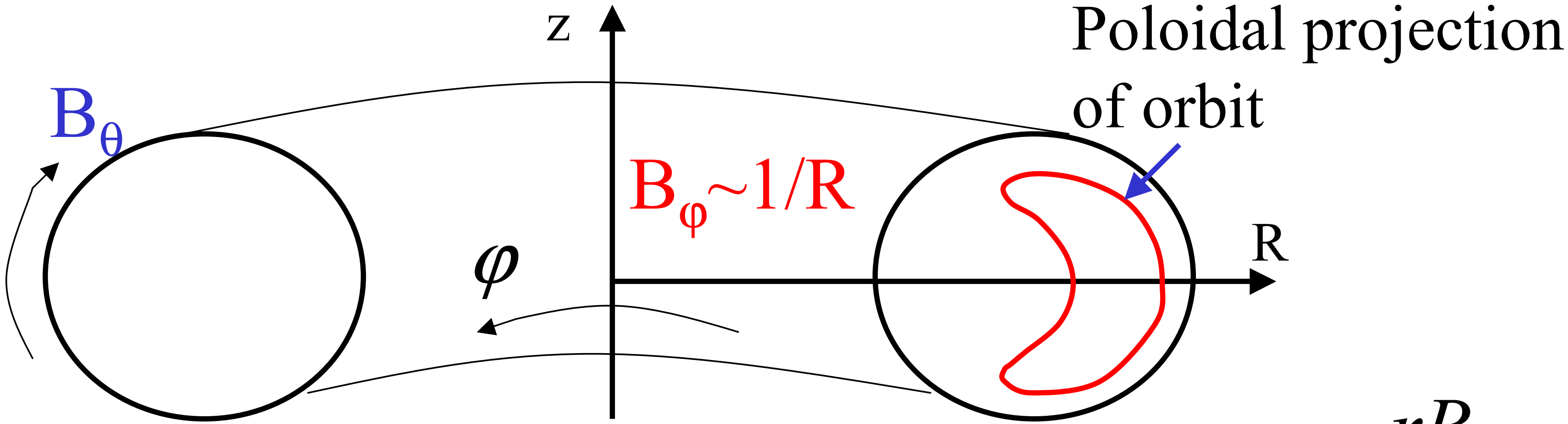

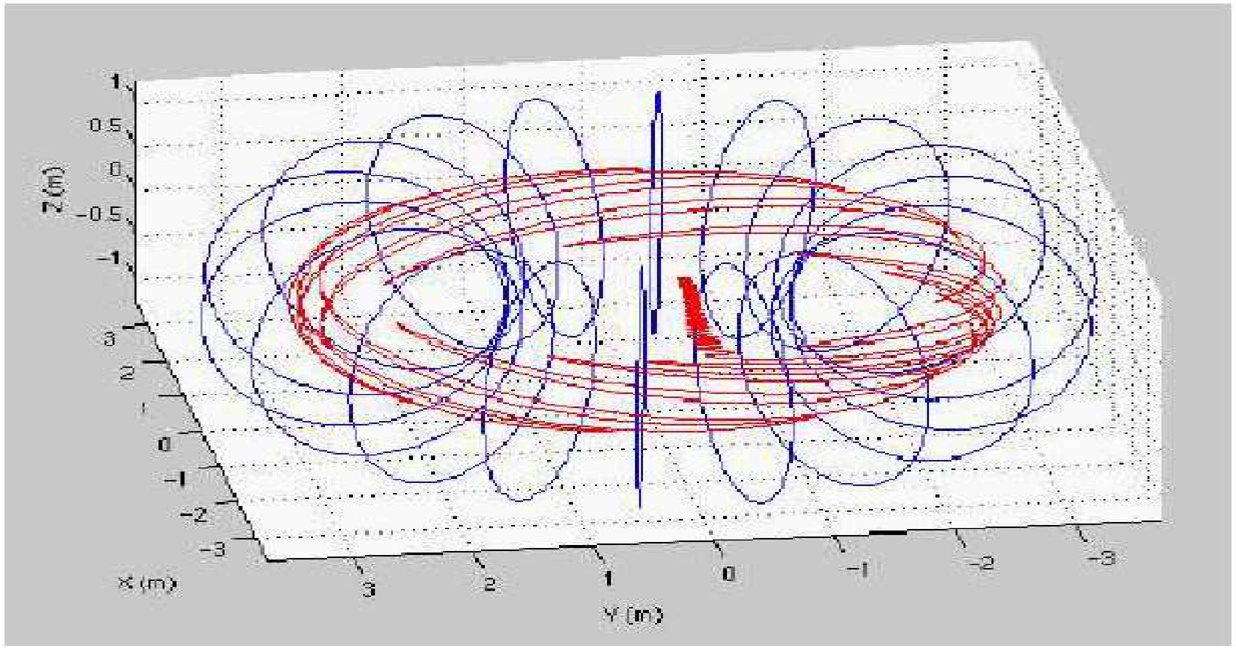

$$q \approx \frac{rB_\varphi}{RB_\theta}$$

Fast ion orbits in tokamaks

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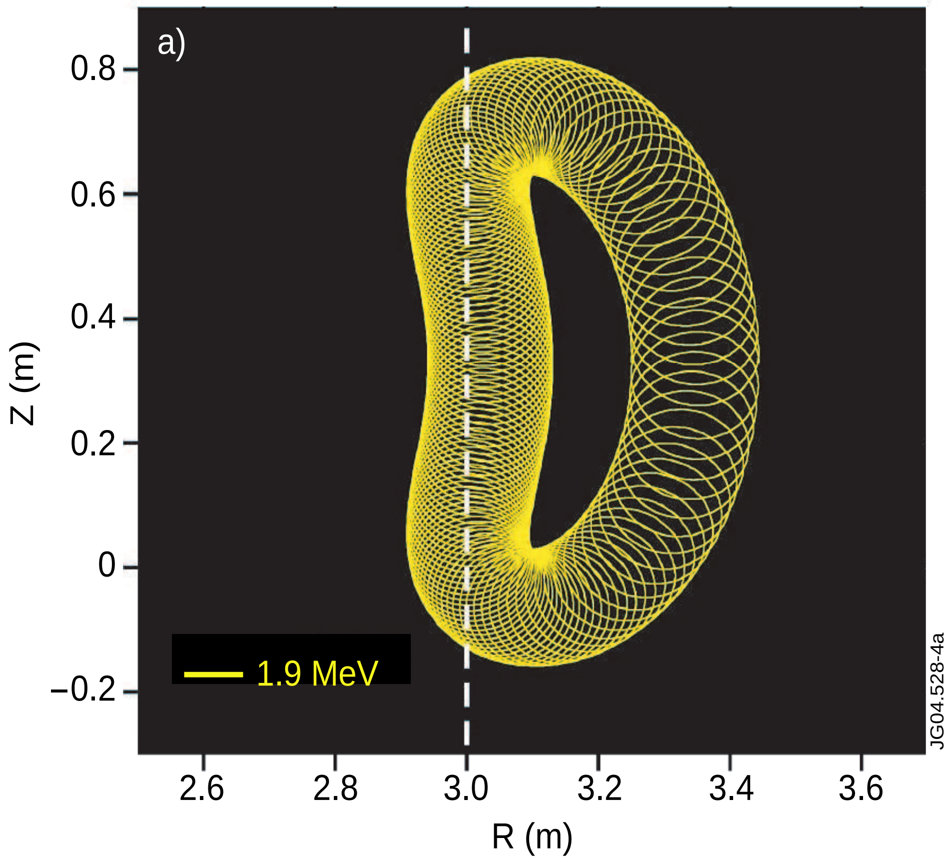
- The orbits of fast ions are central to understanding their confinement.
- In the case of energetic ions the situation is quite complicated, with a variety of possible orbits

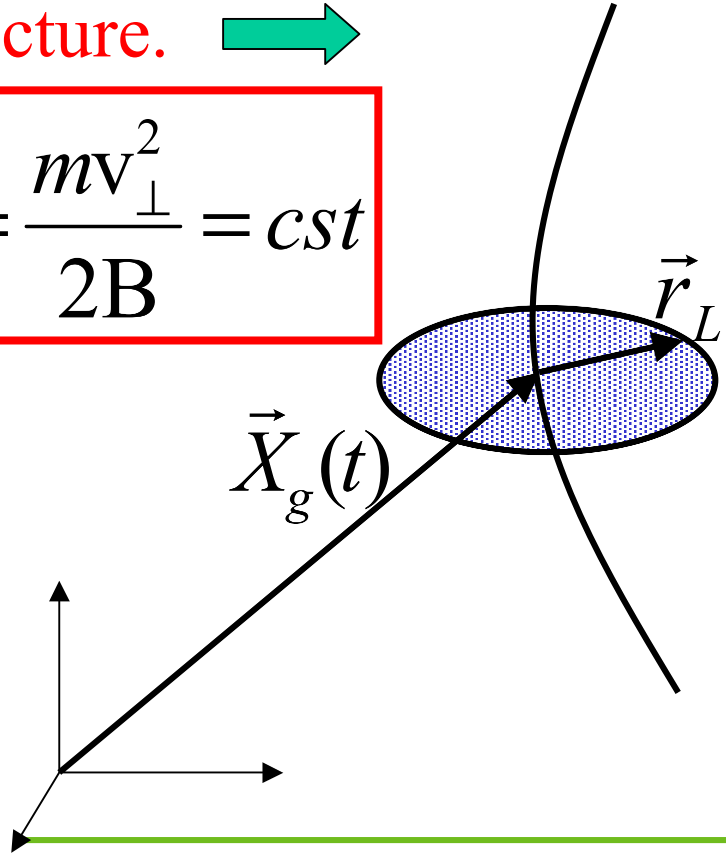


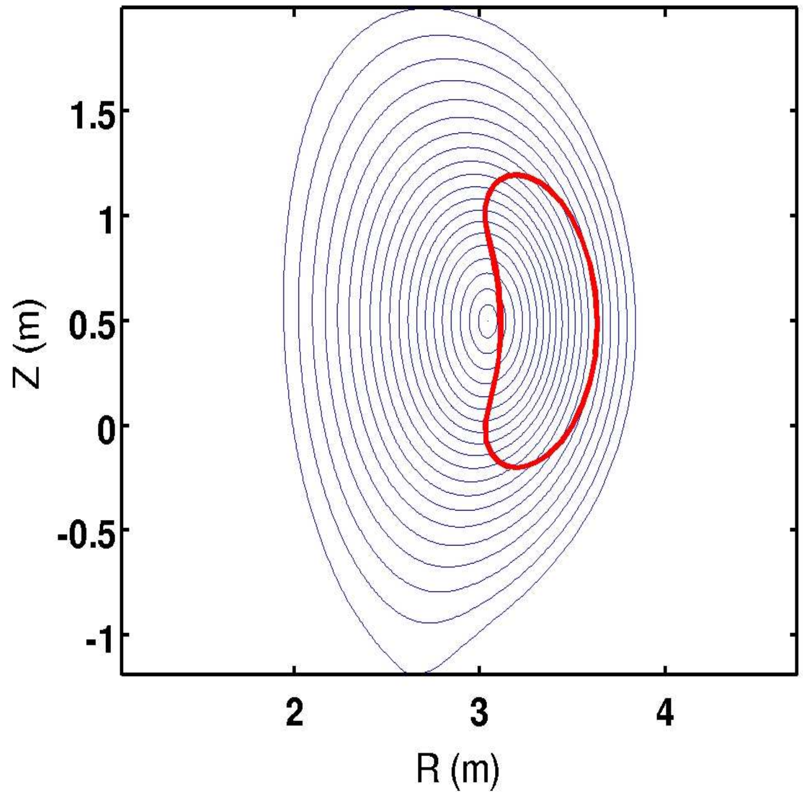
1.9 MeV alpha particle (^4He) in JET

Guiding centre orbits

- The particle motion can be viewed as being composed of two parts: the guiding centre and the gyro motion.
- We will concentrate on guiding centre orbits in this lecture.

$$\mu = \frac{m v_{\perp}^2}{2B} = cst$$





Small banana width limit

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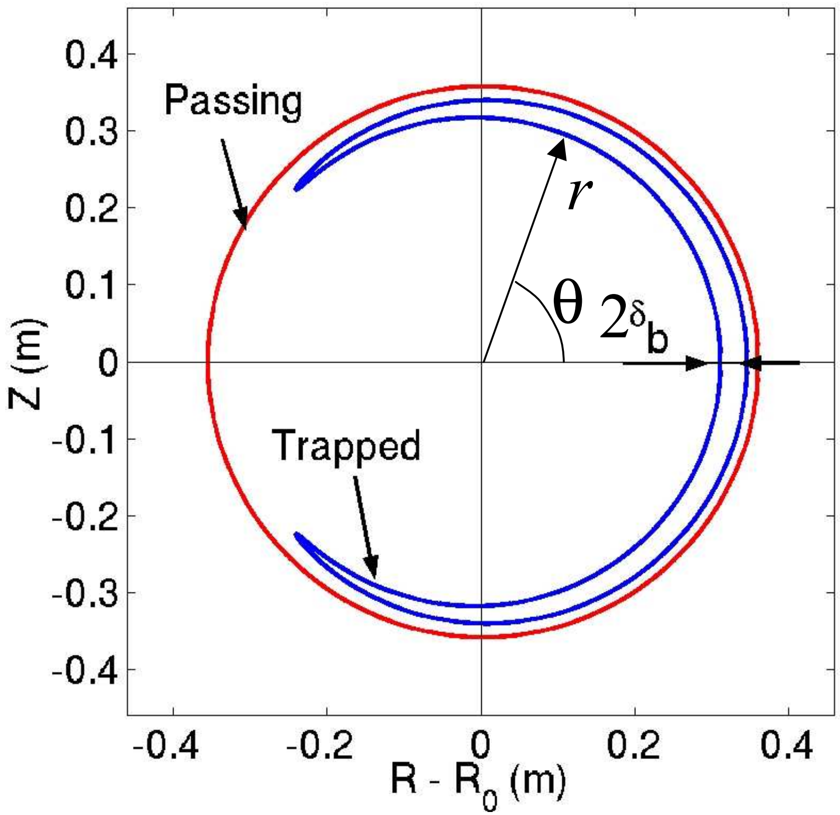
- In the small banana width limit there are two types of guiding centre orbits: passing and trapped (also called banana orbit).
- The banana-width is of the order,

$$\delta_b \sim \frac{v_{\|0}}{\omega_{B_\theta}} = \frac{v}{\omega_c} q \left(\frac{r}{R}\right)^{-1/2}$$

$\delta_b << r$



- The "fun" starts when $\delta_b \geq r$!

Example

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- Take a 3.5 MeV alpha particle, $q = 1, r / R = 0.1$

JET: B~3T → $2\delta_b / a \sim 0.8$

Essential to take finite orbit width effects into account

ITER: B~6T → $2\delta_b / a \sim 0.2$

Finite orbit width effects needed for detailed calculations.

Orbits with $\delta_b \geq r$ are discussed in e.g.,

Stringer Plasma Phys **16**, 651 (1974)

Rome and Peng, Nuclear Fusion 19, 1193 (1979)

Porcelli, Eriksson, Berk, EPS 1994.

Doloc and Martin, Phys. Plasmas **2**, 3655 (1995).

Egedal Nuclear Fusion **40**, 1597 (2000).

Eriksson and Porcelli PPCF **43**, R145 (2001) .

Equation of motion for the G.C.

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- For simplicity we will consider a tokamak with circular flux surfaces.

$$\frac{dr}{dt} = -\mathrm{v}_D \sin\theta$$

$$r\frac{d\theta}{dt} = \mathrm{v}_\parallel \frac{B_\theta}{B} - \mathrm{v}_D \cos\theta$$

$$R\frac{d\varphi}{dt} = \mathrm{v}_\parallel \frac{B_\varphi}{B}$$

$$m\frac{d\mathrm{v}_\parallel}{dt} = -\mu\nabla_\parallel B$$

grad-B + curvature drift:

$$\mathrm{v}_D \approx \frac{\frac{1}{2}\mathrm{v}_\perp^2 + \mathrm{v}_\parallel^2}{\omega_c R}$$

$$\mu = \frac{m\mathrm{v}_\perp^2}{2B}$$

Invariants

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- The guiding centre motion can often be more conveniently analysed with the aid of invariants of the motion:

$$E = \frac{1}{2} m\mathrm{v}^2$$ (Energy)

$$\mu = \frac{m\,\mathrm{v}_\perp^2}{2B}$$ (Magnetic momentum)

$$p_\varphi = \frac{Ze}{2\pi}\psi - mR\mathrm{v}_\varphi$$ (Negative of the physical canonical momentum)

ψ is the poloidal flux: $\psi = 2\pi \int_0^r RB_\theta dr'$

$\psi \geq 0$; plasma current in positive φ direction

Orbit equation

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With $\Lambda = \dfrac{\mu B_0}{E}$ and $\mathrm{v}_\varphi = \dfrac{B_\varphi}{B}\mathrm{v}_\parallel$

the parallel velocity can be expressed as,

$$\mathrm{v}_\parallel = \pm \mathrm{v}\sqrt{1 - \Lambda \frac{\mathrm{B}}{\mathrm{B}_0}} \quad ; \quad \mathrm{v}_\parallel = \frac{1}{mR}\frac{B}{B_\varphi}\left[\frac{Ze}{2\pi}\psi - p_\varphi\right]$$

$$\pm\sqrt{1-\Lambda\frac{B}{B_0}} = \frac{1}{m\mathrm{v}}\frac{B}{g(\psi)}\left[\frac{Ze}{2\pi}\psi - p_\varphi\right]$$

$$g(\psi) = RB_\varphi \;, \quad B = B(r,\theta), \quad \psi = \psi(r)$$

Simplifications

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- Assume low beta tokamak with circular flux surfaces.
- The the inverse aspect ratio, $\varepsilon = r/R$, to be small.
- $B_\varphi / B \approx 1,\ B \approx B_0 / (1 + \varepsilon\cos\theta) \approx B_0 (1 - \varepsilon\cos\theta)$

$$\pm\sqrt{\lambda + \varepsilon\cos\theta} = \frac{Ze}{2\pi m R_0 \mathrm{v}_{\perp 0}}\left[\psi - \psi_\varphi\right]$$

Where, $\lambda = \frac{1}{\Lambda} - 1$, $\psi_\varphi = \frac{2\pi}{Ze} p_\varphi$, $\mathrm{v}_{\perp 0} = \mathrm{v}\sqrt{\Lambda}$

- Solution can yield two orbits; an additional parameter $\sigma = \mathrm{sign}(\mathbf{v}\cdot\mathbf{I}_p)$ at $R = R_{max}$, needed for full description.

λ ↗ ⇨ "$\mathrm{v}_{\|}/\mathrm{v}_{\perp}$" ↗ ; λ ↘ ⇨ "$\mathrm{v}_{\|}/\mathrm{v}_{\perp}$" ↘

Banana width

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- We assume low shear, i.e. q constant over an orbit $\Rightarrow$

$$\psi = 2\pi \int_0^r RB_\theta dr' = \pi r^2 B / q$$

- Consider a trapped particle with its turning point at $R=R_0(\theta \pm \pi/2) \Rightarrow \lambda = 0$, $\varepsilon_{t.p.}^2 = q\psi_\varphi / \pi B R_0^2$

$$\pm\sqrt{\varepsilon \cos\theta} = \frac{\omega_c R_0}{2q v_{\perp 0}} \left[\varepsilon^2 - \varepsilon_{t.p.}^2\right]$$

- Assuming $|(\varepsilon - \varepsilon_{t.p.})| / \varepsilon_{t.p} << 1 \Rightarrow \varepsilon^2 - \varepsilon_{t.p.}^2 \approx 2\varepsilon_{t.p.}(\varepsilon - \varepsilon_{t.p.})$

$$\delta_b = R_0\left[\varepsilon(\theta = 0) - \varepsilon_{t.p.}\right] \approx \frac{q v}{\omega_c \sqrt{\varepsilon}}$$

Potato width

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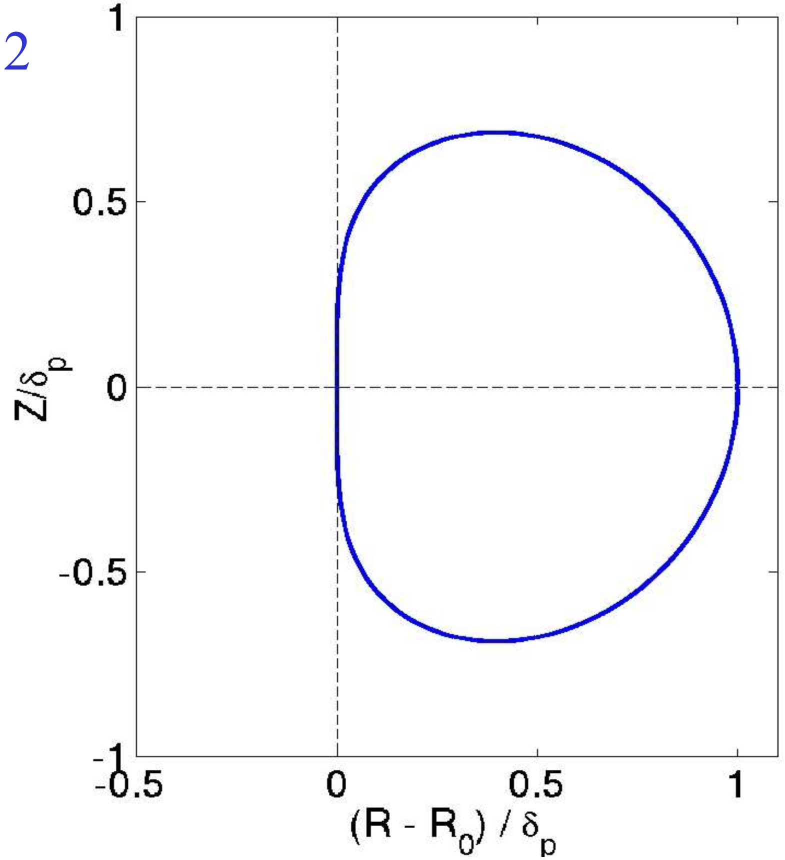
- We now consider a case with r < δ_b.
- Take a particle with $v_\parallel = 0$ at $r = 0 \Rightarrow \lambda = 0,\ \psi_\varphi = 0$.

$$\pm\sqrt{\varepsilon\cos\theta} = \frac{\omega_c R_0}{2 q v_{\perp 0}}\varepsilon^2$$

$\theta = 0$

$$\delta_p = \left(\frac{2 q v}{R_0 \omega_c}\right)^{2/3} R_0$$

δ_p is the characteristic length scale for non-standard orbits in the potato regime



Normalised orbit equation

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- We introduce the normalised quantities:

$$\hat{r} = r / \delta_p \,, \quad \hat{\lambda} = \lambda \frac{R_0}{\delta_p} \,, \quad \hat{\psi}_\varphi = \frac{2q}{B_0 \delta_p^2} \psi_\varphi$$

$$\pm\sqrt{\hat{\lambda} + \hat{r}\cos\theta} = \hat{r}^2 - \hat{\psi}_\varphi$$

- In the normalised variables the velocity no longer appears explicitly in the orbit equation.
- We only have a two parameter space to explore.

Orbit classification

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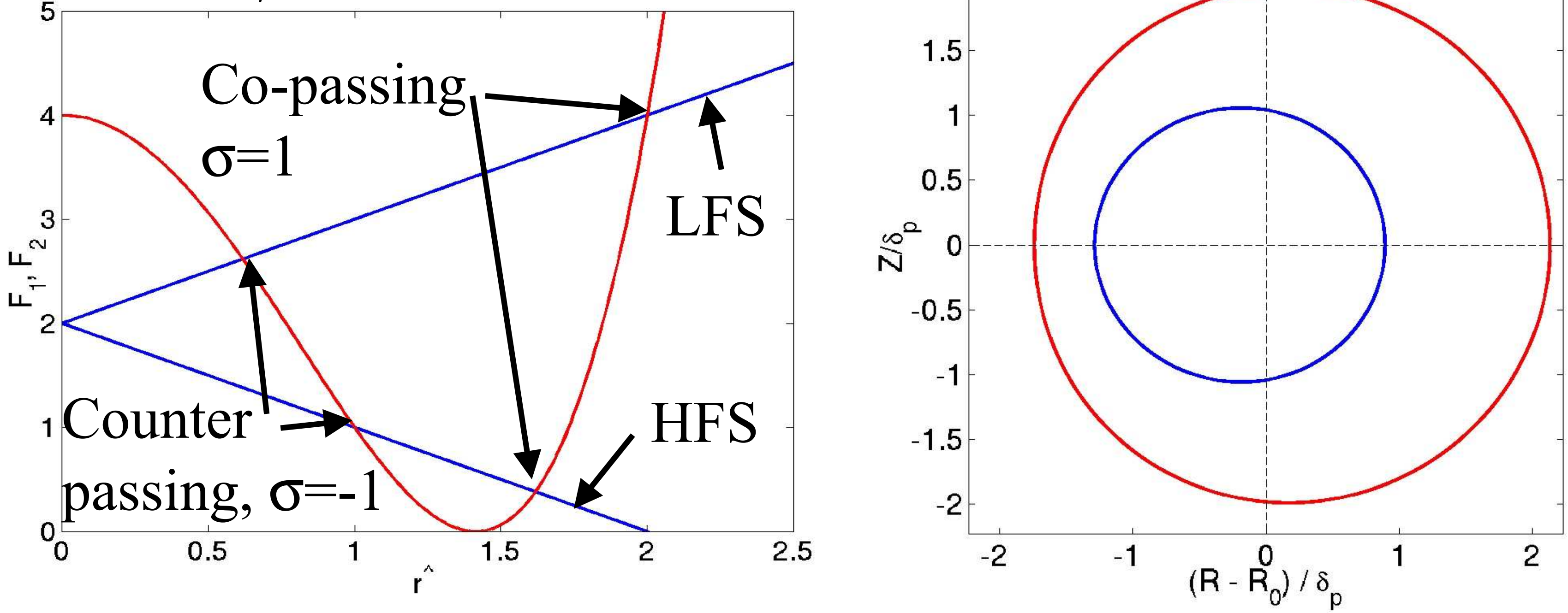
- In order to make an orbit classification it is helpful to study the intersections with the mid-plane

$$F(\hat{r}) = F_1(\hat{r}) - F_2(\hat{r}) = (\hat{\lambda} \pm \hat{r}) - \left(\hat{r}^2 - \hat{\psi}_\varphi\right)^2 = 0$$

$\hat{\lambda} = 2,\ \hat{\psi}_\varphi = 2$ two orbits



Stagnation and pinch orbits

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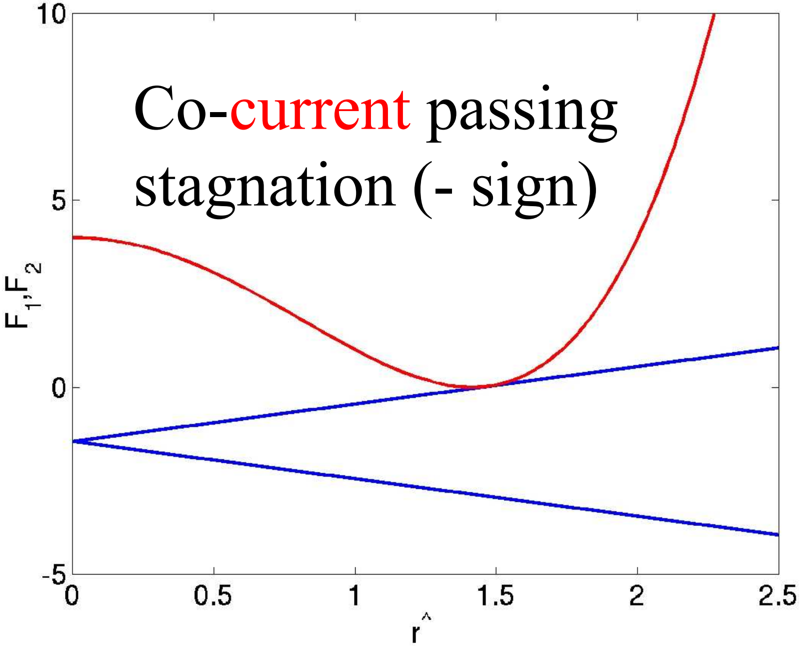
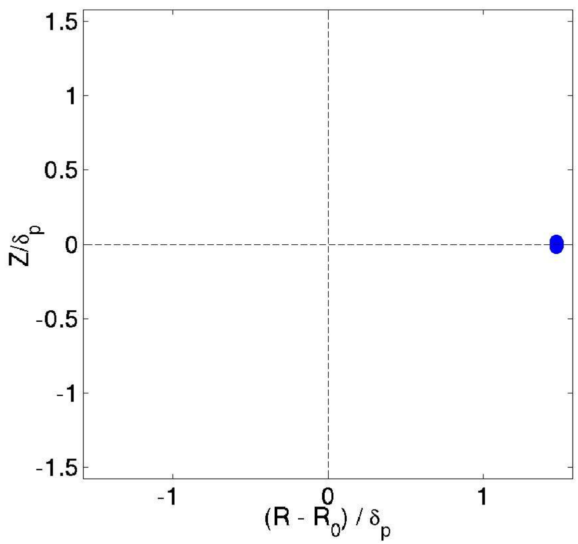
- The locus of stagnation and pinch orbits is given by:

$$\begin{cases} F = 0 \\ \partial F / \partial \hat{r} = 0 \end{cases} \Rightarrow \begin{cases} \hat{\lambda} = \dfrac{1}{16\,\hat{r}^2} \pm \hat{r} \\ \hat{\psi}_\varphi = \hat{r}^2 \pm \dfrac{1}{4\,\hat{r}} \end{cases}$$

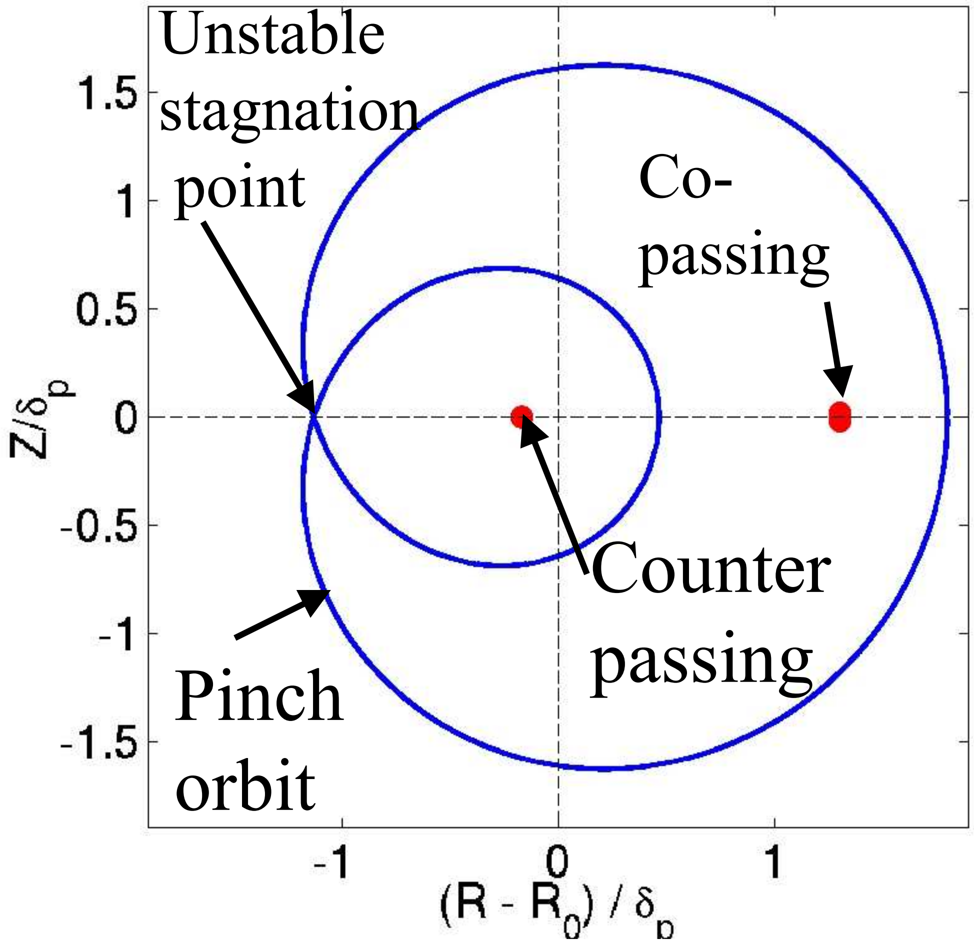




$$\begin{cases} \hat{\lambda} = \dfrac{1}{16\hat{r}^2} \pm \hat{r} \\ \hat{\psi}_\varphi = \hat{r}^2 \pm \dfrac{1}{4\hat{r}} \end{cases}$$

- - ⇒ co-passing stagnation orbits
- + ⇒ two branches:
  (i) counter-passing stagnation
  (ii) Pinch orbits

$$\hat{\psi}_\varphi = 1.5$$



Physical picture of stagnation orbit

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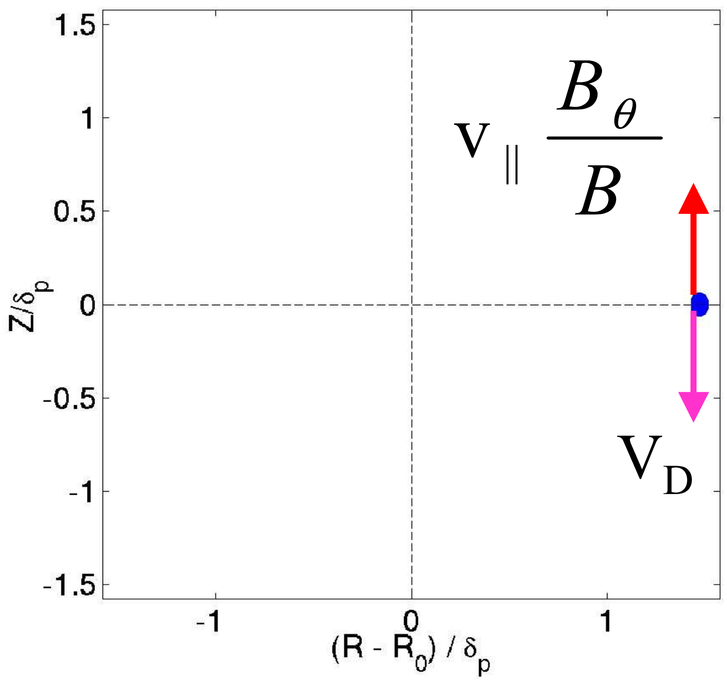
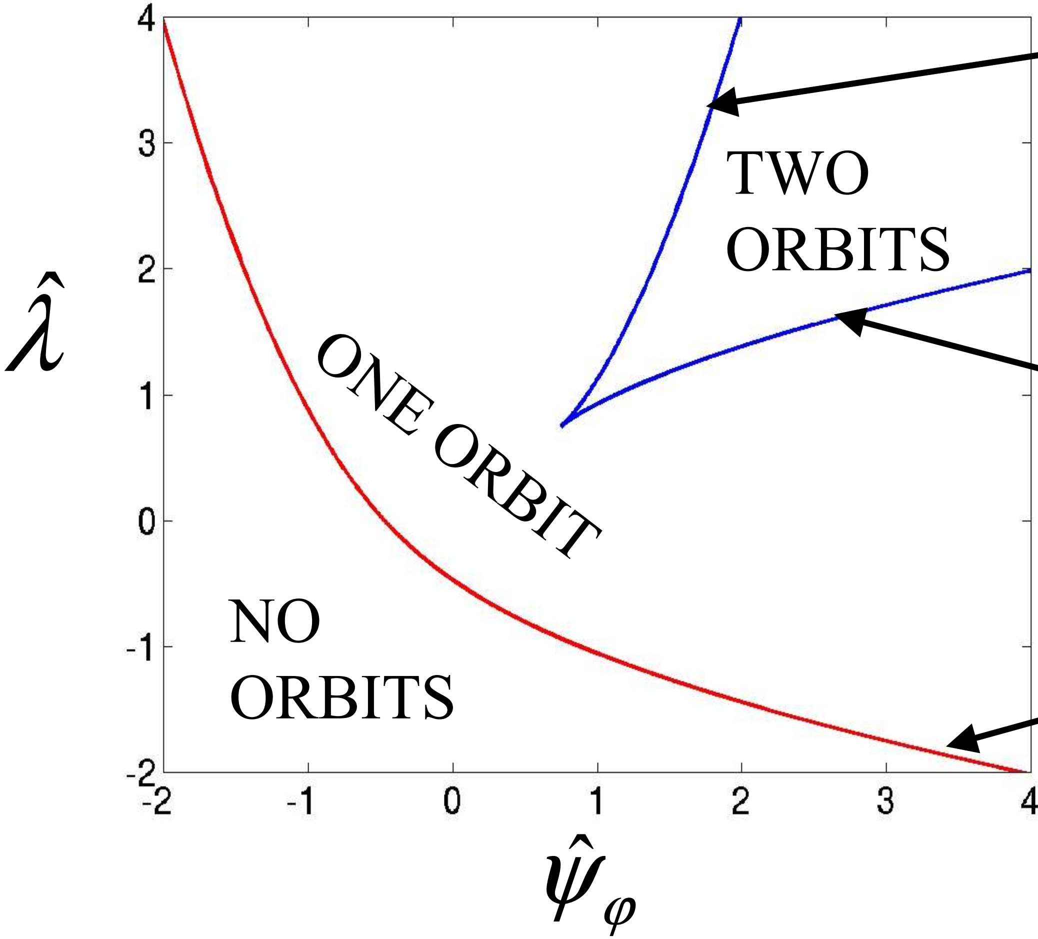
- Stagnation points corresponds to cases where the drift velocity exactly cancels the vertical motion along the field line in the mid-plane

$$0 = v_z = v_D - v_{\parallel} \frac{B_\theta}{B}$$



Orbit map I



Counter-passing stagnation orbits

Pinch orbits

Co-passing stagnation orbits

Further limiting orbits

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- The phase space is further subdivided by:
  (i) orbits tangent to the magnetic axis
  (ii) orbits with turning points ($v_{||} = 0$), at $\theta = 0$ or π, i.e barely trapped orbits, to be distinguished from the pinch orbit

$$\pm\sqrt{\hat{\lambda}+\hat{r}\cos\theta} = \hat{r}^2 - \hat{\psi}_\varphi$$

(i) ⇒ $$\hat{\lambda} = \hat{\psi}_\varphi^2$$

(ii) ⇒ $$\hat{\lambda} = \pm\sqrt{\hat{\psi}_\varphi}$$

Orbit map II (complete)

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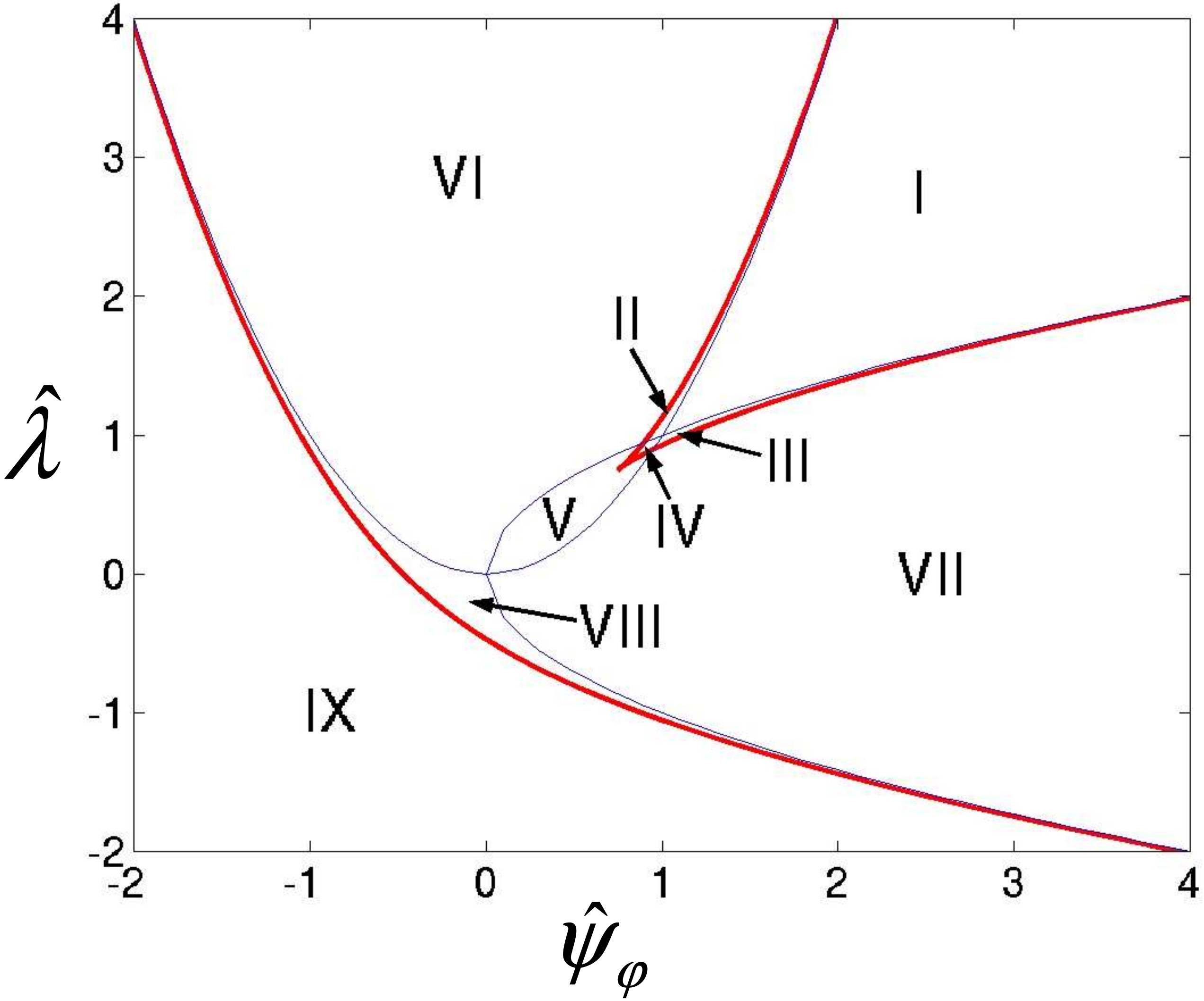
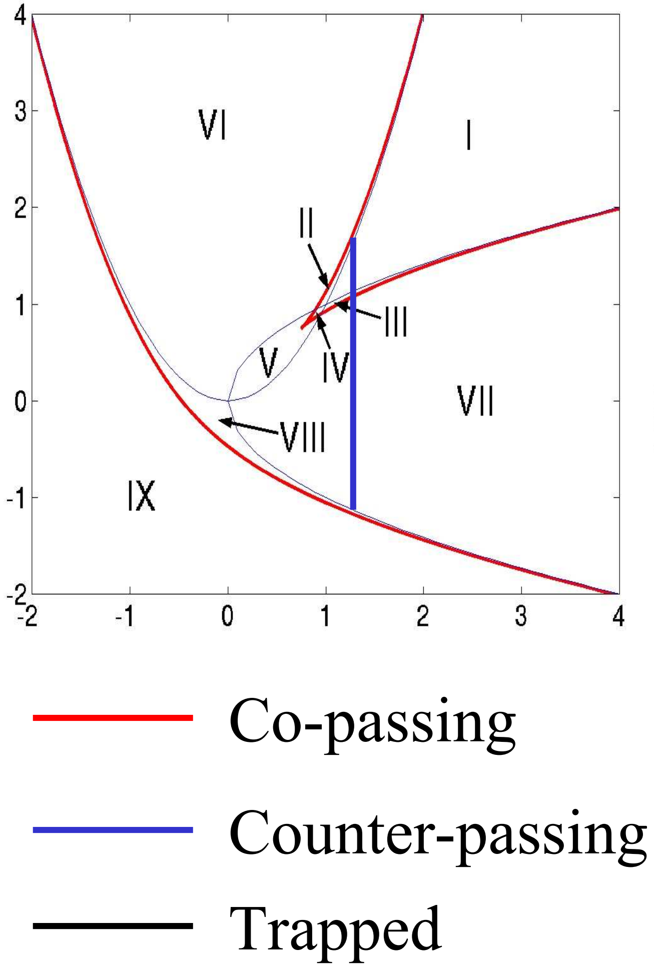
Reg.	Orbits
I	two
II	two
III	two
IV	two
V	one
VI	one
VII	one
VIII	one
IX	none

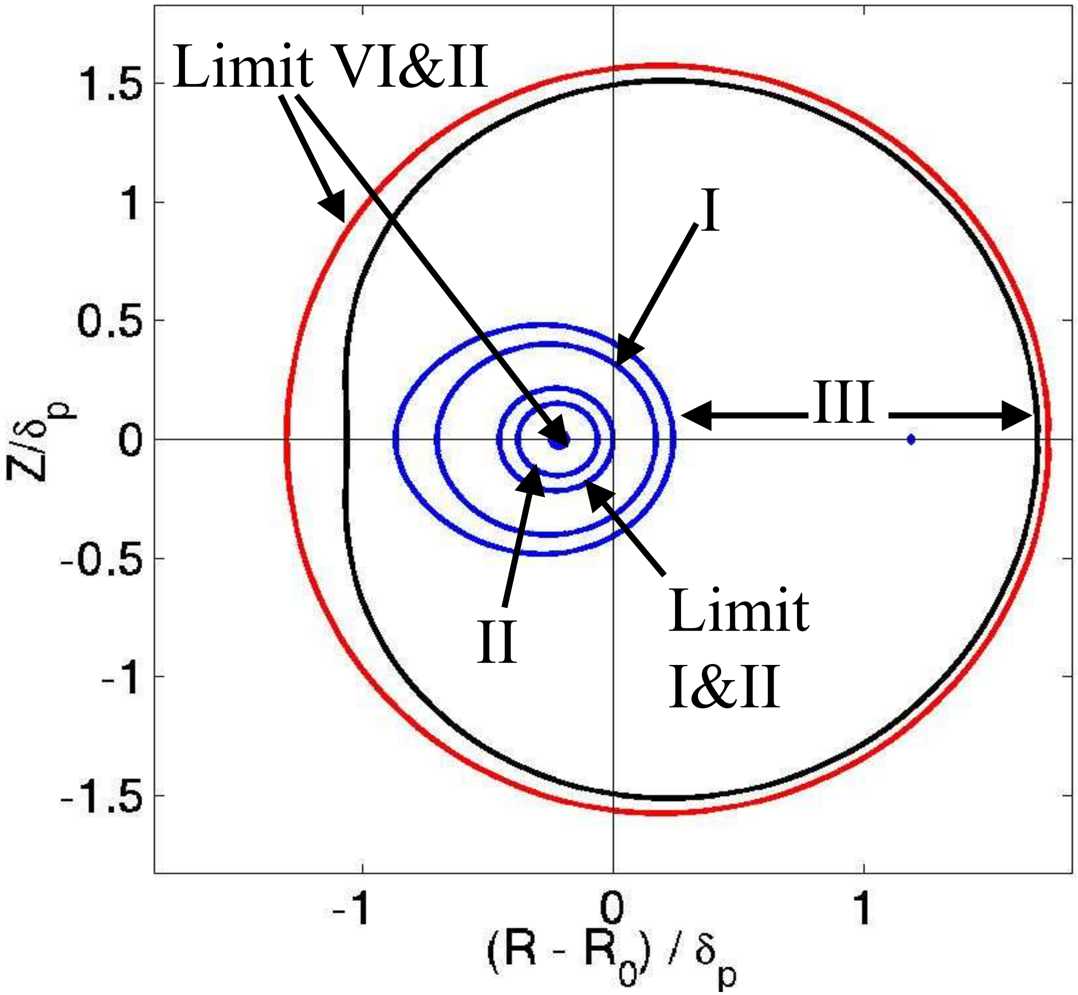


Orbit types

Reg.	Orbits	Type
I	two	Co-/counter passing encircling the axis
II	two	Co-passing encir. axis, counter-pass. HFS
III	two	Trapped encir. axis, counter-pass encir. axis
IV	two	Trapped encircling axis, counter-pass. HFS
V	one	Trapped encircling axis
VI	one	Co-passing encircling axis
VII	one	Trapped not encircling axis
VIII	one	Co-passing LFS
IX	none	

Orbits for $\hat{\psi}_\varphi = 1.2$





Orbits for $\hat{\psi}_\varphi = 1.2$ Cont.

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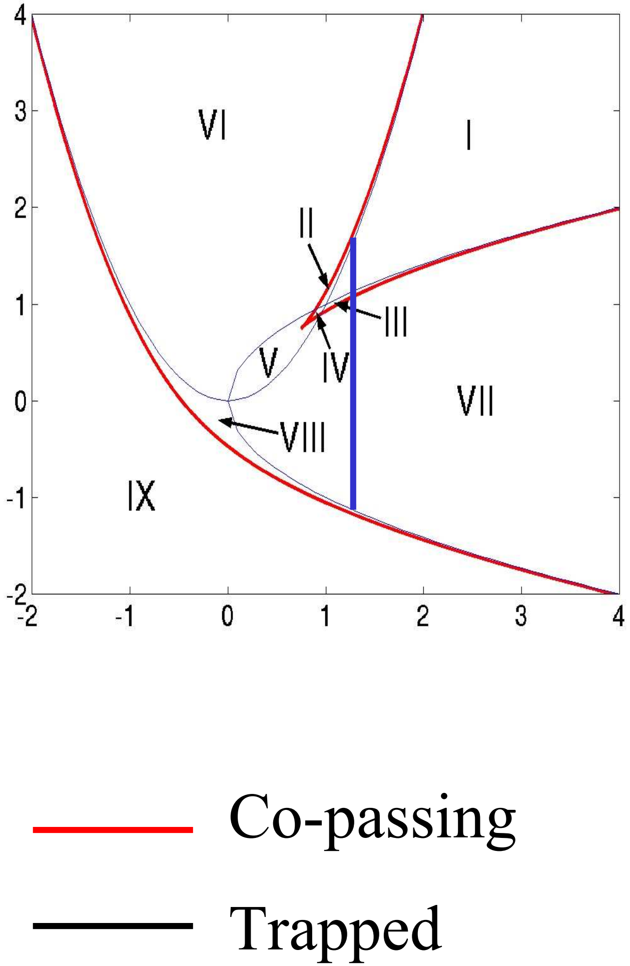
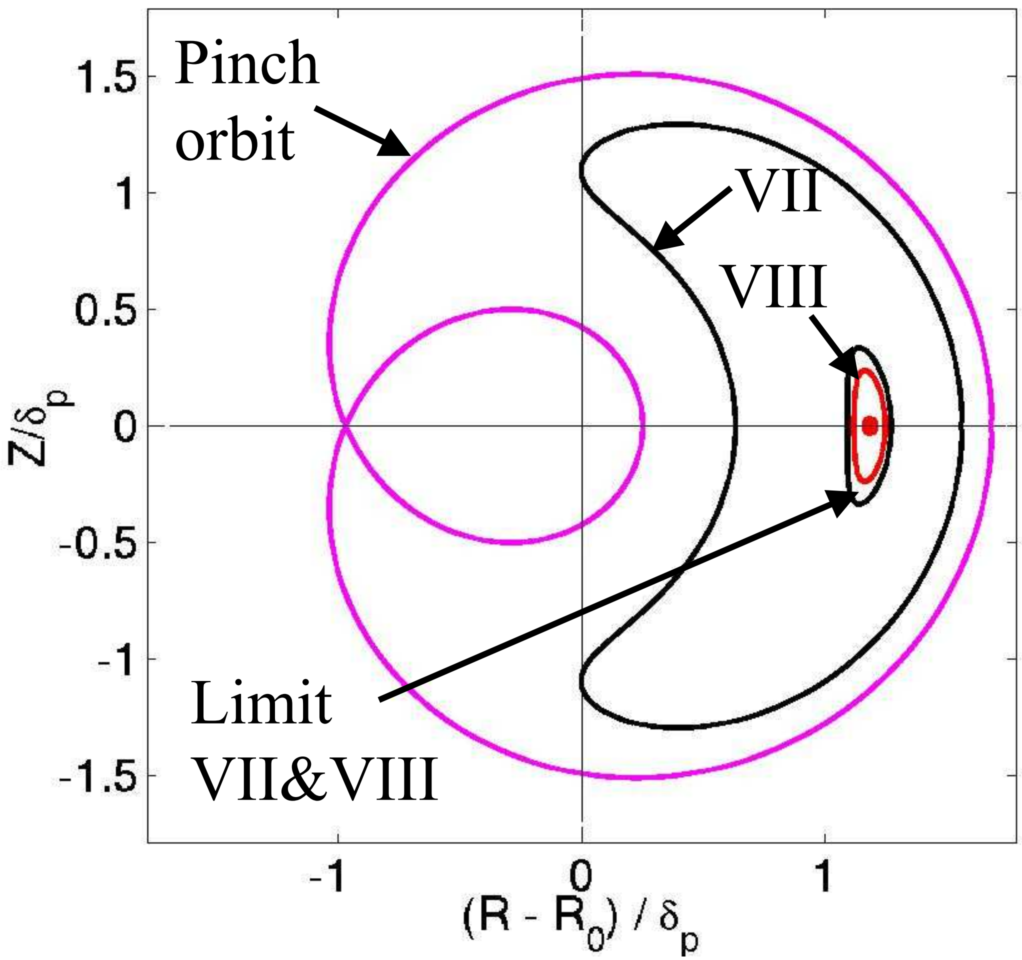
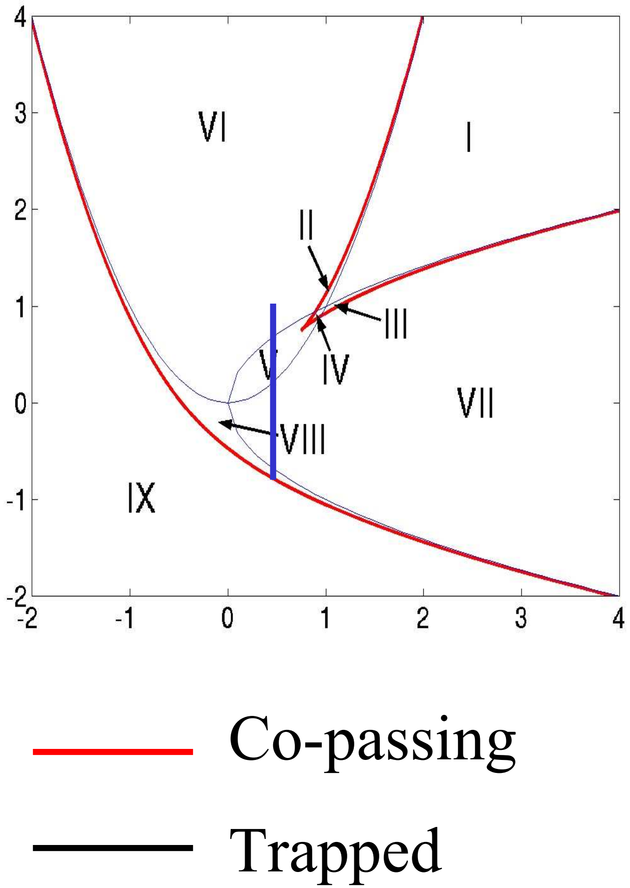
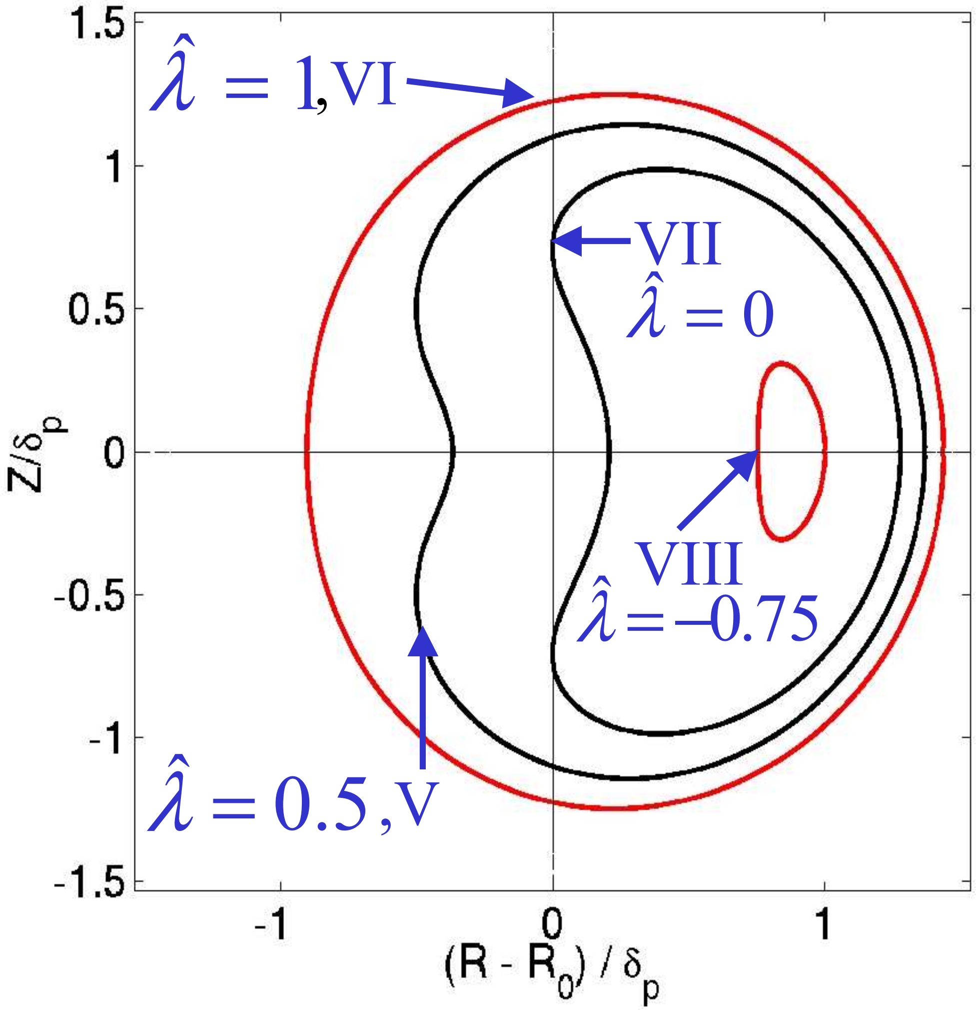




Orbits for $\hat{\psi}_\varphi = 0.5$





Orbit averaged Fokker-Planck Eq.

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- To study collisional slowing of fast ions and their interaction with different waves we need of a Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \dot{z}^i \frac{\partial f}{\partial z^i} = C(f) + Q(f) + S \qquad \mathbf{z}=(\mathbf{r},\mathbf{v})$$

- Axisymmetry → Unperturbed motion integrable → action angle variables ($\mathbf{J}$, $\boldsymbol{\theta}$) exist* such that $H_0=H_0(\mathbf{J})$,

$$\dot{\theta}_i = \frac{\partial H_0}{\partial J^i} = \Omega_i(\mathbf{J})$$

$$J^1 = m\mu / Ze, \quad J^3 = P_\varphi = mRv_\parallel \frac{B_\varphi}{B} - Ze\psi_p$$

$$\Omega_1 = \langle \omega_c \rangle, \quad \Omega_2 = 2\pi / \tau_b, \quad \Omega_3 = \langle \dot{\varphi} \rangle$$

*A.N. Kaufman Phys. Fluids. 1972.

3D orbit averaged equation

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- We can change to another set of invariants **I=I(J)** →

$$\frac{\partial f}{\partial t} + \Omega_i(\mathbf{I})\frac{\partial f}{\partial \theta_i} = C(f) + Q(f) + S \qquad \text{6D}$$

- Since τ_b/τ_c (bounce time over collsional time) <<1 we can expand $f = f_0 + (\tau_b/\tau_c) f_1 + \ldots.$
- Lowest order: $\Omega_i \partial f_0 / \partial \theta_i = 0$ → $f_0 = f_0(\mathbf{I})$ →

$$\frac{\partial f_0}{\partial t} = \langle C(f_0)\rangle + \langle Q(f_0)\rangle + \langle S\rangle \qquad \text{3D}$$

Axisymm. + small ρ

$$\langle\cdots\rangle = (2\pi)^{-3}\iiint(\cdots)\, d\theta_1 d\theta_2 d\theta_3 = \frac{1}{\tau_b}\int(\cdots)\, d\tau_b$$

Methods of solution

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- Direct Finite Difference/FEM[1]
- Statistical approach, i.e. Monte Carlo method.
  - Direct solution of the orbit averaged equation[2].
  - Orbit following Monte Carlo

$$\frac{\partial f_0}{\partial t} = \sum_{\substack{\text{arc-length} \\ i \text{ along orbit}}} \langle \Delta C_i(f_0) \rangle + \ldots$$

Monte Carlo operator for each arc-length of orbit + accelerated collisions, i.e one calculated orbit corresponds to many poloidal orbits in reality.

[1]F.S.Zaitsev et al., Phys. Fluids 1993.

[2]Eriksson and Helander PoP 1994; FIDO code: J. Carlsson et al., Proc. Varenna Theory of Fusion Plasmas (1995).

[4]TRANSP code, Goldston et al., J. Comp. Phys. (1981).

Slowing down of fast ions

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- The most important collisional process for fast ions is slowing down on the electrons.

In the variables r, θ, v, $\xi = (v_{\|}/v)$ we have

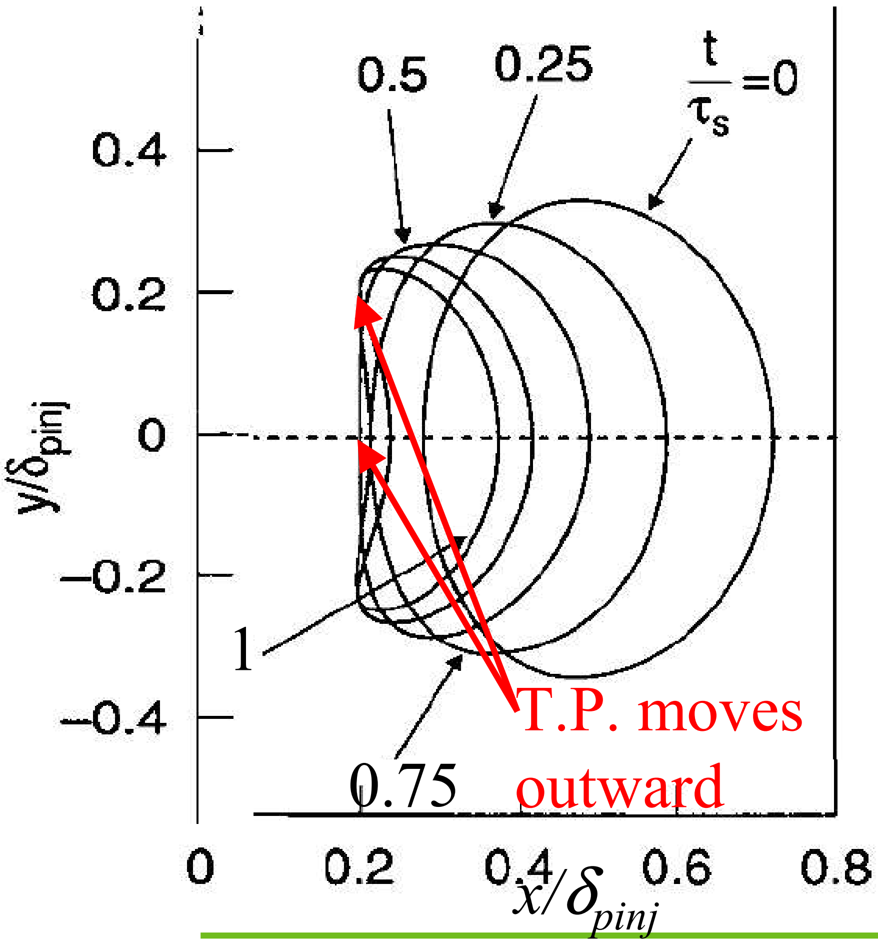
$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^3}{t_s} f\right] + S$$

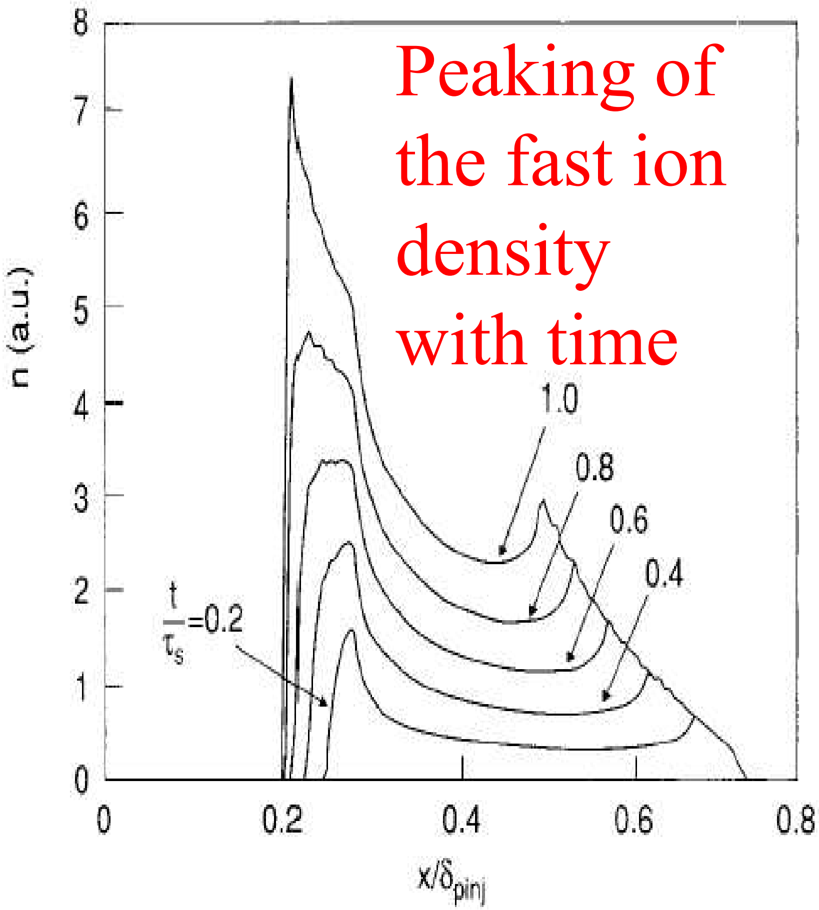
Transformation to v, Λ, P_φ, θ and orbit averaging ➡

$$\frac{\partial f_0}{\partial t} = \frac{v}{t_s}\frac{\partial f_0}{\partial v} + \frac{1}{t_s}\left\langle P_\varphi - Ze\psi \right\rangle \frac{\partial f_0}{\partial P_\varphi} + \left\langle S \right\rangle$$

Example

- We consider an artificial source of mono-energetic particles ionised at the same point in space (reg. VIII).



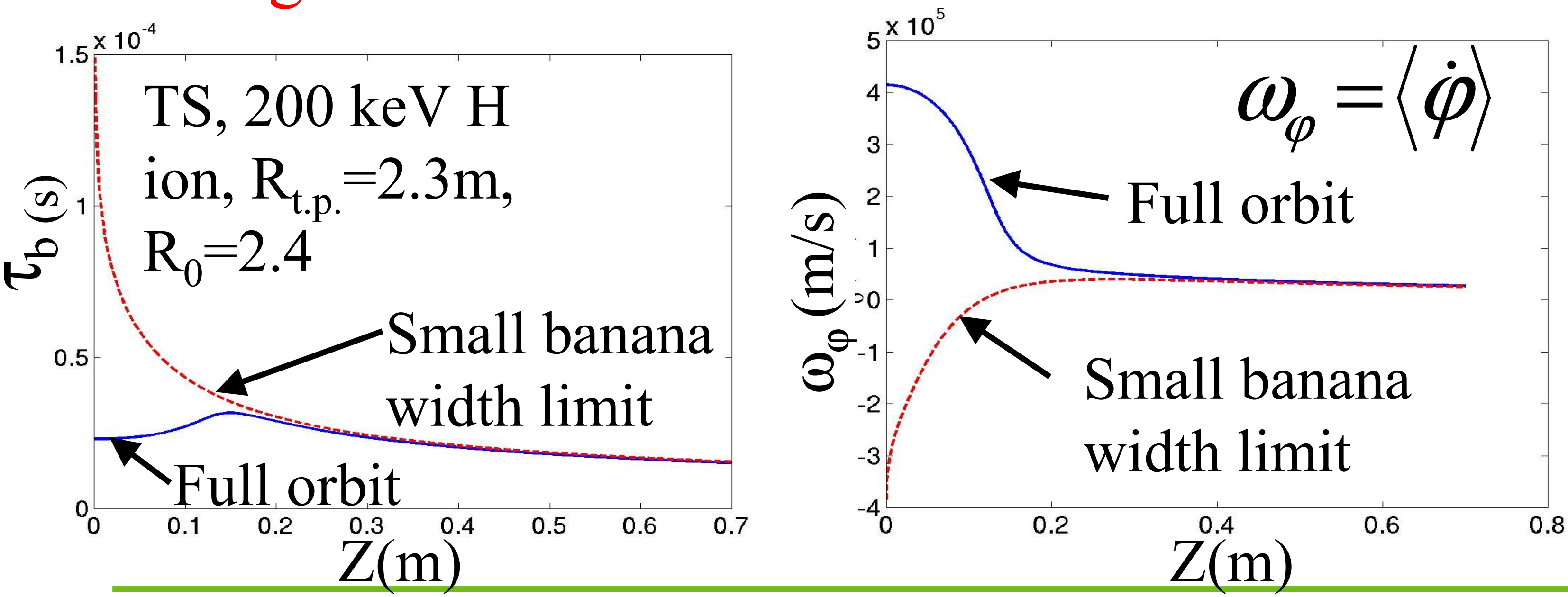


Bounce time/precession freq.

- Particles have resonant interactions with wave-perturbations (e.g. MHD) when,

$$\omega - n\langle\omega_c\rangle - l\frac{2\pi}{\tau_b} - N\omega_\phi = 0$$

- Orbits in the potato regime leads to significant changes of the relevant frequencies.



Fast ions and ripple

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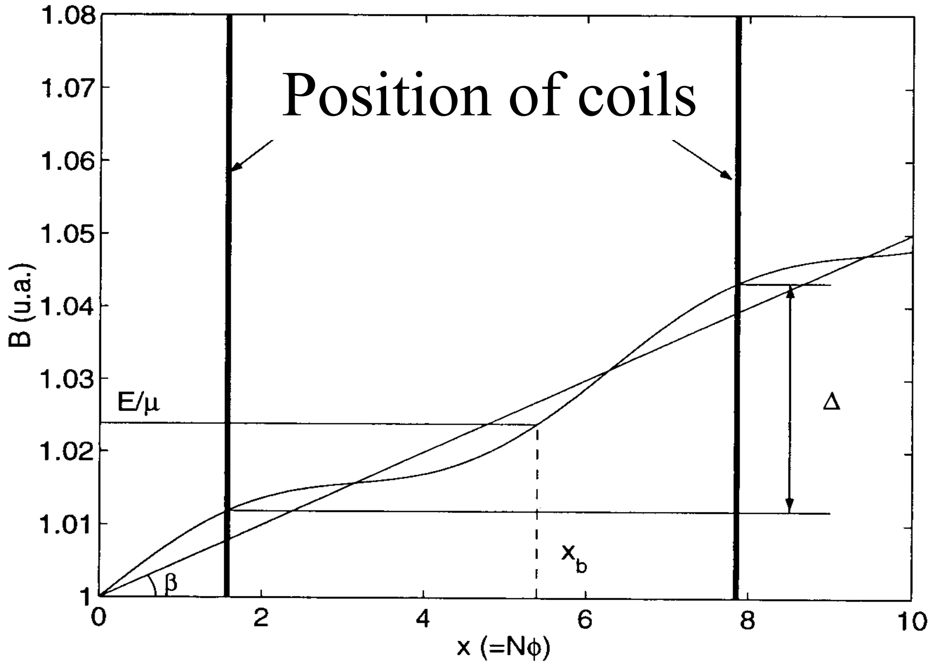
- The ripple in the magnetic field caused by the finite number of toroidal field coils leads to:
  - Trapping of ions in local magnetic wells.
  - Stochastic diffusion.



$$B=\frac{B_0R_0}{R}\left[1+\delta\cos(N\varphi)\right]\approx$$

$$B_0(1-\varepsilon\cos\theta)\left[1+\delta\cos(N\varphi)\right]$$

$$\delta=(B_{max}-B_{min})/(B_{max}+B_{min})$$

Ripple wells

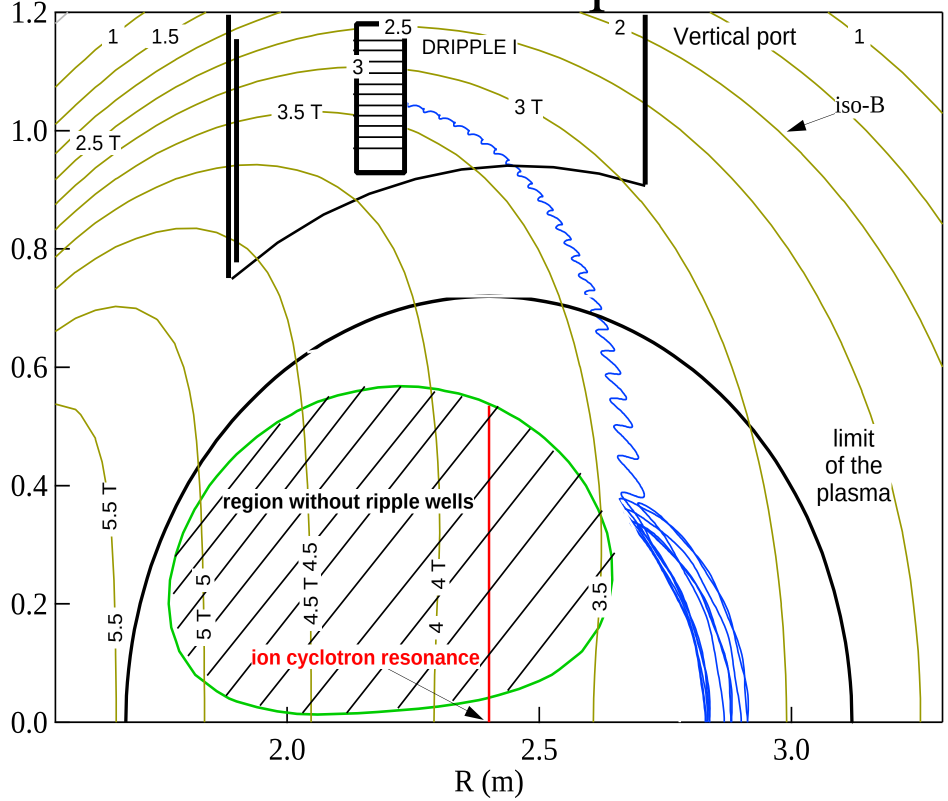
- Ripple wells, characterised by non-monotonic B along a field line, exist when,

$$\left.\frac{dB}{d\varphi}\right|_{\text{along field line}} =$$

$$\frac{\partial B}{\partial \varphi} + \frac{B_\theta}{\varepsilon B}\frac{\partial B}{\partial \theta} = 0$$

$$\upsilon = \frac{BN\delta}{B_\theta|\sin\theta|} = \frac{BN\delta}{B_R} > 1$$

Tore Supra



Goldston and Towner, J. Plasma Phys. 1981.

Ripple well trapping

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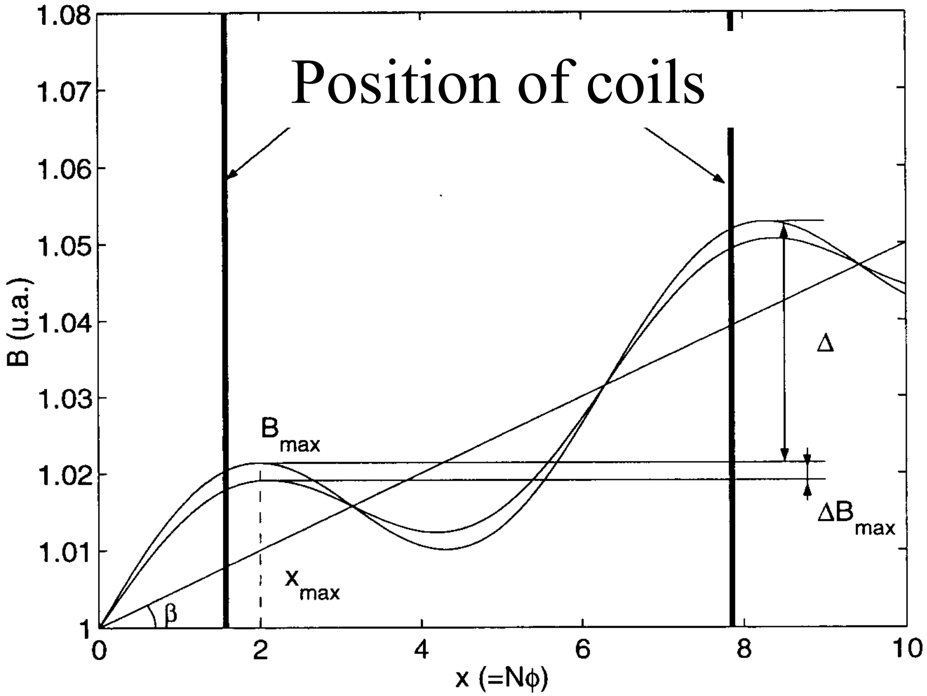
- There are two mechanisms for trapping ions in local magnetic ripple wells:
  - Collsional
  - Non-collisional (more important for really fast ions)



Non-Collisional mechanism:

When $d\upsilon/ds > 0$ (s is along v_D), after a bounce a trapped ion finds a deeper well on its return and can be locally trapped.

Loss via pitch-angle scattering

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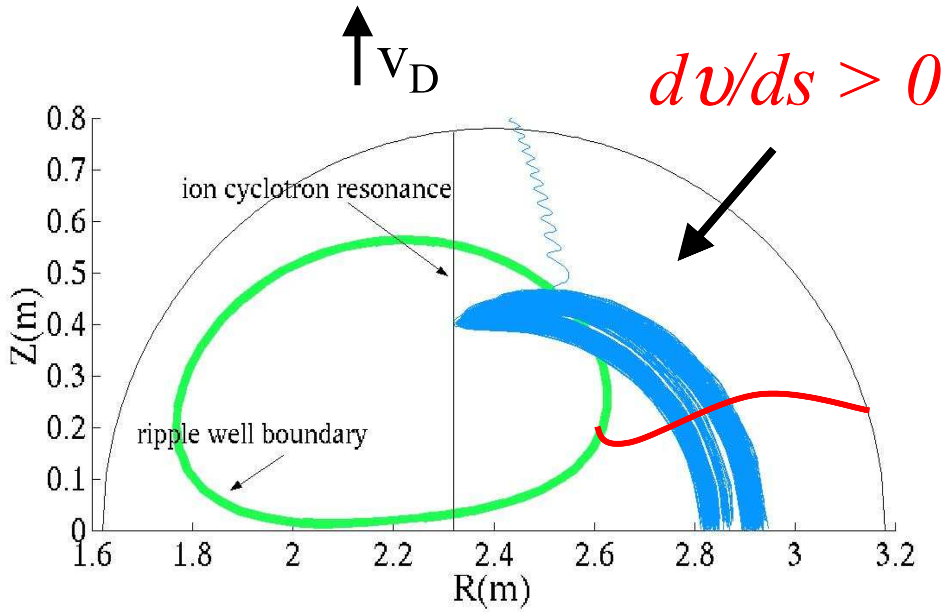
- Example of a medium fast ion initially confined.
- Such ions are subject to fairly strong pitch angle scattering.
- It leads to collisional scattering of trapped particle turning points.



Orbit following Monte Carlo including collisions

- In the end turning point is scattered into the region $\upsilon > 1$, $d\upsilon/ds > 0$; it is then quickly ripple well trapped and leaves the plasma.

Stochastic diffusion

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- In regions without ripple wells, the motion of a particle is still perturbed.
- Guiding centre Lagrangian*

$$\overline{L} = \frac{m\mathrm{v}_{\parallel}^{2}}{2} + Ze\mathbf{A}\cdot\mathbf{v}_{g} - \mu B \quad \Rightarrow \quad \dot{P}_{\varphi} = \frac{\partial \overline{L}}{\partial \varphi} \approx -\mu \frac{\partial B}{\partial \varphi}$$

$$\Rightarrow \quad \Delta P_{\varphi} = \mu B_{0} \int \delta(R,z)\sin(N\varphi)dt$$

- Stationary (i.e turning) points, $\dot{\varphi} = 0$, give contribution

$$\Delta P_{\varphi} \approx \mu \delta B_{0} \sqrt{2\pi N / |\ddot{\varphi}_{b}|} \sin(N\varphi_{b} + 4\ddot{\varphi}_{b} / \pi|\ddot{\varphi}_{b}|)$$

*J.B. Taylor, Physics of Fluids 1964.

Chirikov criterion

- We can view the problem as a mapping of consecutive bounces

$$(P_\varphi)_{n+1} = (P_\varphi)_n + \Delta P_\varphi \cos(N\varphi_{b,n} + \alpha)$$

$$(\Delta\varphi_b)_{n+1} = (\Delta\varphi_b)_n + \Delta P_\varphi \frac{\partial(\Delta\varphi_b)}{\partial P_\varphi}$$

- The motion becomes stochastic when the following Chirikov criterion is fulfilled:

$$\gamma = N\Delta P_\varphi \frac{\partial \Delta\varphi_b}{\partial P_\varphi} >> 1$$

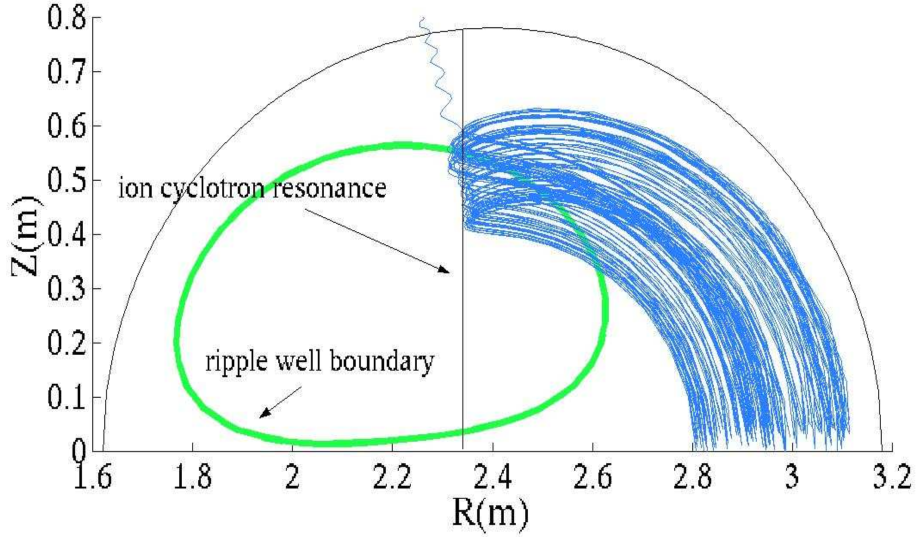
GWB criterion

- For $\theta = \pi/2$ one obtains the GWB criterion*

$$\delta_{crit} = \left(\frac{\varepsilon}{\pi N q}\right)^{3/2} \frac{1}{\rho\, dq/dr}$$

- The inverse dependence the Larmor radius means that only fast ions are affected.

Example of a fast ion in TS which first is subjected to stochastic diffusion and then is ripple well trapped.



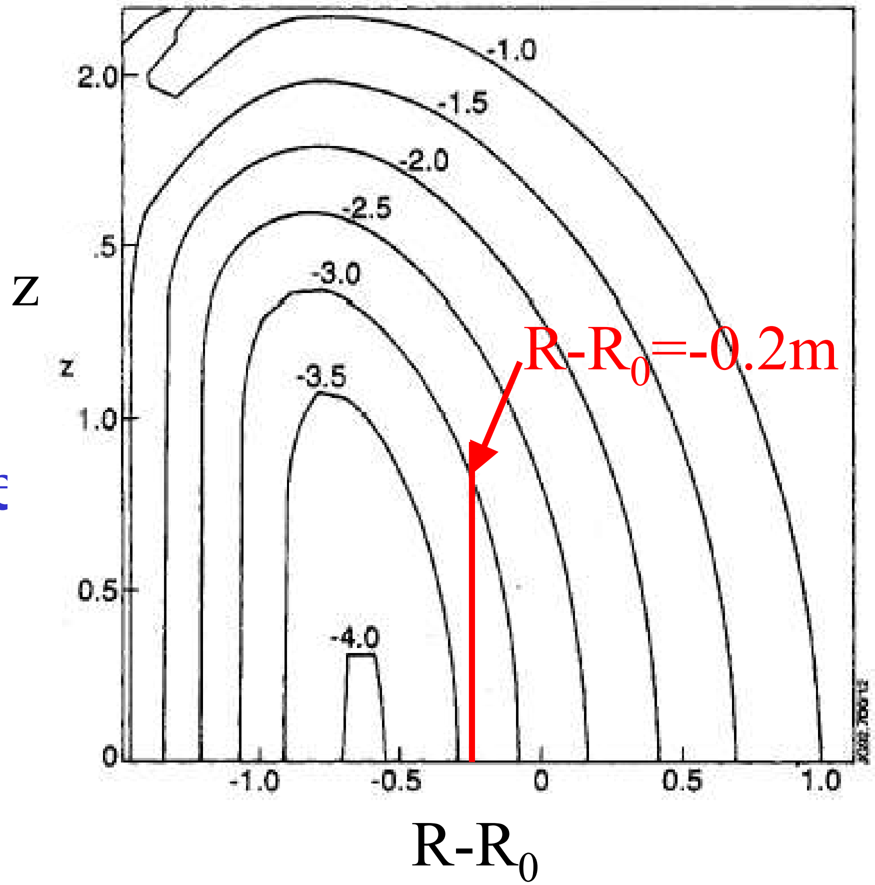
*Goldston,White and Boozer, Phys Rev. Lett. **47**, (1981)

Influence of orbits in potato regime

Association Euratom-Cea

- A number of JET experiments were carried out with enhanced ripple.
- Fast ions in the MeV range were created by ICRF heating
- An analysis of the threshold stochastic ripple shows it to be significantly influenced by orbits in the potato regime.

1 MeV protons with turning points along the red line is studied



Influence of potato orbits cont.

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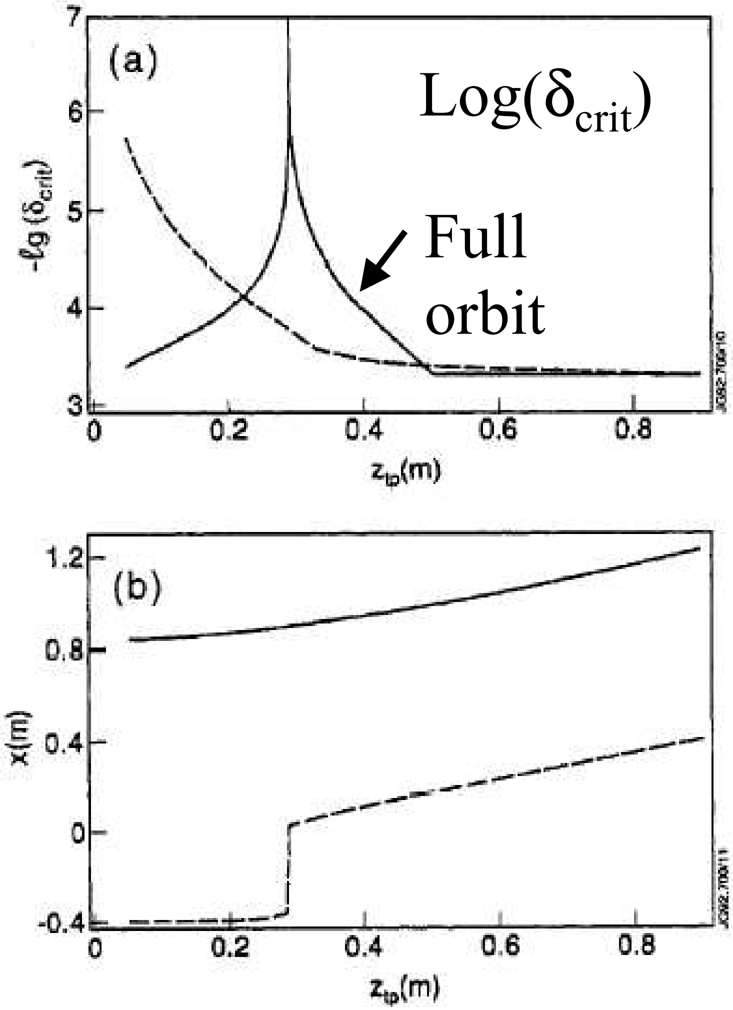
- The result from small banana width theory was compared to an actual integration of the real orbit.
- Significant differences for z up to 0.5 m found*.
- Effect due to difference and change in orbit topology



*L-G. Eriksson and P. Helander Nucl. Fus. 1993.

Orbits before and after transition

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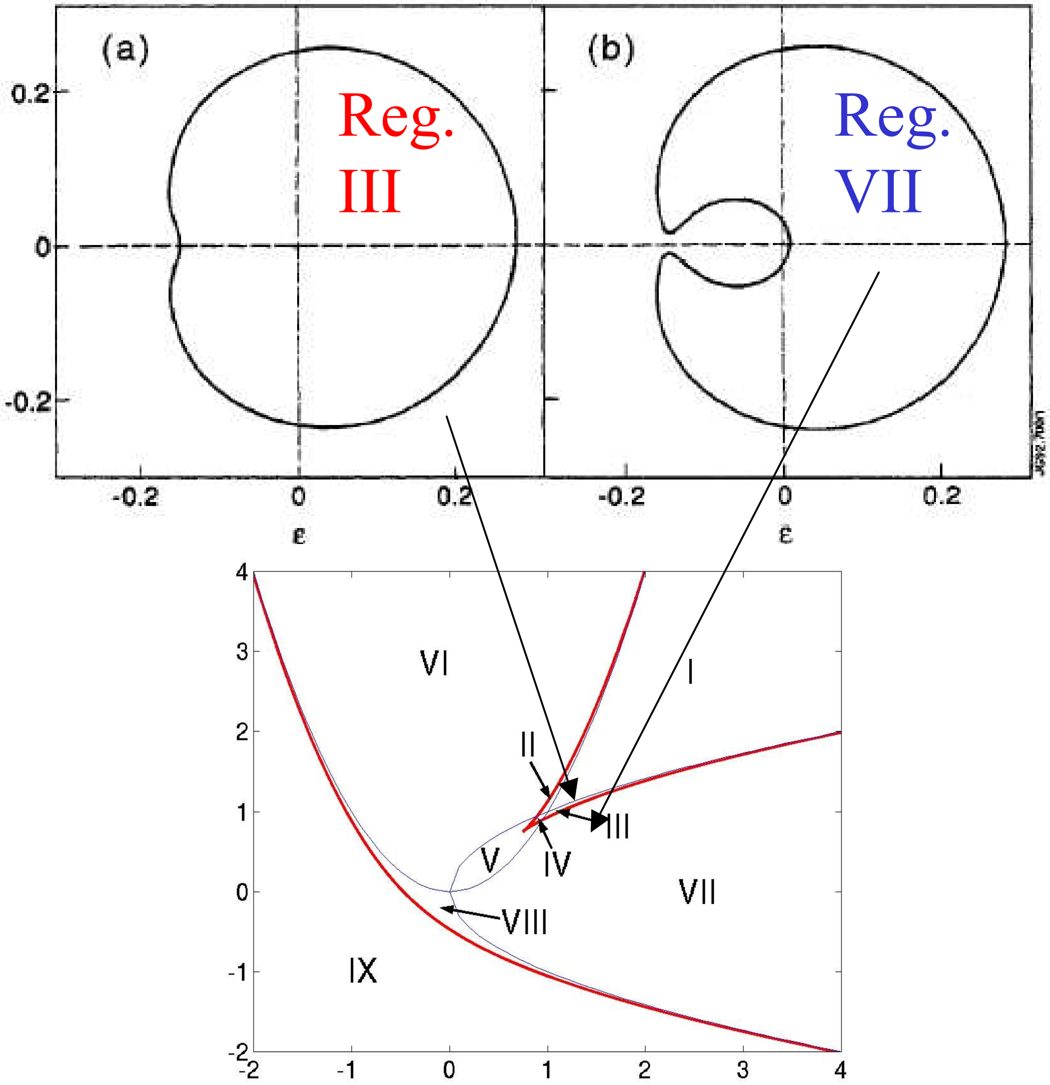
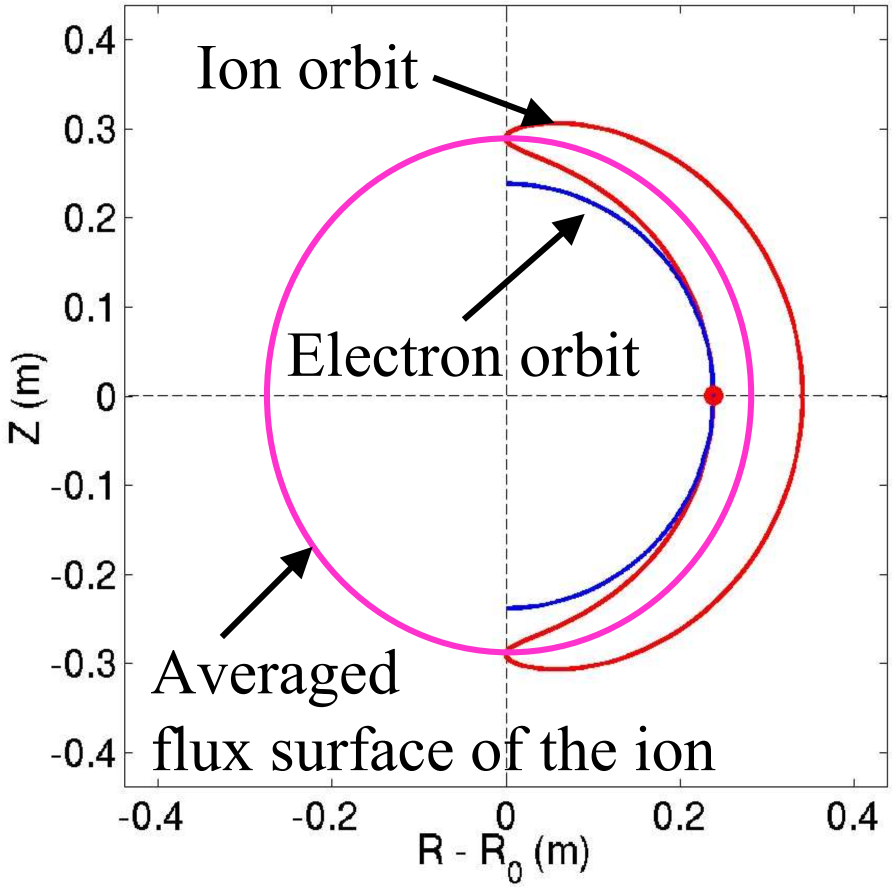


- The transition corresponds to crossing the pinch orbit.
- In spite of region III looking small, all the trapped ions up to z=0.3 are in that region for $R\text{-}R_0$=-0.2m

Fast ions and rotation

Association Euratom-Cea

- Consider Neutral Beam Injection of ions into trapped orbits.
- The injected trapped ions carry little momentum on average.
- How do the injected ions transfer their initial momentum to the bulk plasma?



Suckwer et al. PRL, 1979; Hinton and Rosenbluth Phys. Lett A 1999

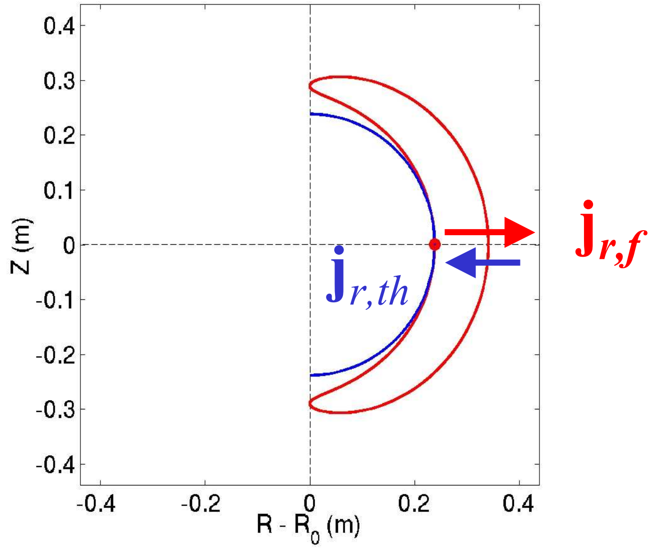
Radial currents

Association Euratom-Cea

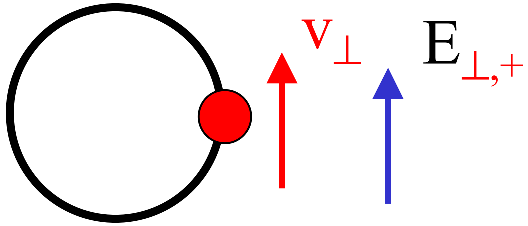
- The difference in average radial position of the fast ion and the electron leads to charge separation.
- Steady injection → radial fast ion current arises.
- The plasma prefers (like the Swedes) to be quasi-neutral, and will respond with an opposite current in the thermal plasma.
- This gives to a torque on the thermal plasma,

$$T_{\varphi} = (\mathbf{j}_{r,th} \times \mathbf{B}) \cdot \hat{\varphi} \approx -j_{r,f} B_{\theta}$$

- Thus, the injected ion transfers its initial toroidal momentum via $\mathbf{j}_{r,f}$ on the bounce time scale.

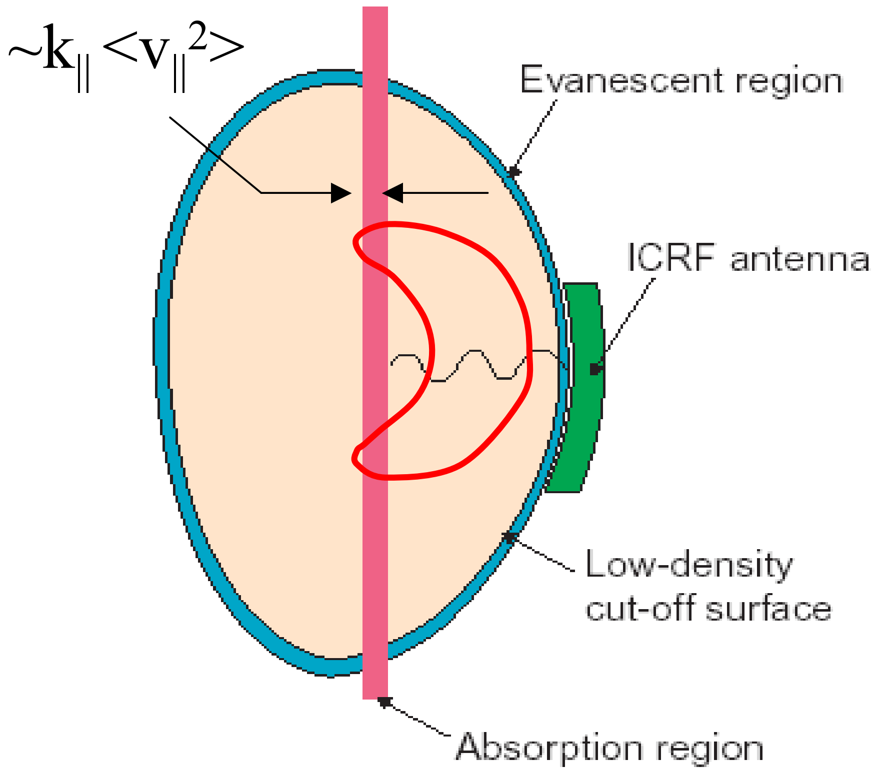


ICRF heating



Resonance condition:

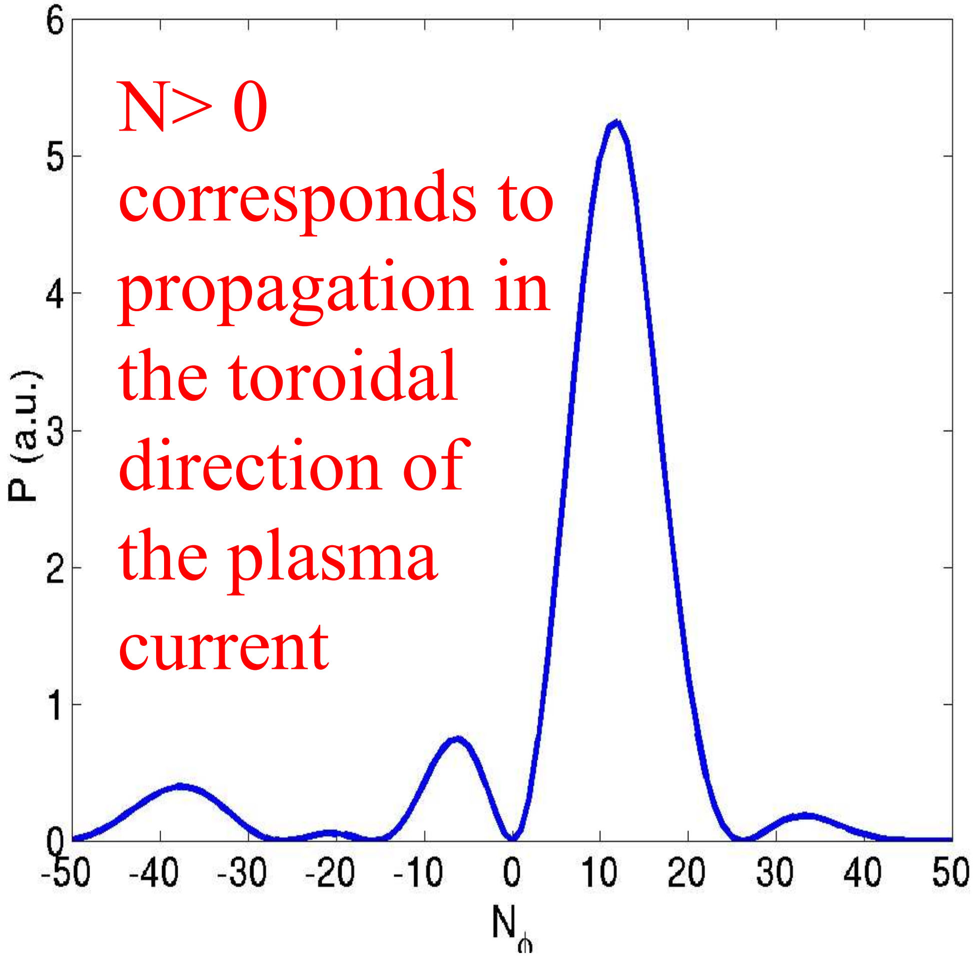
$$\omega - k_{\|} v_{\|} = n\omega_{ci}$$



- ICRF heating increases mostly $v_\perp$ of resonating ions.
- The number of resonating ions is normally small → they become very energetic during high power heating.
- Resonating ions tend to be trapped with turning points close to $\omega = \omega_{ci}$.

JET four strap antennas with +90^o phasing

Association Euratom-Cea



$$\langle N \rangle = \frac{\sum_N N P(N)}{\sum_N P(N)}$$

$$\Rightarrow \quad \boxed{\langle N \rangle \approx 8}$$

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- From quantum mechanical equations we obtain for a particle absorbing a wave quantum: $\Delta E = \hbar\omega$

$$\Delta P_{\varphi} = R\hbar k_{\varphi} = \hbar N \quad \Rightarrow \quad \Delta P_{\varphi} = \frac{N}{\omega}\Delta E$$

- Thus the toroidal momentum imparted to the plasma per unit time is given by:

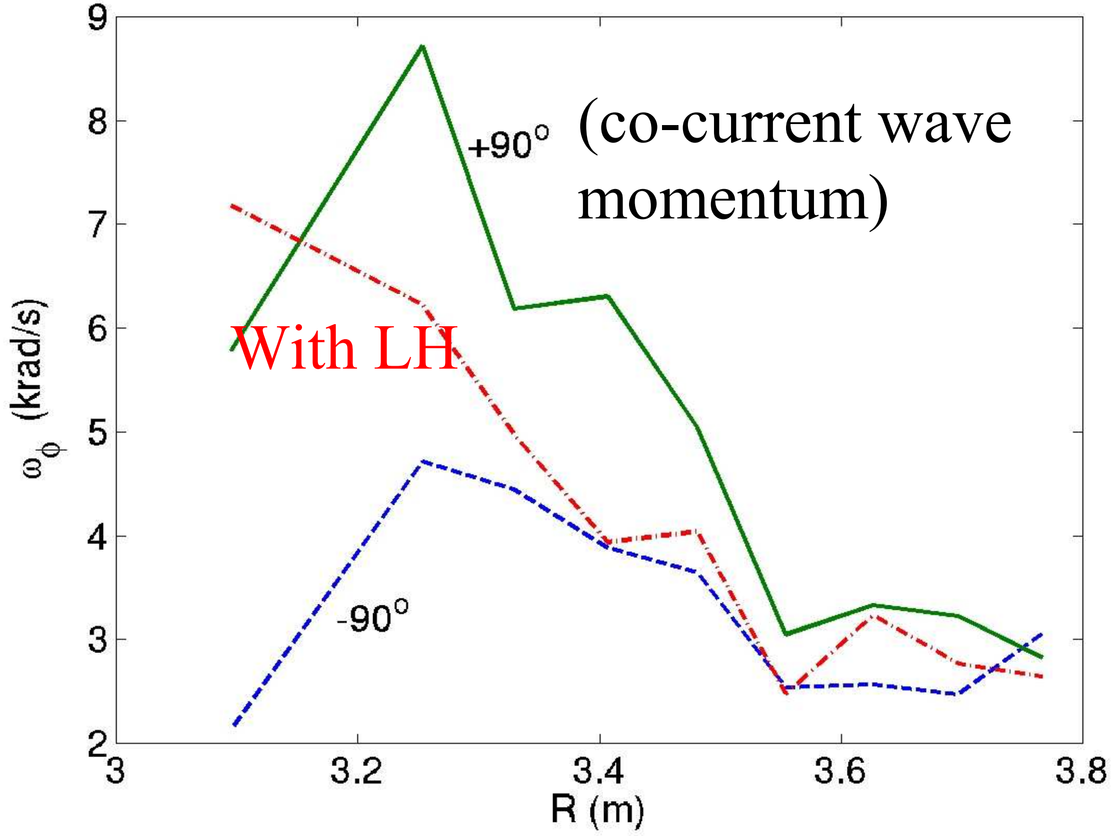
$$\frac{dP_{\varphi}}{dt} = \frac{\langle N \rangle}{\omega} P_{ICRF}$$

- (The same result is of course also obtained from a combination Maxwell's equations and the equation of motion)

Carbon rotation profiles in JET experiment

Association Euratom-Cea

6MW ICRF power, ^{3}He minority, $\omega = \omega_{ci}$, 15 cm HFS



L-G. Eriksson et al. PRL, 2004

- The +90° discharge rotates more strongly in the centre than –90°, consistent with absorption of wave momentum!
- LH discharge shows the difference is not due to modified heating efficiency

^3He accelerated from 100 to 850 keV

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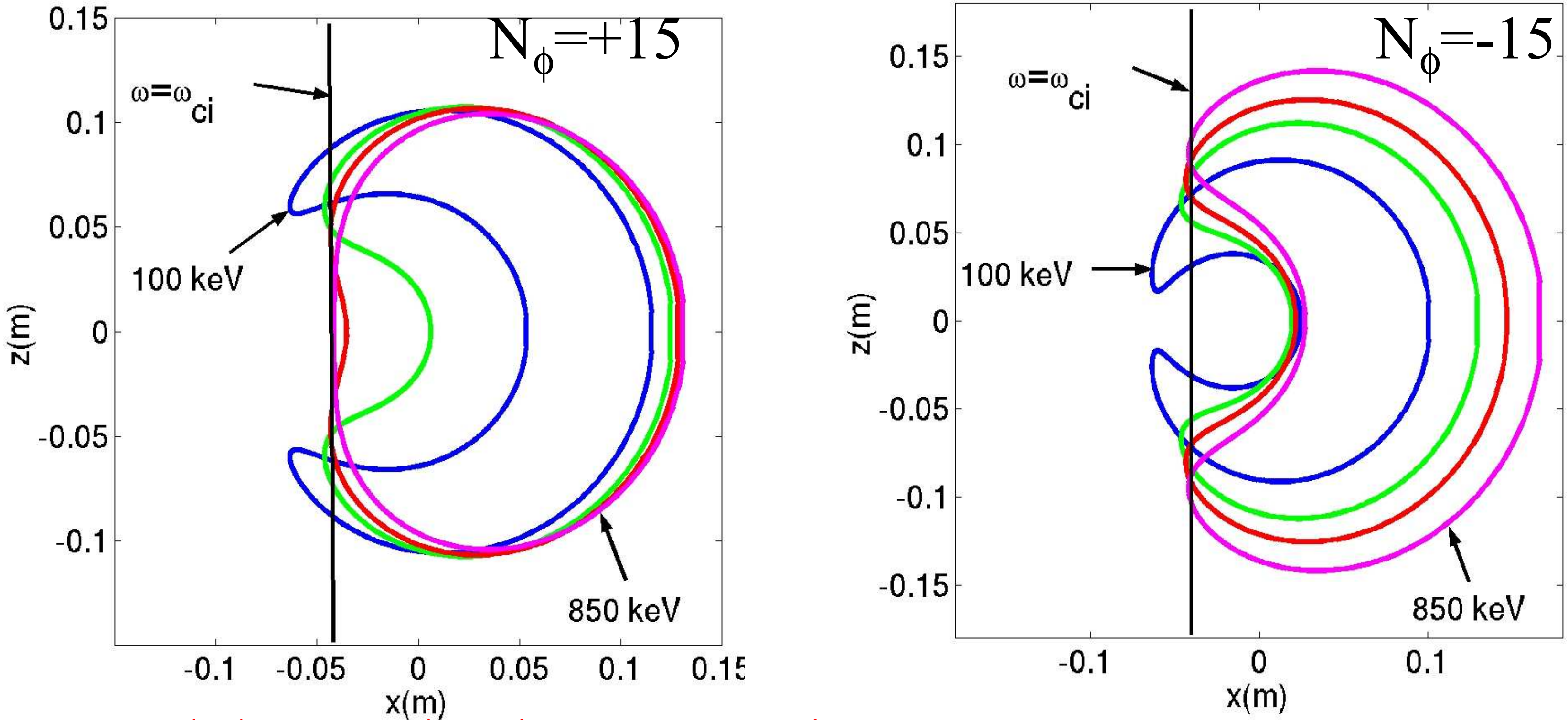

Eventual de-trapping into co-passing orbit in the potato regime. Co-current torque from inward fast ion current and collisions with fast passing ions

Outward fast ion current ⇒ counter current torque

3D simulation

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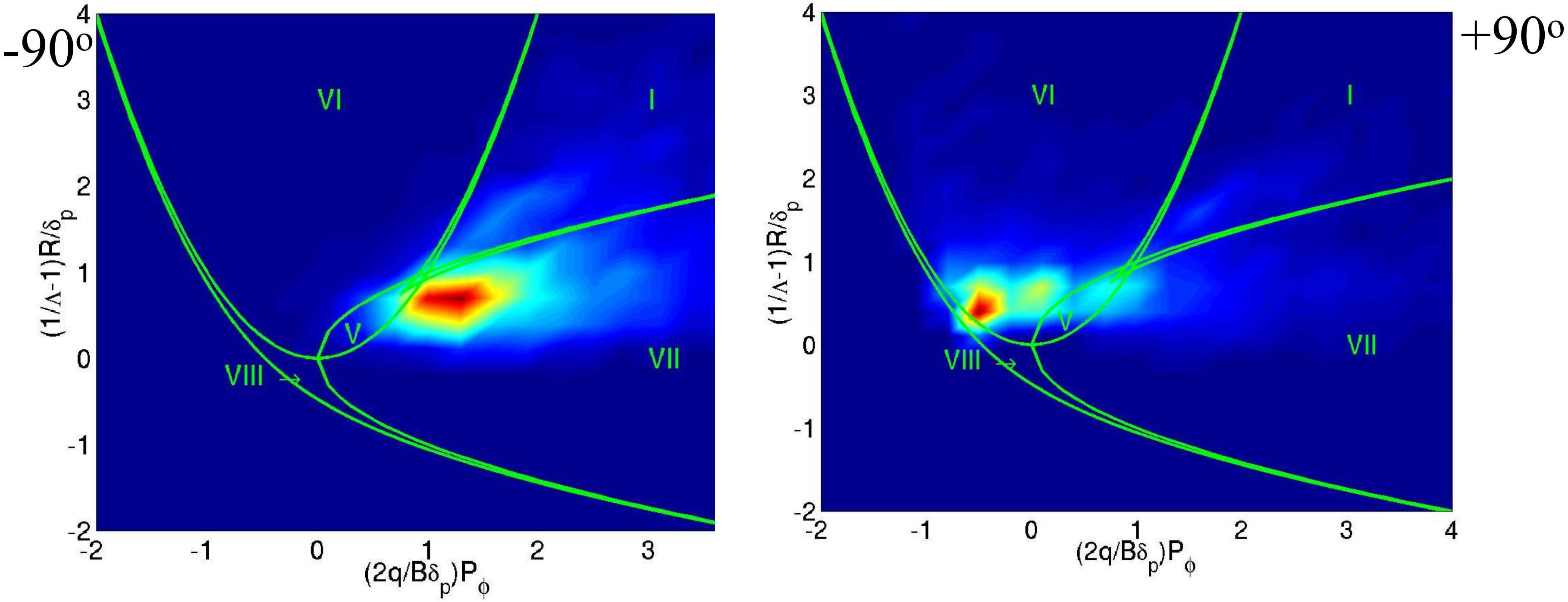
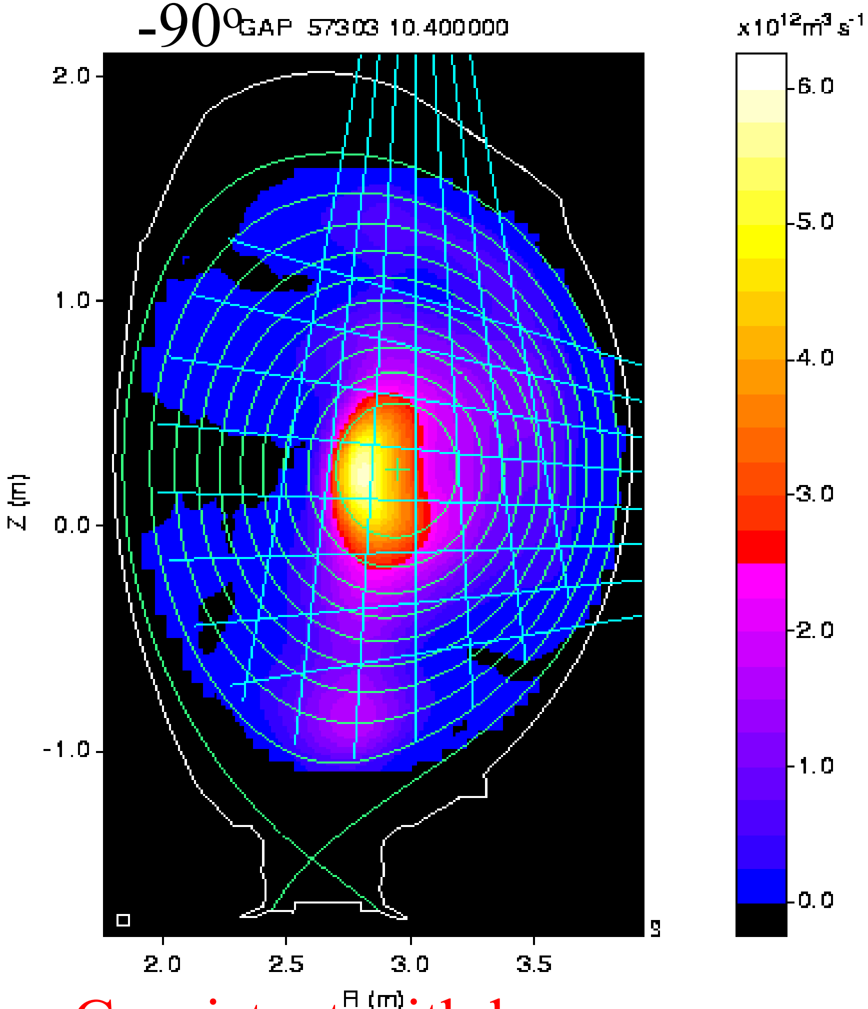
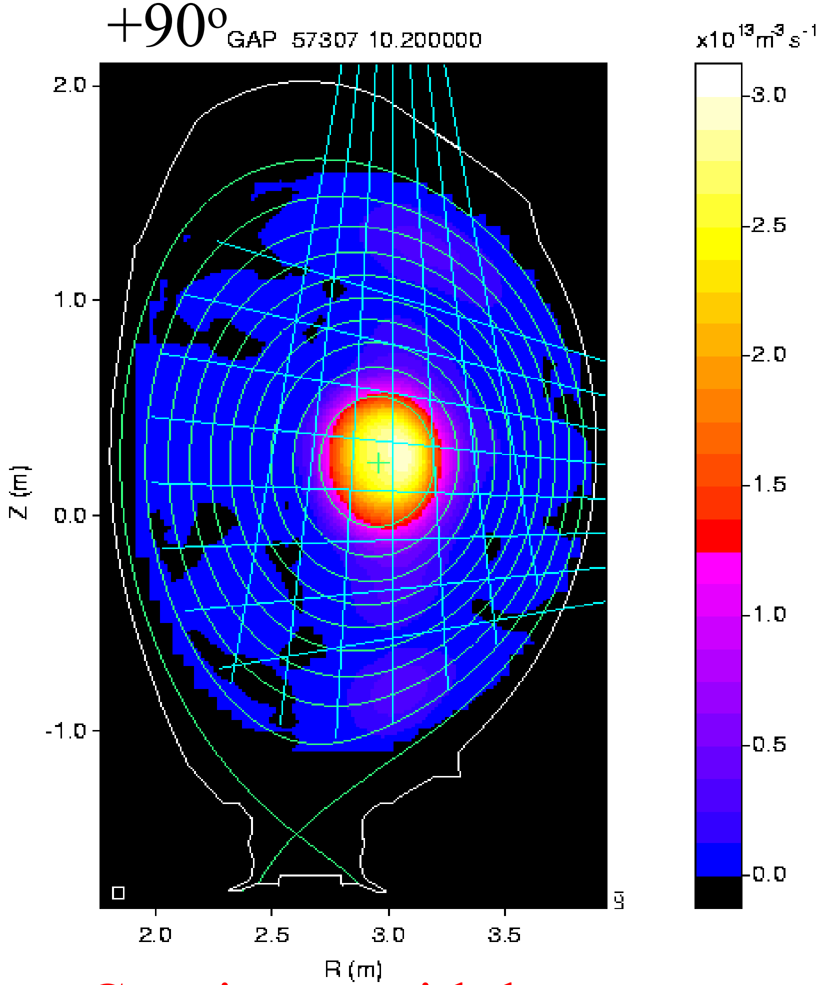
Simulation with SELFO (Monte Carlo code FIDO + full wave code LION) solving orbit averaged Fokker Planck equation self consistently with the wave field; MC particles in an orbit classification diagram; E > 500 keV



Gamma-rays, interaction fast ^{3}He and C/Be impurities

(V. Kiptily)





Consistent with large trapped ion fraction

Consistent with large fraction of co-passing ions

Sawtooth redistribution

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- Fast ions are redistributed during sawooth crashes (periodic reconnections, N=1, events characterised by a sudden drop of the central plasma temperature).
- Fast ions that deviate little from a field line are strongly redistributed during crash phase.
- Trapped fast ions do not stick to a field line and have significant precession → little redistribution.
- A group of particles with a small ratio of $v_{||}/v_{\perp}$ resonant with the mode causing the sawooth exist, and they are are strongly redistributed. Region VIII particles.

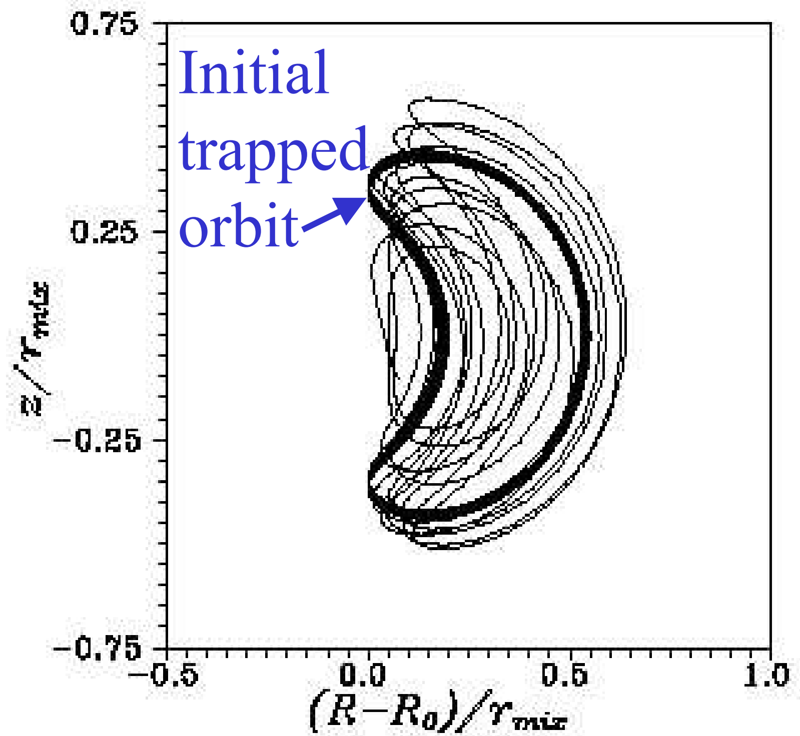
Kolesnichenko, et al., Nuclear Fusion 2000.

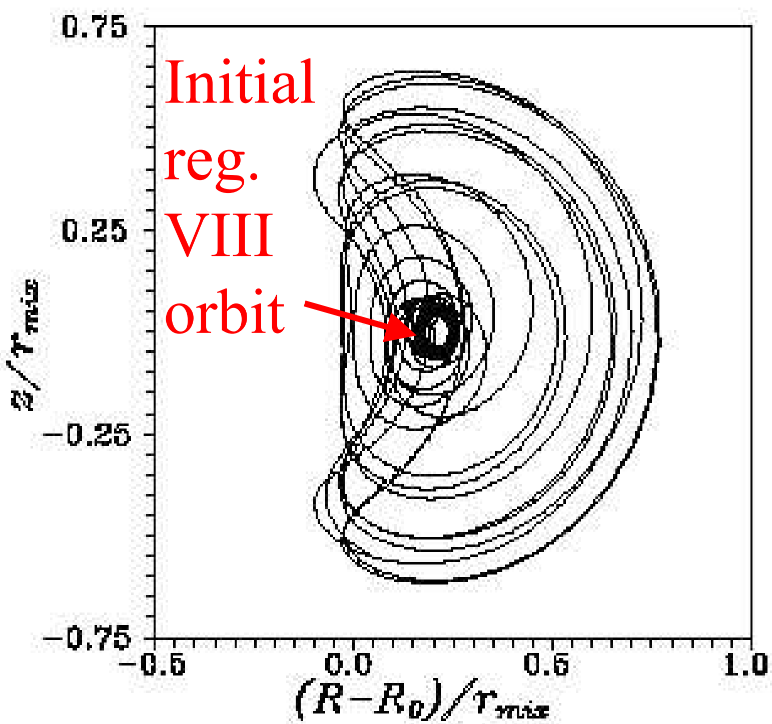
Simulated orbit evolution

Association Euratom-Cea

Resonance with perturbation:

$\omega_\varphi \approx \omega_b$





Kolesnichenko, et al., Nuclear Fusion 2000.

Orbits in current hole plasmas

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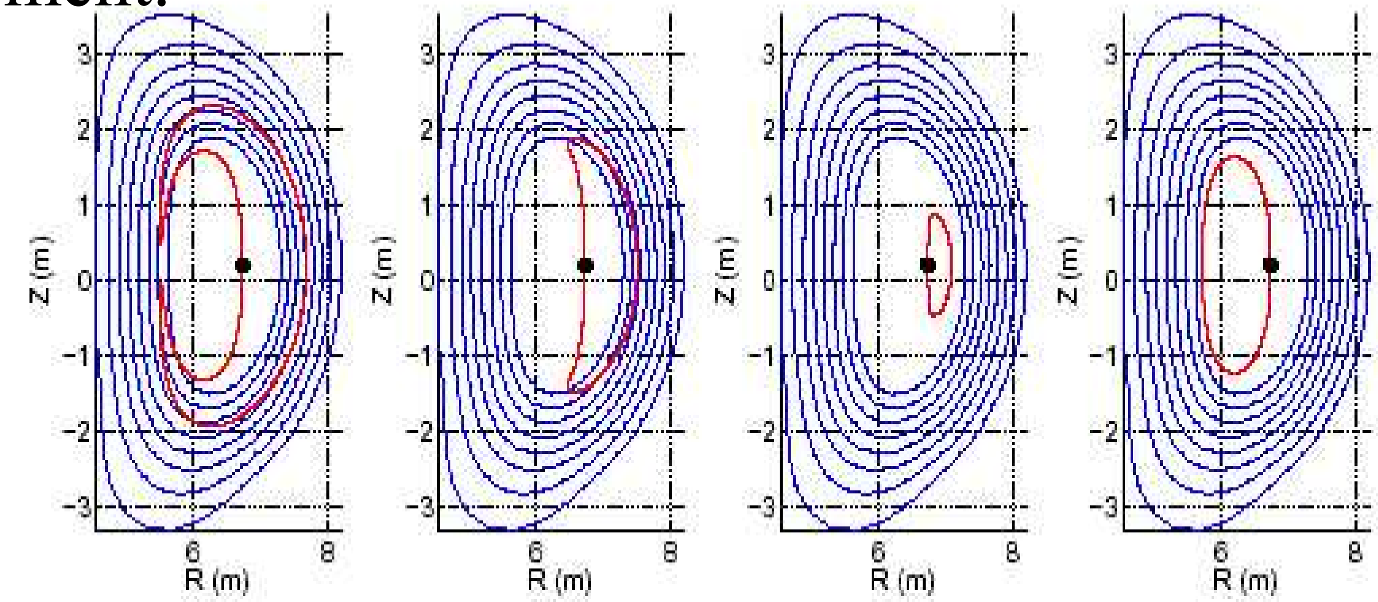
- Current holes, a central region with virtually no current, appear in some ITB plasmas.
- This has significant consequences for the fast ion confinement.

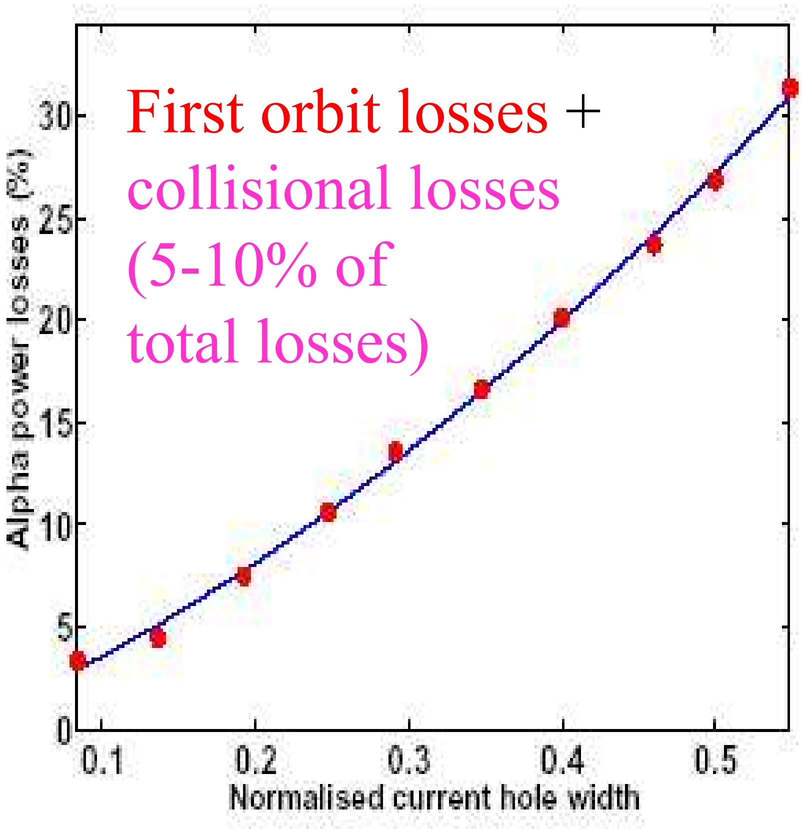


Barley trapped | Trapped | Co-passing | Counter-passing

Tobita PPCF 2005; Schneider ICCP, Nice 2005

Enhanced losses

- The alpha particle confinement is degraded in the presence of a current hole
  - Significant effect for JET
  - Small effect in ITER (<0.5% losses)



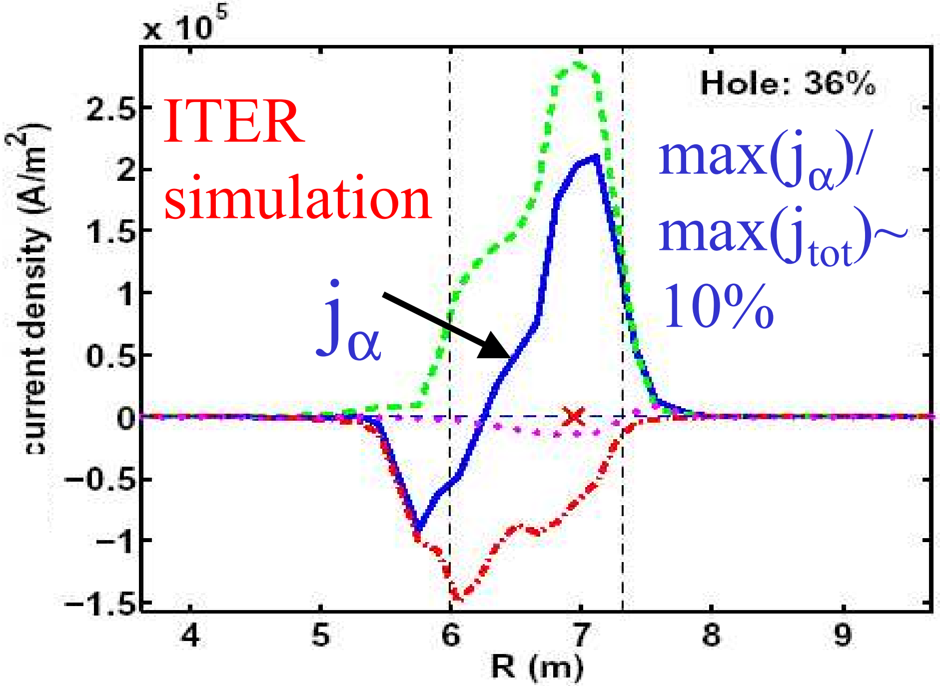
Schneider ICCP, Nice 2005; Orbit following Monte Carlo code

Asymmetries/fast ion current

- Conservation of P_φ → a particle born with a co-current velocity in the current hole is never trapped.

$$P_\varphi = mR\mathrm{v}_\varphi - Ze\psi/2\pi ,$$

$\mathrm{v}_\varphi > 0$: when ψ becomes finite v_φ increases



Schneider 2005, submitted to PPCF

- A non-negligible alpha particle current arises due to the asymmetries.
- The dynamics of the current hole could be affected.
- Especially if G. Huysmans model, PRL 2001, is relevant.

Conclusions

- Fast ion orbits play an important role in many of today's tokmak discharges.
- It is in general not possible to use small banana width limit theory for analysing fast ions effects when the ions have energies in excess of a few hundred keV in JET, Tore Supra …
- The situation should be much less complicated for ITER.
- However, there is hope for those of us who are attached to non-standard orbits: the possibility of current holes in ITER

Orbit integration in toroidal geometry

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- In many cases it is necessary to calculate accurate orbits in general, not necessarily axi-symmetric, geometries.
- Orbit following codes using an efficient method which conserves the invariants very well is needed.
- Straight forward integration of the guiding centre velocity,

$$\mathbf{v} = \mathrm{v}_{\|}\hat{b} + \frac{\mathrm{v}_{\perp}^2}{2\omega_{\mathrm{c}}}\hat{b}\times\nabla\ln B + \frac{\mathrm{v}_{\|}^2}{\omega_{\mathrm{c}}}\hat{b}\times\boldsymbol{\kappa}, \quad \boldsymbol{\kappa} \approx \frac{\nabla_{\perp}B}{B}$$

is not optimal in this respect.

Boozer coordinates*

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- How much simpler life would be if the magnetic field lines were straight (sorry I am exaggerating).
- Well they can be!
- In Boozer coordinates the magnetic field is given by,

$$\mathbf{B} = \nabla\varphi \times \nabla\psi_p + q(\psi_p)\nabla\psi_p \times \nabla\theta$$

$$\mathbf{B} = \overline{B}_{\psi_p}(\psi_p, \theta, \varphi)\nabla\psi_p + \overline{B}_\theta(\psi_p)\nabla\theta + \overline{B}_\varphi(\psi_p)\nabla\varphi$$

- These are contra and co-variant representations of the magnetic field, respectively ($2\pi\psi_p = \psi$).
- Note $\overline{B}_\theta$ is generally different from $B_\theta = \mathbf{B}\cdot\nabla\theta/|\nabla\theta|$

*A.H. Boozer Phys. Fluid. 1981.

Orbit integration/Boozer θs

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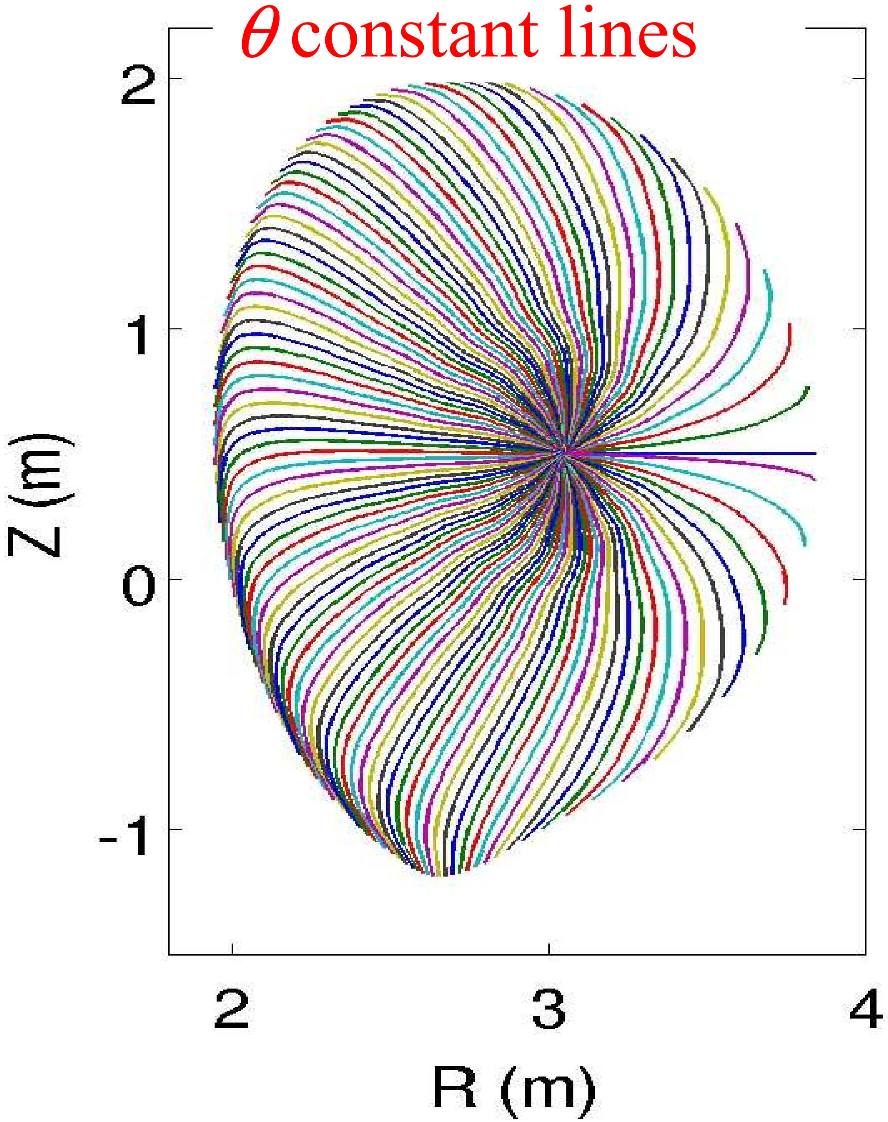

- A direction along field line

$$\frac{d\varphi}{d\theta} = \frac{\mathbf{B}\cdot\nabla\varphi}{\mathbf{B}\cdot\nabla\theta} = q(\psi_p)$$

hence "straight field line".

Equilibrium codes can supply:

$$\theta = \theta(R,Z) \quad , \quad I(\psi_p) = \overline{B}_\theta$$

$$\psi_p = \psi_p(R,Z) \ , \ g(\psi_p) = \overline{B}_\varphi$$

The aim is to follow orbits in θ and ψ_p, and then map to R and Z

Principle

- The Hamiltonian is given by,

$$H = \frac{m \mathrm{v}_{\parallel}^{2}}{2} + \mu B + Ze\Phi$$

- Equations of motion,

$$\frac{d\theta}{dt} = \dot{\theta} = \frac{\partial H}{\partial p_\theta}, \qquad \frac{d\varphi}{dt} = \dot{\varphi} = \frac{\partial H}{\partial p_\varphi}$$

$$\dot{p}_\theta = \frac{dp_\theta}{dt} = -\frac{\partial H}{\partial \theta}, \qquad p_\varphi = \frac{dp_\varphi}{dt} = -\frac{\partial H}{\partial \varphi}$$

- Conjugate momenta p_θ and p_φ can be found, leading to,

Equations of motion $(\Phi = 0)$

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$$\rho_\parallel = \mathrm{v}_\parallel / \omega_c, \quad I(\psi_p) = \mu_0 I_p(\psi_p)/2\pi, \quad g(\psi_p) = RB_\varphi \text{(Axisymm.)}$$

$$D = gq + I + \rho_\parallel [gI' - g'I]$$

$$\frac{d\theta}{dt} = \frac{Ze}{m}\rho_\parallel B^2 \frac{1-\rho_\parallel g'}{D} + \frac{\mu + (Ze)^2 \rho_\parallel^2 B^2/m}{ZeD} g \frac{\partial B}{\partial \psi_p}$$

$$\frac{d\varphi}{dt} = \frac{Ze}{m}\rho_\parallel B^2 \frac{q+\rho_\parallel I'}{D} - \frac{\mu + (Ze)^2 \rho_\parallel^2 B^2/m}{ZeD} I \frac{\partial B}{\partial \psi_p}$$

$$\frac{d\psi_p}{dt} = \frac{1}{ZeD}\left[I \frac{\partial B}{\partial \theta} - g \frac{\partial B}{\partial \varphi} \right] \left[\mu + (Ze)^2 \rho_\parallel^2 B^2/m \right]$$

$$\frac{d\rho_\parallel}{dt} = \frac{-1}{ZeD}\left[(q+\rho_\parallel I')\frac{\partial B}{\partial \varphi} + (1-\rho_\parallel g')\frac{\partial B}{\partial \theta} \right] \left[\mu + (Ze)^2 \rho_\parallel^2 B^2/m \right]$$

White and Chance Phys. Fluid. 1984, White and Boozer Phys. Fluid. 1995

$$\mathbf{B} = \nabla\varphi \times \nabla\psi_p + q(\psi_p)\nabla\psi_p \times \nabla\theta$$

$$\mathbf{B} = \overline{B}_{\psi_p}(\psi_p, \theta, \varphi)\nabla\psi_p + \overline{B}_\theta(\psi_p)\nabla\theta + \overline{B}_\varphi(\psi_p)\nabla\varphi$$

- A direction along field line is given by,

$$\frac{d\varphi}{d\theta} = \frac{\mathbf{B}\cdot\nabla\varphi}{\mathbf{B}\cdot\nabla\theta} = \frac{q(\psi_p)\nabla\theta \times \nabla\varphi}{\nabla\theta \times \nabla\varphi} = q(\psi_p)$$

i.e. it is constant on a flux surface, hence “straight field line”.

- One can also show that,

$$\overline{B}_\theta = \mu_0 \frac{I_\varphi(\psi_p)}{2\pi} \quad , \quad q = \frac{d\psi_t}{d\psi_p}$$

$$\left(\begin{array}{c} \overline{B}_\varphi = RB_\varphi \\ \text{Axi-symm.} \end{array} \right)$$

Preliminaries to orbit following

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- In order to follow an orbit in Boozer coordinates we must we must evaluate

$$\frac{d\theta}{dt} = \dot{\theta} = \frac{\partial H}{\partial p_\theta}, \qquad \frac{d\varphi}{dt} = \dot{\varphi} = \frac{\partial H}{\partial p_\varphi}$$

$$\dot{p}_\theta = \frac{dp_\theta}{dt} = -\frac{\partial H}{\partial \theta}, \qquad p_\varphi = \frac{dp_\varphi}{dt} = -\frac{\partial H}{\partial \varphi}$$

- We have the Hamiltonian,

$$H = \frac{m \mathrm{v}_\parallel^2}{2} + \mu B + Ze\Phi$$

- Thus,we have to find the conjugate momenta p_θ and p_φ

Guiding centre Lagrangian

- The guiding centre Lagrangian can be written as,

$$\overline{L} = \frac{m \mathrm{v}_{\parallel}^{2}}{2} + Ze\,\mathbf{A} \cdot \mathbf{v}_{g} - \mu B - Ze\,\Phi$$

- The parallel velocity is given by,

$$\mathrm{v}_{\parallel} = \frac{\mathbf{v} \cdot \mathbf{B}}{B} = \frac{1}{B}\left(\overline{B}_{\psi_p}\dot{\psi}_{p} + \overline{B}_{\theta}\dot{\theta} + B_{\varphi}\dot{\varphi}\right)$$

- However, $B_{\psi_p}\dot{\psi}_p / B_{\varphi}\dot{\varphi} \sim \mathrm{v}_D / \mathrm{v}_{\parallel} << 1$
- Moreover,

$$\mathbf{A} = \psi_t \nabla\theta - \psi_p \nabla\varphi \;\Longrightarrow\; \mathbf{B} = \nabla\varphi \times \nabla\psi_p + \nabla\psi_t \times \nabla\theta$$

Guiding centre Lagrangian

- The guiding centre Lagrangian can be written as*,

$$\overline{L} = \frac{m \mathrm{v}_{\parallel}^2}{2} + Ze\,\mathbf{A}\cdot\mathbf{v}_g - \mu B - Ze\,\Phi$$

$$\overline{L} = \frac{m}{2B^2}\left(\overline{B}_\theta\dot{\theta} + \overline{B}_\varphi\dot{\varphi}\right)^2 + Ze\left(\psi_t\dot{\theta} - \psi_p\dot{\varphi}\right) - \mu B - Ze\Phi$$

- Where we have assumed $\dfrac{B_\psi \mathrm{v}_D}{B_\varphi \mathrm{v}_\varphi} << 1$

*J.B. Taylor, Physics of Fluids 1964.

Canonical conjugate momenta

$$\overline{L} = \frac{m}{2B^2}\left(I\dot{\theta} + g\dot{\varphi}\right)^2 + Ze\left(\psi_t\dot{\theta} - \psi_p\dot{\varphi}\right) - \mu B - Ze\Phi$$

Where we have introduced $I(\psi_p) = \overline{B}_\theta$ and $g(\psi_p) = \overline{B}_\varphi$

- The canonical conjugate momenta are now given by,

$$p_\theta = \frac{\partial \overline{L}}{\partial \dot{\theta}} = m v_\parallel \frac{I}{B} + Ze\,\psi_t$$

$$p_\varphi = \frac{\partial \overline{L}}{\partial \dot{\varphi}} = m v_\parallel \frac{g}{B} - Ze\,\psi_p = mR v_\parallel \frac{B_\varphi}{B} - Ze\,\psi_p$$

Axi-symm.

$$= const\,.$$

Equations of motion $(\Phi = 0)$

Association Euratom-Cea

- From $\partial H / \partial p_\theta, \partial H / \partial p_\varphi, \partial H / \partial \theta$ and $\partial H / \partial \varphi$ one obtains

$$\frac{d\theta}{dt} = \frac{Ze}{m}\rho_\parallel B^2 \frac{1-\rho_\parallel g'}{D} + \frac{\mu + \frac{(Ze)^2}{m}\rho_\parallel^2 B^2}{ZeD} g \frac{\partial B}{\partial \psi_p}$$

$$\frac{d\varphi}{dt} = \frac{Ze}{m}\rho_\parallel B^2 \frac{q+\rho_\parallel I'}{D} - \frac{\mu + \frac{(Ze)^2}{m}\rho_\parallel^2 B^2}{ZeD} I \frac{\partial B}{\partial \psi_p}$$

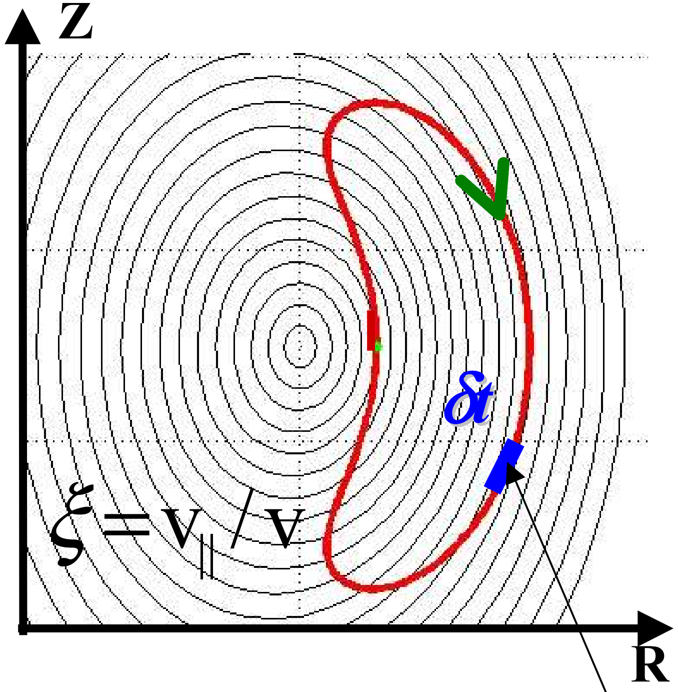
$$\frac{d\psi_p}{dt} = \frac{1}{ZeD}\left[I \frac{\partial B}{\partial \theta} - g\frac{\partial B}{\partial \varphi}\right]\left[\mu + \frac{(Ze)^2}{m}\rho_\parallel^2 B^2\right]$$

$$\frac{d\rho_\parallel}{dt} = \frac{-1}{ZeD}\left[(q+\rho_\parallel I')\frac{\partial B}{\partial \varphi} + (1-\rho_\parallel g')\frac{\partial B}{\partial \theta}\right]\left[\mu + \frac{(Ze)^2}{m}\rho_\parallel^2 B^2\right]$$

White and Chance Phys. Fluid. 1984, White and Boozer Phys. Fluid. 1995

Principle of OFMC with acceleration

Association Euratom-Cea



$$C_{p.a.s}(f) = \frac{\gamma(v)}{v^2}\frac{\partial}{\partial \xi}\left[(1-\xi^2)\frac{\partial f}{\partial \xi}\right]$$

MC operator for collisions applied at every time step:

Orbit integration time: δt

Accelerated time: $\Delta t = N_{acc}\delta t$

$$\Delta\xi = -\frac{\gamma}{2v^2}\xi N_{acc}\delta t + \lambda\sqrt{\frac{\gamma}{2v^2}\left(1-\xi^2\right)N_{acc}\delta t}$$

$$\Delta v = \ldots$$