PV Dynamics for Drift Wave Turbulence

→ Consider H-W system: → $\left\{ \begin{array}{l} C \sim k_{\parallel}^2 v_{th}^2/\nu \\ k_{\parallel} \text{ const/ local equivalent} \end{array} \right.$

$$\left(\frac{\partial}{\partial t} + \underline{\nabla}\hat{\phi}\times\hat{z}\cdot\underline{\nabla}_\perp\right)(\nabla^2\hat{\phi}) = -C\frac{\partial^2}{\partial z^2}(\hat{\phi}-\hat{n}) + \nu\nabla^2\nabla^2\hat{\phi}$$

$$\left(\frac{\partial}{\partial t} + \underline{\nabla}\hat{\phi}\times\hat{z}\cdot\underline{\nabla}_\perp\right)(n_0+\hat{n}) = -C\frac{\partial^2}{\partial z^2}(\hat{\phi}-\hat{n}) + D\nabla^2\hat{n}$$

↳ poorly understood parallel electron dynamics.

adding ⇒ (C cancels, even if $k_\parallel^2 v_{th}^2/\omega\nu \gg 1$)

$$\left(\frac{\partial}{\partial t} + \underline{\nabla}\hat{\phi}\times\hat{z}\cdot\underline{\nabla}_\perp\right)\left(\underbrace{n_0+\hat{n}-\nabla^2\hat{\phi}}_{q}\right) = D\nabla^2\hat{n} - \nu\nabla^2\nabla^2\hat{\phi}$$

more? → curvature

$$\begin{aligned} u &= n_0+\hat{n}-\nabla^2\hat{\phi} \quad \to \text{PV} \\ &= \langle u\rangle + q \end{aligned}$$

$$\frac{du}{dt} = D\nabla_\perp^2\hat{n} - \nu\nabla^2\nabla^2\hat{\phi} \to \underline{\text{PV equation}}$$

<u>Averaging</u> PV equation:

$$\left(\frac{\partial}{\partial t} + \underline{\nabla}\hat{\phi}\times\hat{z}\cdot\underline{\nabla}_\perp\right)(\hat{n}-\nabla^2\hat{\phi}) + \tilde{v}_r\frac{\partial n_0}{\partial r} = D\nabla^2\hat{n} - \nu\nabla^2\nabla^2\hat{\phi}$$

$$\boxed{\Gamma_u = \langle \tilde{v}_r \tilde{n} \rangle - \partial_x \langle \tilde{v}_{xE} \tilde{v}_{yE} \rangle = \text{const.}}$$

⇒ PV const

⇒ conservation of PV links particle flux (relaxation) with divergence of Reynolds stress (flow drive)

Can bound flow production

Γ_0

Fluct'n/Flux

Flows

radient flux

$$\Gamma_0 = \langle \tilde{v}_r \tilde{n} \rangle - \partial_x \langle \tilde{v}_{xE} \tilde{v}_{yE} \rangle$$

Now, mean flow obeys:

$$\frac{\partial}{\partial t} \langle v_y \rangle = -\underline{\underline{\mu}} \langle v_y \rangle - \partial_x \langle \tilde{v}_{xE} \tilde{v}_{yE} \rangle$$

at stationarity: $\underline{\underline{\mu}} \langle v_y \rangle = \partial_x \langle \tilde{v}_{xE} \tilde{v}_{yE} \rangle$

$$\frac{\partial}{\partial t}\langle u\rangle + \frac{\partial}{\partial x}\langle \tilde{V}_x(\tilde{n} - \nabla^2\tilde{\phi})\rangle = 0$$

at stationary state, with $D, \nu \to 0$,

$$\frac{\partial}{\partial x}\langle \tilde{V}_x(\tilde{n} - \nabla^2\tilde{\phi})\rangle = 0$$

$$\boxed{\Gamma_u = \text{const.}}$$

$\boxed{\text{PV flux constant, in absence sources, dissp'n.}}$

$$\Gamma_u = \underbrace{\Gamma_n}_{\text{particle flux}} + \underbrace{\Gamma_\omega}_{\text{vorticity flux}} = \text{const.}$$

Now,
$$\begin{aligned}\Gamma_\omega &= -\langle \tilde{V}_x \nabla^2\tilde{\phi}\rangle \\ &= +\langle \partial_y\tilde{\phi}(\partial_x^2\tilde{\phi} + \partial_y^2\tilde{\phi})\rangle \\ &= \partial_x \langle \partial_y\tilde{\phi}\, \partial_x\tilde{\phi}\rangle \\ &= -\partial_x\langle \tilde{V}_x \tilde{V}_y\rangle\end{aligned}$$

$\hookrightarrow$ Reynolds stress

$$\Gamma_u = \Gamma_n - \partial_x\langle \tilde{V}_{xE}\tilde{V}_{yE}\rangle = \text{const.}$$

2/ ,

→ maximum flow induced by fluctuations in region from PV conservation

$$\Gamma_0 = \underline{\underline{u}} \langle V_y \rangle$$

i.e. → all incident PV flux converted to flow

→ $\Gamma_{n,\text{turb}} = 0$

∴ if u = scalar
$= u(x)$

$$\langle V_y \rangle = \Gamma_0 / u(x)$$

u profile sets

→ realization of flow needs additional criterion

i.e. $(\Gamma_0/u^2)(-u')$ vs $\Delta\omega_n$ determines if dynamically significant flow generated.

→ threshold: $\langle vn \nabla^2\phi \rangle$ vs $\langle vn \rangle$

$$\partial_x \langle \tilde{V}_x \tilde{V}_y \rangle$$

$$\boxed{\delta\left(\delta V_y / \delta\phi\right) \quad \text{vs.} \quad \rho_s^2 \frac{k_x}{L_\varepsilon}}$$

dimensionless ratio

→ threshold criterion (ZF dominance)

Note: Homogenization → Mixing Length Theory Predictions

Homogenization ⇒ $\underline{\nabla}\, q \to 0$

$$\underline{\nabla}\left(\ln n_0 + \frac{\tilde{n}}{n_0} - \rho_s^2 \nabla^2 \tilde{\phi}\right) \to 0$$

no ZF ⇒ $\dfrac{\tilde{n}}{n_0} \sim \dfrac{1}{k_\perp L_n}$ ✓ $k_\perp \sim 1/\ell_{cell}$

with all ZF ⇒ $\dfrac{e\tilde{\phi}}{T} \sim \dfrac{1}{(k_\perp^2 \rho_s^2)}\,\dfrac{1}{k_\perp L_n}$ upper limit on ZF potential

⇒ suggests PV homogenized in streamers .... **eddys...**