'Modelization' of Turbulence Spreading and Entrainment in Fluids and Plasmas

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Series of Papers from 2002 →

Outline

I.) Spreading Happens........

Motivation from Basic Examples (Fluids, MFE...)

II.) Theoretical Perspectives on Spreading ......

→ turbulence evolution in space

→ approaches to description, basic processes in M.F.E.

III.) Towards a Simple Theoretical Modelization structure

→ energy evolution model

→ numerical studies, including edge invasion of core

∴ conventional multi-zone models?

IV.) Some Ongoing Theoretical Work.....

→ General: Foundations of the k-ϵ industry ?!

– eddy displacement pdf ?

→ Specific: Zonal Flows and the turbulent energy flux ....

V.) Discussion

I.) Spreading Happens......

→ Some Examples from Fluid and M.F.E. Dynamics

Spreading has a history ......

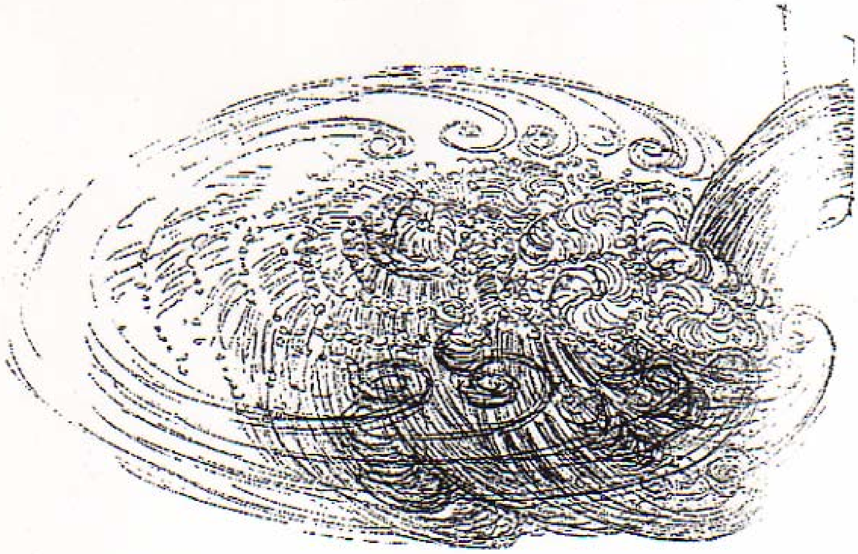


Plate 3 Copy of Leonardo's famous sketch of water falling into a pool. Note the different scales of motion, suggestive of the energy cascade. See the discussion in Section 1.6. [Courtesy of F.C. Davidson.]

A Simple Study of Turbulence

Spreading ..... From Italy, B.C.E.

(Before Coppi's Era)

From: "Turbulence," P.A. Davidson

III.) Towards a Simple, Theoretical Modelization

→ energy evolution model (simple)

→ numerical studies, incl. edge environment of core

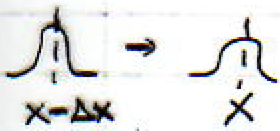
→ ∴ is the conventional wisdom re: multi-zone confinement models any more than convention?

II.) Towards a Simple, Theoretical Model

→ Fokker-Planck Theory (calculates $\langle \tilde{v}\tilde{\mathcal{E}} \rangle$)

$\mathcal{E}(x,t)$ → local turbulence energy density

$T(x,\Delta x,t)$ → transition probability for step of size Δx in time Δt → due NL couplings.



i.e. - require: $\Delta x > \Delta x_c$, $\Delta t > \tilde{\tau}_c$

- in inhomogeneous system; {spectral transfer, spatial transport} coupled

↔ nonlinear decay rate has form:

$$\frac{\mathcal{E}_k(x)}{\tilde{\tau}_{c k}} \cong \sum_{k'} (\mathbf{k}\cdot\mathbf{k}'\times\hat{z})^2 \frac{c^2}{B^2} |\phi_{k'}|^2 R(k,k')\, \mathcal{E}_{k'}(x)$$

$$\cong -\partial_x D_k(\mathcal{E}) \frac{\partial \mathcal{E}_k}{\partial x} + k_\theta^2 D_{\theta k}(\mathcal{E})\, \mathcal{E}_k$$

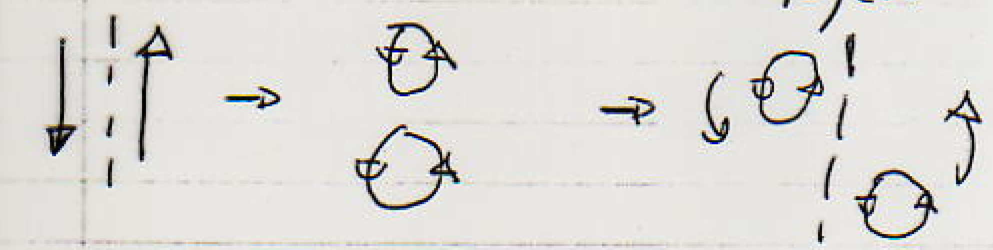
{spatial transport of energy via diffusion} {local transfer in k}

∴ {expect NL interaction works to relax intensity gradients

Analogy : **Eddy fragmentation** $\leftrightarrow$ KH

Spreading $\leftrightarrow$ 'Thickening' of shear layers

"Spreading via pairings"



so

local, linear growth

$$\mathcal{E}(x, t+\Delta t) = \mathcal{E}(x,t) + \left[\gamma(x)\,\mathcal{E}(x) - \gamma_{NL}\,\mathcal{E}^{\alpha+1}(x)\right]$$

$$+ \int d(\Delta x)\, T(x, \Delta x, \Delta t)\, \mathcal{E}(x-\Delta x, t)$$

radial steps

local NL damping

what is T, really?

usual $\Rightarrow$ (Richardson)

$\to \gamma_{NL}(x)$

$\to \alpha = 1 \to$ W.T.

$\alpha = 1/2 \to$ S.T.

$$\boxed{\frac{\partial \mathcal{E}(x,t)}{\partial t} = \left(\gamma(x) - \gamma_{NL}(x)\,\mathcal{E}^{\alpha}\right)\mathcal{E}(x,t) - \frac{\partial}{\partial x}\left(V_{\mathcal{E}}\,\mathcal{E}(x,t)\right) + \frac{\partial^2}{\partial x^2}\left[D_{\mathcal{E}}(\mathcal{E})\,\mathcal{E}(x,t)\right]}$$

$$\left.\begin{aligned} V_{\mathcal{E}} &= \langle \Delta x/\Delta t \rangle \\ D_{\mathcal{E}} &= \langle \Delta x \Delta x / 2\Delta t \rangle \end{aligned}\right\}\ \text{both intensity dependent.}$$

→ Computing D_ε, V_ε

For energy evolution:

$$\frac{dx}{dt} = \underbrace{V_{gr}}_{\substack{\text{radial}\\\text{group}\\\text{velocity}}} + \underbrace{\langle V_r \rangle}_{\substack{\text{mean}\\\text{radial energy}\\\text{flow (coherent)}}} + \delta V_r$$

δV_r → fluctuating large scale flow
↔ random couplings

standard ⇒ $\langle V_r \rangle = 0$

$$\begin{cases} D_\varepsilon = \underbrace{D_{0,\alpha}}_{\text{spatially dependent.}} \, \varepsilon^\alpha \\ V_\varepsilon = V_{gr}(x) + \hat{V}_\varepsilon \end{cases}$$ where:

drift $\hat{V}_\varepsilon = \partial/\partial x \left(D_{0,\alpha} \varepsilon^\alpha \right)$, in Stratonovich calculus ....

nb:

* Key Question : Does $\langle \Delta x \Delta x \rangle$ exist (finite) for self-similar process?
[redacted] not ↔ Fractional Kinetics.
→ validity of k-ϵ Model foundations?

⇒ finally

C.f. Richardson Law

$$\boxed{\frac{\partial}{\partial t} \varepsilon(x,t) + \frac{\partial}{\partial x}\left[V_{gr} \varepsilon \right] - \frac{\partial}{\partial x} D_{0,\alpha} \varepsilon^\alpha \frac{\partial \varepsilon}{\partial x} = \left[\gamma(x) - \gamma_{NL} \varepsilon^\alpha \right] \varepsilon}$$

$$\Rightarrow \frac{\partial \mathcal{E}(x,t)}{\partial t} + \overset{\text{wave radiation}}{\frac{\partial}{\partial x}\left[V_g(x)\,\mathcal{E}(x,t)\right]} - \frac{\partial}{\partial x} D_0 \mathcal{E}^{\alpha} \frac{\partial \mathcal{E}}{\partial x}$$

$$= \left[\gamma(x) - \gamma_{NL}(x)\,\mathcal{E}^{\alpha}\right] \mathcal{E}(x,t)$$

↳ prevents $\left\{ \begin{array}{l} \text{blow up} \\ \text{"exponential spr"} \end{array} \right.$

- D_0, $\gamma(x)$, $\gamma_{NL}(x)$ from local turbulence model

for local GB{Drift - ITG:

$$D_0 = \rho_s^2 c_s / L_\perp$$

$$\gamma(x) = \frac{c_s}{L_\perp} f\left(L_{T0}/L_T\right) \quad \ldots$$

- $V_g(x)$ captures some toroidal coupling effects i.e. $V_g \sim V_d \sim \frac{\rho_i}{R} v_{Ti} \sim \epsilon v_{Ti}$

[redacted] ....

- recovers $K-\epsilon$ type phenomenology systematically: Familiar, reaction-diffusion type equation!

Key Fundamental Issues:

- scales controlling $\langle \Delta x \Delta x \rangle$
- convergence of $\langle \Delta x \Delta x \rangle$

} Physics of T →

c.f. later

→ **Energy Theorem**

- can write in conservative form

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial}{\partial x}\Gamma_{\mathcal{E}} = S_{\mathcal{E}}$$

$$\Gamma_{\mathcal{E}} = V_{gr}\mathcal{E} - (D_{0,\alpha}\mathcal{E}^{\alpha})\frac{\partial \mathcal{E}}{\partial x} \quad \rightarrow \text{intensity flux}$$

∴ for total energy in interval $[x_-, x_+]$, E

$$\frac{\partial E}{\partial t} = -\Gamma_{\mathcal{E}}\Big|_{x_-}^{x_+} + \int_{x_-}^{x_+} dx\, S_{\mathcal{E}} \qquad \left\{\begin{array}{l}\text{growth} \\ \text{via } S_{\mathcal{E}} \text{ or} \\ \Gamma_{\mathcal{E}}\end{array}\right.$$

and

$$\Gamma_{\mathcal{E}}\Big|_{x_-}^{x_+} = V_{gr}\mathcal{E}\Big|_{x_-}^{x_+} + \underbrace{\Delta'_{\mathcal{E}}\,\mathcal{E}}_{\text{energy flux}}$$

$$\boxed{\Delta'_{\mathcal{E}}\,\mathcal{E} = \frac{\partial}{\partial x}\left(\frac{D_{0,\alpha}}{1+\alpha}\mathcal{E}^{1+\alpha}\right)\Big|_{x_-}^{x_+}} \qquad \left\{\begin{array}{l}\text{set by} \\ \text{intensity} \\ \text{profile}\end{array}\right.$$

i.e. for $S, V_{gr} \rightarrow 0$

$\Delta'_{\mathcal{E}} > 0 \rightarrow$ region pumped (influx)

$\Delta'_{\mathcal{E}} < 0 \rightarrow$ region damped (outflow)

∴ Growth set by $\left\{\begin{array}{l}\text{local sources,} \\ \text{energy density profile} \\ \text{structure ...}\end{array}\right.$

Analyzing the Dynamics

→ have toy model:

$$\frac{\partial}{\partial t}\mathcal{E}(x,t) + \frac{\partial}{\partial x}(V_g\mathcal{E}) - \frac{\partial}{\partial x}\left(D_0\,\mathcal{E}^\alpha\frac{\partial\mathcal{E}}{\partial x}\right)$$
$$= \gamma(x)\,\mathcal{E} - \gamma_{NL}(x)\,\mathcal{E}^{\alpha+1}$$

Consider:

a.) $\alpha=1$, $\gamma=0$, $\gamma_{NL}=0$, $V_g=0$, D_0 const.

⇒ self-similar spreading

b.) $\alpha=1$, $\gamma=\text{const}$, $V_g=0$, $\gamma_{NL}=\text{const}$, $D_0=\text{const}$,

⇒ front solution

c) $\alpha=1$, $\gamma(x)$, $V_g=0$, $\gamma_{NL}=\text{const}$, $D_0=\text{const}$,

⇒ tunneling, gaps, barriers, etc.

a.) $\alpha = 1$; $\gamma = \gamma_{NL} = 0$; $V_g = 0$

$$\frac{\partial \mathcal{E}(x,t)}{\partial t} - \frac{\partial}{\partial x} D_0 \mathcal{E} \frac{\partial \mathcal{E}}{\partial x} = 0$$

$\leftarrow \}\} \rightarrow$

NL diffusion $\Rightarrow$

$$\mathcal{E}(x,t) = \frac{A}{t^{1/3}} \left(1 - x^2/d(t)^2\right) \Theta\left(|d(t) - x|\right)$$

$$d(t) = \left(6 A D_0\right)^{1/2} t^{1/3}$$

simple scaling

self-similar expansion ...

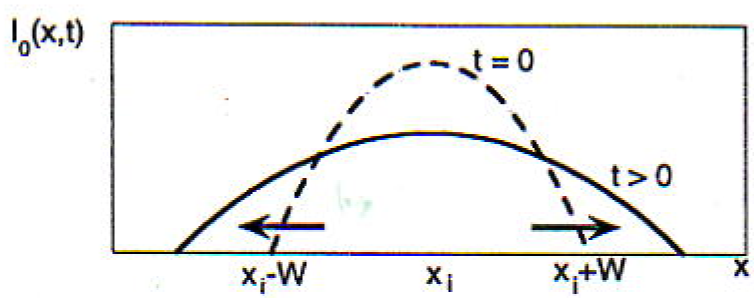
$A = \int dx\, \mathcal{E}(x,0)$ $\rightarrow$ initial 'impulse' of patch

$\rightarrow$ sub-diffusive evolution, i.e.

- for $\alpha = 1/2$, $d(t) \sim t^{2/5}$
- generally: $d(t) \sim \left[D_0 A^{\beta} t\right]^{1/2+\beta}$

for $d(t)^2 \sim D_0 I^{\beta} t$

$\rightarrow$ spreading via 'pulse ~~front~~ propagation':



b.) Fisher - KPP Fronts

$V_g = 0$

rescaling $\Rightarrow$

$\gamma \sim$ const.

$$\frac{\partial \mathcal{E}}{\partial t} - \frac{1}{2}\frac{\partial}{\partial x}\mathcal{E}\frac{\partial \mathcal{E}}{\partial x} = \mathcal{E}(1-\mathcal{E})$$

$\rightarrow$ Fisher eqn. with NL diffusivity

vs.

'Classic Fisher':

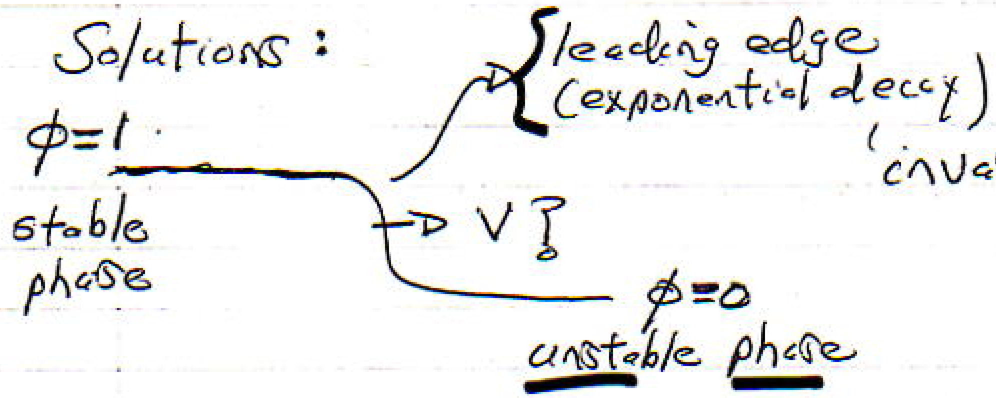
const.

$$\frac{\partial \phi}{\partial t} - D_0 \frac{\partial^2 \phi}{\partial x^2} = \gamma_0 \phi (1-\phi)$$

$\rightarrow$ logistic + diffusion

$\rightarrow$ similar TDGL

Solutions:



Issue: Speed of leading edge ? – front

Demand marginal stability in co-moving frame

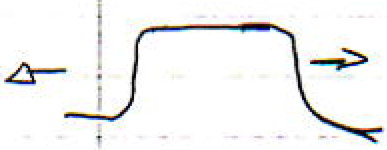
$$\Rightarrow \quad V = (2\gamma_0 D_0)^{1/2}$$

Note: Geometric Mean of $\left\{\begin{array}{l}\text{reaction} \\ \text{diffusion}\end{array}\right.$ rates

→ But : here diffusion nonlinear!

∴ explore self-similar 'leading edge' solution

leading edge



$$\Rightarrow \mathcal{E}(x,t) = f(t)\Big(1 - \exp[-|x-d(t)|] - \exp[-|x+d(t)|]\Big)$$

i.e.

- localized blob, extent $2d(t)$
- expanding at $\dot{d}(t)$

so obtain:

$$\begin{cases} \dot{d}(t) - \frac{1}{2} + \dfrac{2e^{-d(t)}\cosh^{-1}(e^{d(t)}/2)}{(-4+e^{2d(t)})^{1/2}} = 0 \\[2ex] f(t) = \dfrac{1}{1-4e^{-2d(t)}} - \dfrac{4e^{d(t)}\cosh^{-1}(e^{d(t)}/2)}{(-4+e^{2d(t)})^{3/2}} \end{cases}$$

and implicit solution for $d(t)$ ⇒

$$\sinh\left[2\cosh^{-1}(e^{d(t)}/2)\right] - 2\cosh^{-1}(e^{d(t)}/2) = e^{t}$$

Now, as $t \to \infty$, 'implicit solution' becomes

$$d(t) = t/2 \quad \Rightarrow \text{ ballistically propagating front}$$

(ala' standard Fisher...)

restoring dimensional factors

$$\Rightarrow \quad V = \left(\gamma^2 D_0 / 2\gamma_{NL}\right)^{1/2}$$

'Poor Man's Explanation'

$$\frac{\partial \mathcal{E}}{\partial t} - \frac{1}{4} D_0 \frac{\partial^2}{\partial x^2}\mathcal{E}^2 - \mathcal{E}(\gamma - \gamma_N \mathcal{E}) = 0$$

take usual Fisher scaling:

$$V = (2 D_{eff}\gamma)^{1/2}$$

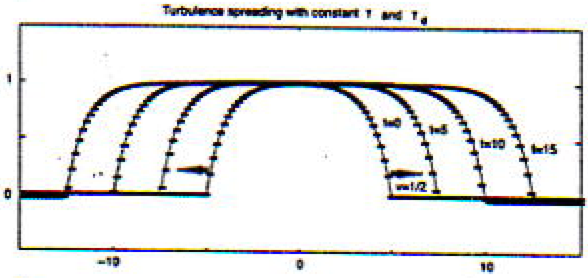
with $\quad D_{eff} = \frac{D_0}{4}\mathcal{E}_{sat} = \frac{D_0 \gamma}{4\gamma_{NL}}$ spr. rel. local satn.

$$\therefore \quad \boxed{V = \left(D_0 \gamma^2 / 2\gamma_{NL}\right)^{1/2}}$$

i.e. → Diffusion set by local saturation level
$\mathcal{E} = \gamma/\gamma_{NL} \leftrightarrow$ top of front

→ agreement with numerical solution **excellent.**

Spreading, invasion front...



- leading edge structure manifested

Comments:

→ $\gamma_{NL} \neq 0$ needed for distinction between:

~ spreading of saturated turbulence

~ spreading of (exponentially) growing turbulence. (trivial)

→ generically, for gyroBohm type model...

$$V_{Front}/V_{gr} \sim O(1/\epsilon_T)$$

(M.L.T. levels)

Z.F.'s

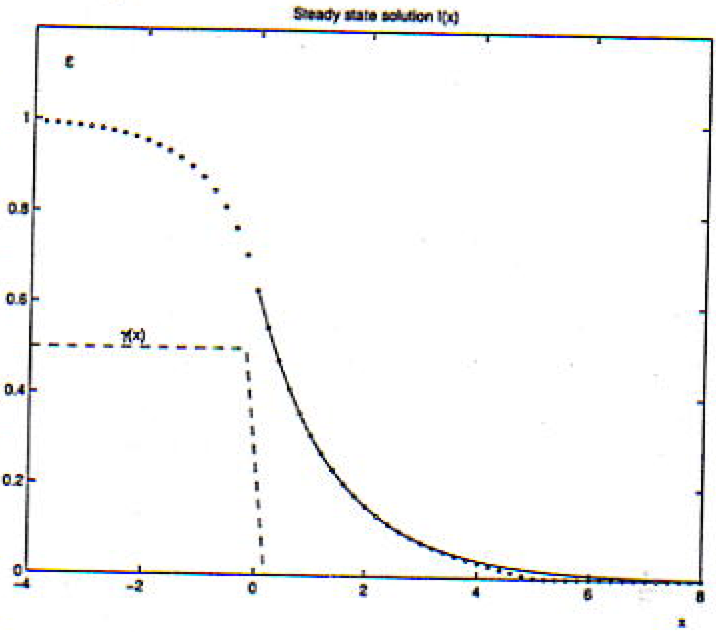
suggests that dynamically induced spreading faster than toroidal coupling induced spreading ... parameter scans needed here ....

→ numerical tests ☺ consistent ↔ see later.

c) Examples – Penetration into $\left\{\begin{matrix}\text{marginal} \\ \text{damped}\end{matrix}\right\}$ regions

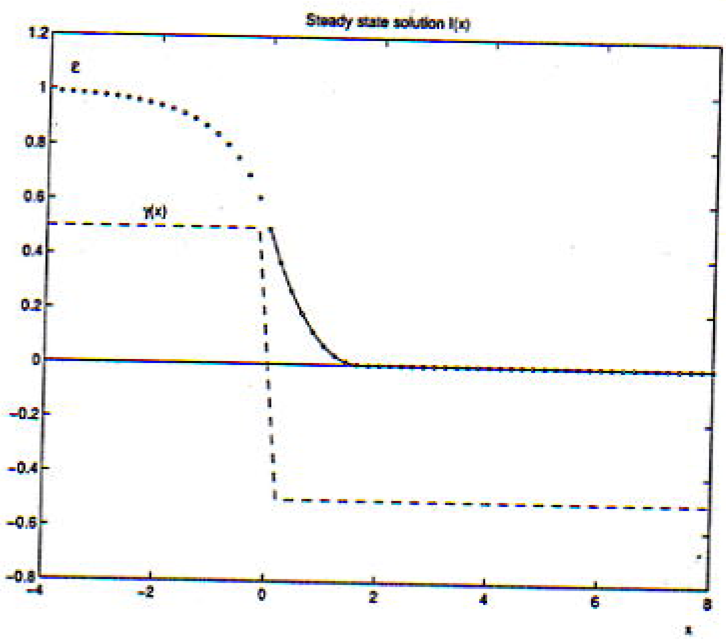
3a

$\gamma \neq$ const.



Marginal

→ significant penetration



damped (heavily)

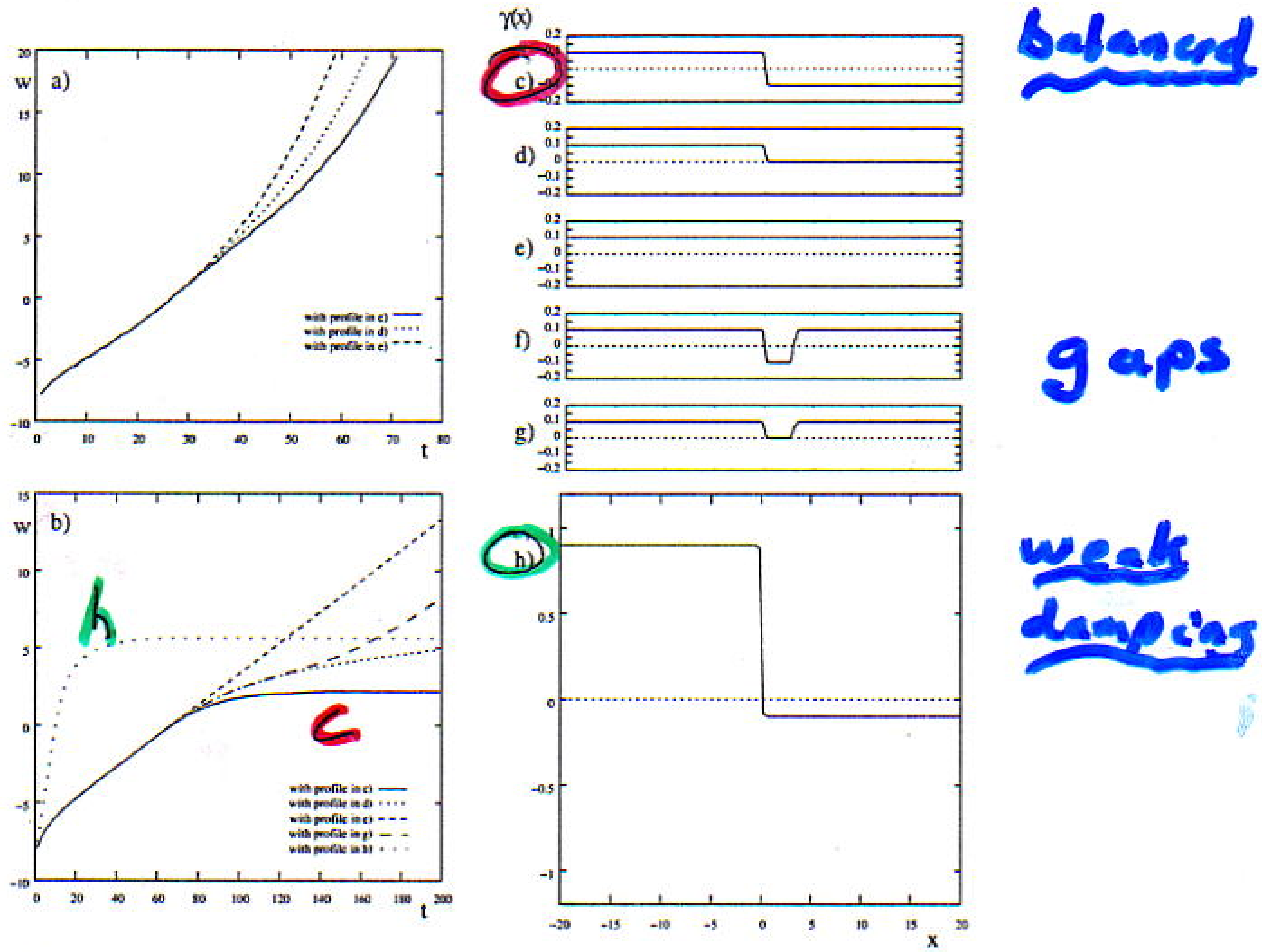
→ modest penetration.

Moral: Likely that there is significant invasion of marginal region adjacent to strongly turbulent region.

i.e. core + edge.

Jumping Barriers, gaps....

31.



Ö.D. Gürcan, P.H. Diamond, T.S. Hahm and Z.Lin Figure 9

Experiment I

Edge Turbulence Spreading to Unstable Core

- **Nonlinear Gyrokinetic Simulations of Ion Temperature Gradient Turbulence:**

  $\frac{R}{L_T} = 6.9$ at core (Cyclone value)

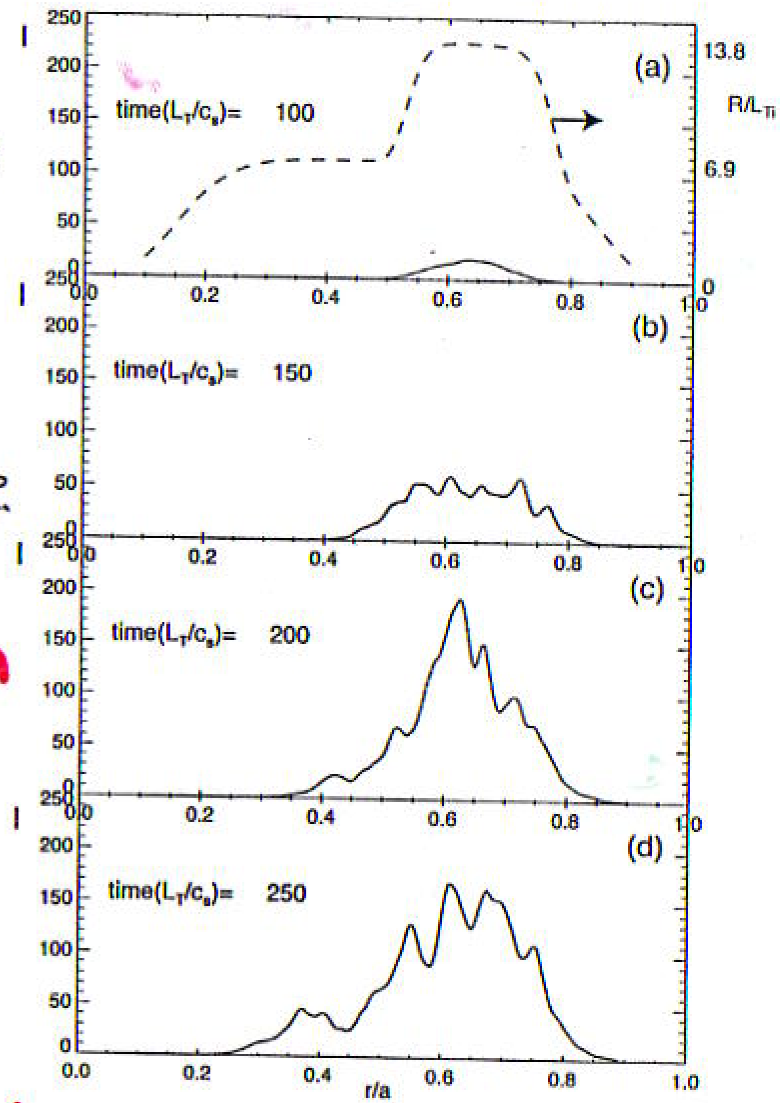
  $\frac{R}{L_T} = 13.8$ at edge   Unstable Core

  \+ Strongly turbulent edge

- Initial Growth at Edge followed by **Ballistic Front Propagation** into Core (track inflection point)

- Saturation Level at Core $\sim 2\times$ Core (only) Result

$$\nabla \cdot \Gamma_I \sim \gamma_{local} I$$



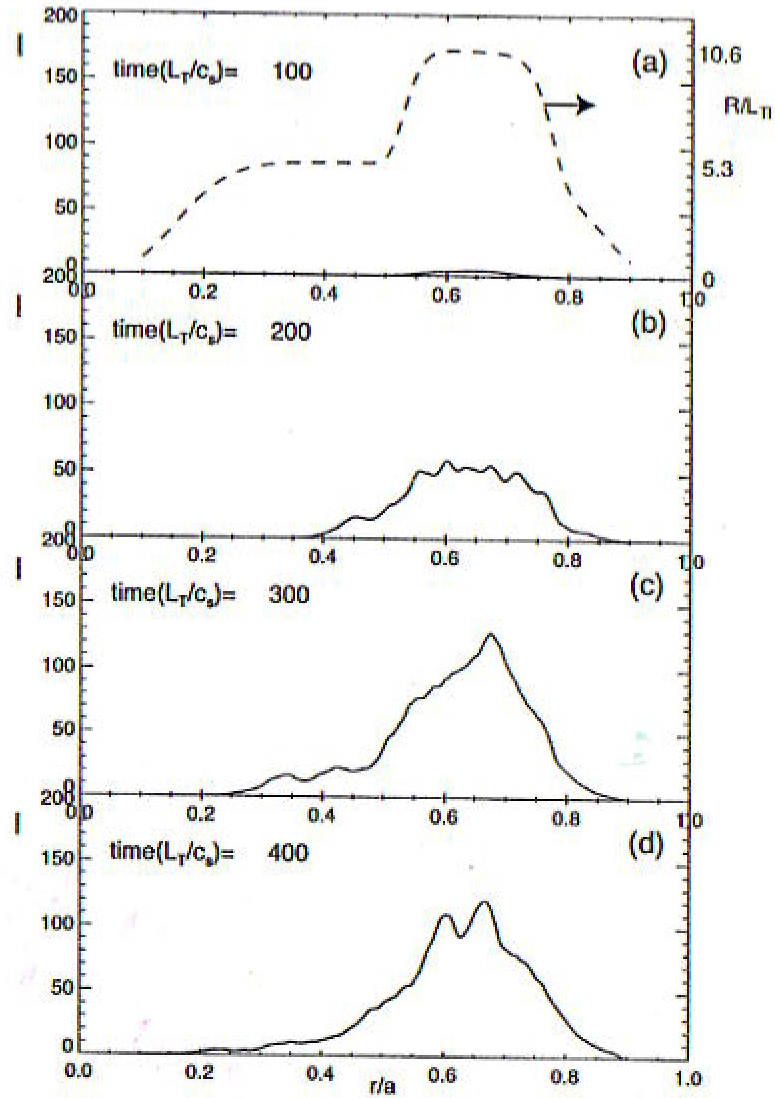
Even with strong, local turbulence invasion from edge makes comparable contribution to core levels.

Experiment II

Turbulence Spreading from Edge to Stable Core

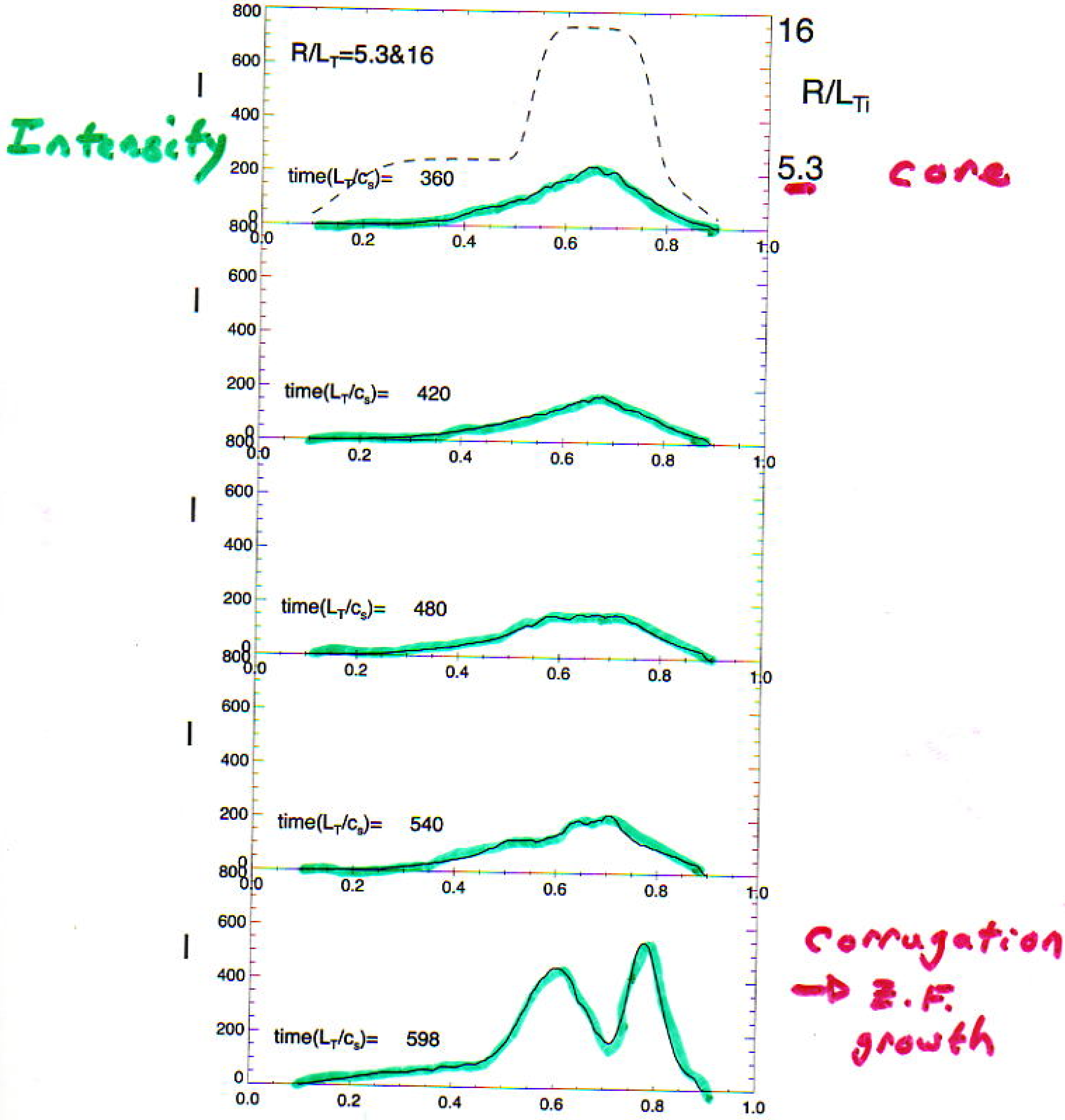
- Nonlinear GTC Simulations of Ion Temperature Gradient Turbulence:
  $\frac{R}{L_T} = 5.3$ at core
  (within Dimits shift regime)
  $\frac{R}{L_T} = 10.6$ at edge:
  ⊖ Marginal core
- Initial Growth at Edge
  $\rightarrow$ Penetration into stable Core
  (Lin-Hahm-Diamond,
  PRL '02, PPCF, PoP '04)
- Saturation Level at Core:
  $\frac{e\delta\phi}{T_e} \sim 3.6\frac{\rho_i}{a}$
  $\rightarrow \boxed{\nabla \cdot \Gamma_I >> \gamma_{local} I}$

invasion front dominates



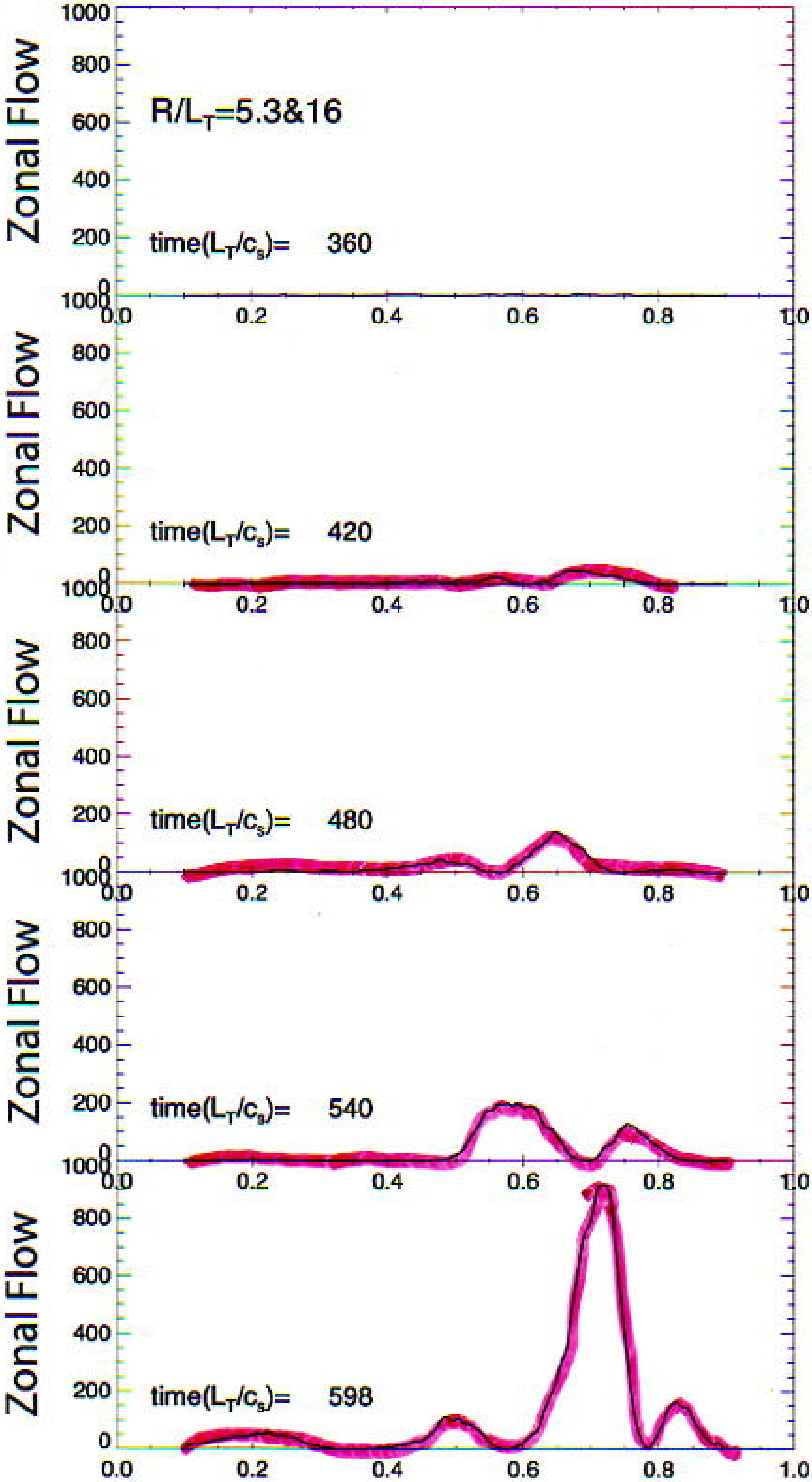
Turbulence front penetrates Z.F.'s

1.8


R/L_T=5.3&16
16
R/L_{Ti}
5.3
I
Intensity
core
time(L_T/c_s)= 360
time(L_T/c_s)= 420
time(L_T/c_s)= 480
time(L_T/c_s)= 540
time(L_T/c_s)= 598
Corrugation → Z.F. growth
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800

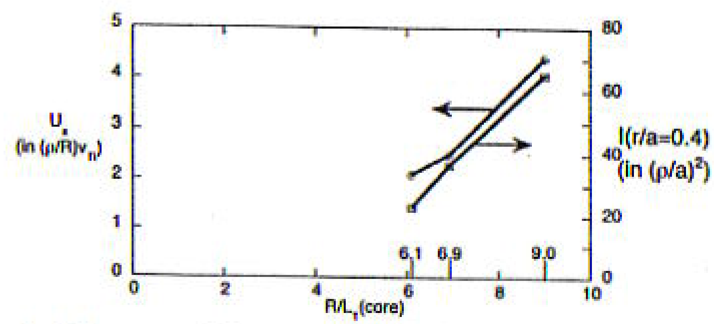
Zonal Flows Lag Advance of Fluctuation Inversion.

Z.F.



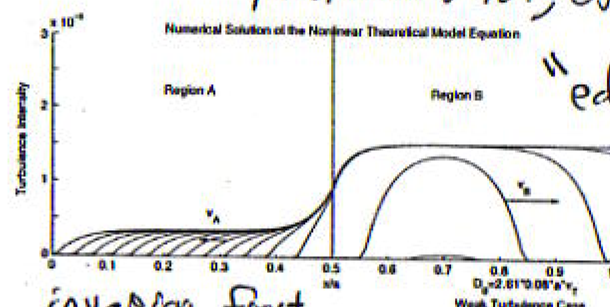
Testing the theory .....

Front Propagation Speed Increases with R/L_T



Suggests NL coupling process.

- From Simulation, U_x and I increase with $(\frac{R}{L_T})$
- Nonlinear Diffusion Model: $U_x \propto (\gamma I)^{1/2}$
  by [Gurcan-Diamond-Hahm-Lin, ~~submitted to PoP '04~~] published PoP '05.



"edge"

invasion front

- Toroidal Linear Coupling dominant Regime: $U_x \sim \frac{\rho_i}{R} v_{Ti}$ — independent R/L_{Ti}
  by [Garbet-Laurent-Samain-Chinardet, NF '94]
- Four Wave Model: Complex Bursty Spreading
  by [Zonca-White-Chen, PoP '04]

[N.B. Actual model single-n T/J]

Relevance to strongly turbulent state dubious.

Upshot

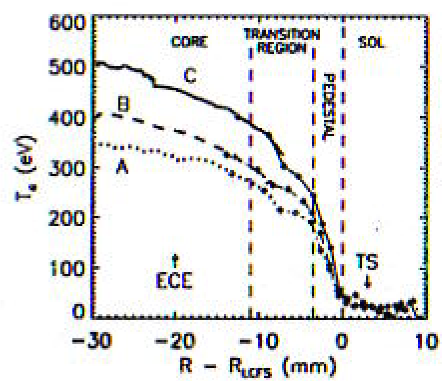
Distinction between "Core" and "Edge" blurred

- *Researchers have frequently divided the tokamak into three zones — a central sawtoothing zone, a middle 'confinement zone', and an edge zone...*
  Goldston-U.S.A. *Kyoto IAEA (1986)*

- *the edge..., often used as a boundary condition for core transport modeling*
  V. Parail, *Plasma Phys. Control. Fusion*, **44**, A63 (2002)

Fact, Fiction, Fantasy ....?

- $\frac{\partial}{\partial x}\gamma(x) \sim \frac{\partial^2}{\partial x^2}P$: large at the top of pedestal →



C-mod, Hughes et al., 2002

– Residual of edge driven surf zone

or

– H-mode converts to core driven surf zone?

....

IV.) Issues in the Theory – Ongoing...

→ Eddy Displacement Transition Pdf
(with F. Otsuka, K. Itoh, S-I. Itoh)

– k-ε industr[illegible] citly a
Fokker-Pl[illegible] ry.....

key: $T(\Delta x, \Delta t, \underline{x})$ → probability of 'step' of size Δx in Δt

?
↓
displacement pdf for ε, K....

$\boxed{\text{No clue..}}$

– key question: $\int d(\Delta x)\,(\Delta x)^2 T < \infty$?

if not, ... Fractional kinetics (Zaslavsky '02)

∴ → modelization structure changes.....

→ what can, in general, be said about T ?

→ how relate T to familiar features of turbulence ?

Clue: Richardson Dispersion

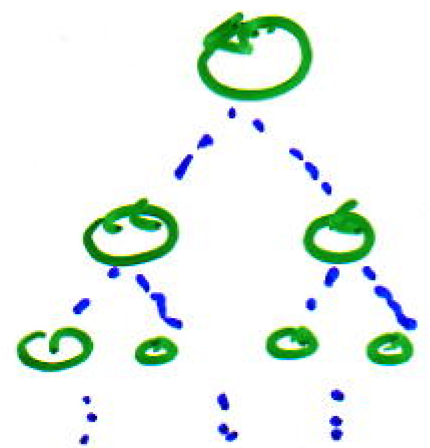
$$\frac{d\ell}{dt} = \epsilon^{1/3}\ell^{1/3} = v(\ell)$$

$$\Rightarrow \quad \ell \sim \epsilon^{1/2} t^{3/2}$$

- power law distributed step size
- super-diffusive scaling

$\therefore$ some simple thoughts .....

"Cascade": hierarchy of eddy fragmentation events



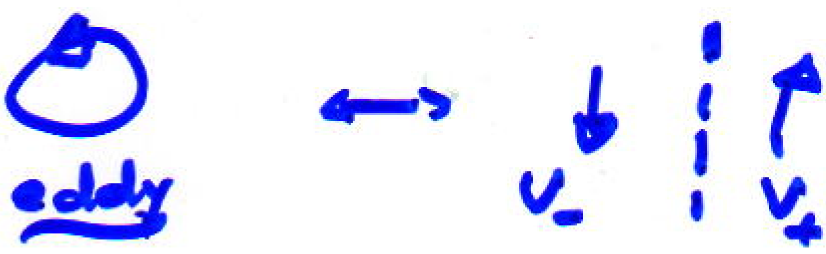
ℓ_0

first instability

ℓ_1

second instability

ℓ_2



shear layer

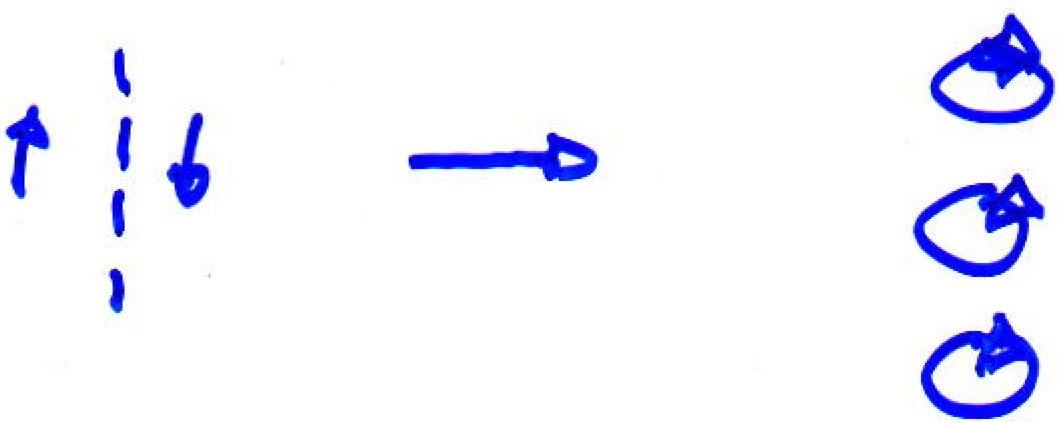
$1/\tau \sim \frac{v(\ell)}{\ell}$

$\gamma_k \sim k \Delta V$

$\Delta V \rightarrow$ jump, differential

$v(\ell) \rightarrow$ velocity differential

$v(\ell) = |v(\underline{r}+\ell) - v(\underline{r})|$

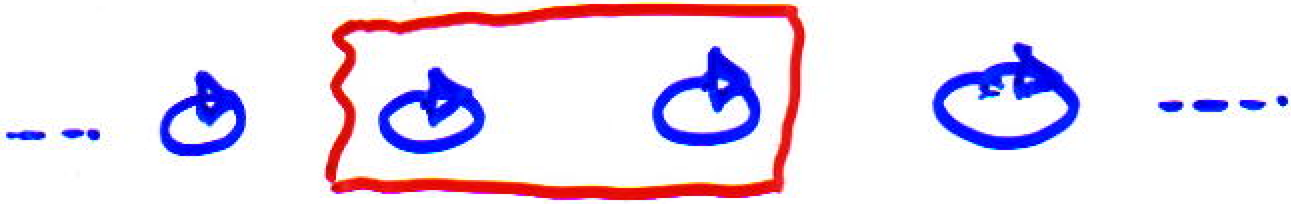


vortex street

— Relation to spreading:

i.e. dynamically, why does spreading happen?

$\leftrightarrow$ consider vortex street vortex pair



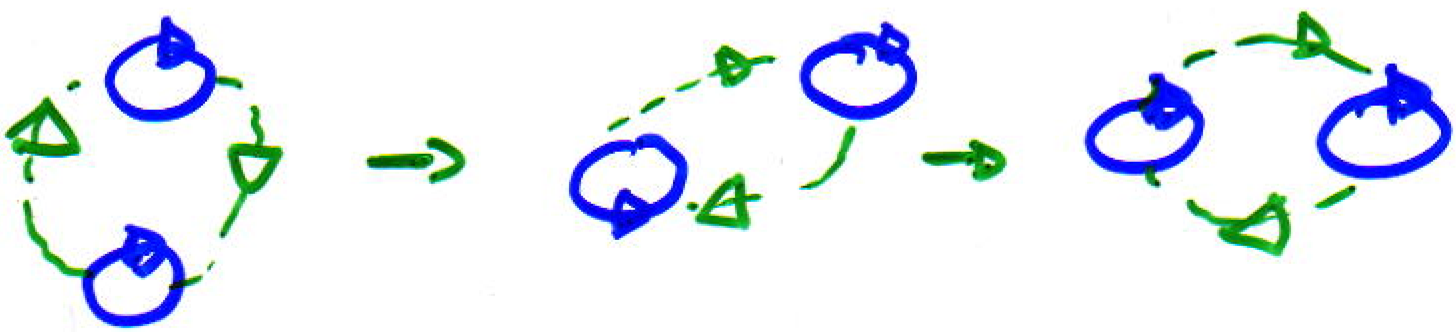
i.e. neighboring vortices interact via

- Magnus force

$$\underline{v}\cdot\underline{\nabla}\,\underline{v} = \underline{\nabla}(v^2/2) - \underline{v}\times\underline{\omega}$$

- Biot Savart Law

∴ like sign pairs (roll up) tend to rotate CM



∴ - Pairing and mutual rotation tend thicken shear layer

- thickness set by $\begin{cases} \text{vortex separation} \\ \lambda_{KH} \end{cases}$

so seems reasonable to hypothesize

$\Delta x \sim \ell$ (scale of frag.)

$\underbrace{\Delta x}_{\text{step size}} \sim \underbrace{\ell}_{\text{eddy size}}$ (what else?)

$\therefore \quad T(\Delta x) \sim P(\ell)$

→ probability of finding an eddy of size ℓ

so, for homogeneous, isotropic, ... turbulence

($\alpha > 1$)

$\therefore \quad \ell_n = \alpha^{-n} \ell_0 \qquad \ell_n \to \ell_n/\alpha$

$P(\ell_n) \sim (\text{\# eddys size } \ell_n)/(\text{total \#})$

$$\sim \alpha^n \Big/ \underbrace{\sum_{p=0}^{p_{max}} \alpha^p}_{\to N}$$

$p_{max} \to$ dissipn. scale

$\alpha^{-n} \ell_0 \sim \ell_n$

$$\therefore\ P(\ell_n) \sim \frac{1}{N}\left(\frac{\ell_0}{\ell_n}\right)$$ → power law

ultra-crude model suggests:

- power law form for T

(no surprise)

- "1/f" type scaling

i.e. small jumps more likely than large jumps

consistent with notion of entrainment as erosion

- integral scale variability, intermittency important here – $\ell_0(\underline{x})$, etc. ?!

– mode structure in M.F.E.

- $\int d\ell\ \ell^2 P(\ell)$ uncertain

controlled by ℓ_0, r

$\therefore$ Caveat Emptor re: $K-\epsilon$ models

Reconsider ala' Fractional Kinetics ??

→ Spreading, Zonal Flows, and the Fluctuation Energy Flux

confluence of 2 lines:

- direct calculation of $\langle \tilde{V}_r \tilde{\mathcal{E}} \rangle$ for 'usual models' i.e. $\left\{ \begin{array}{l} \text{Hasegawa-Mima} \\ \text{H-Wakatani} \\ \text{etc.} \end{array} \right.$

(Ö. Gurcan, P.D., T.S. H.)

- Zonal shears and the "Non-locality length" - thoughts on Waltz, Candy PoP '05.

→ Calculating $\langle \tilde{V} \tilde{\mathcal{E}} \rangle$

– Simplest route forward: simply CALCULATE $\langle \tilde{V} \tilde{\mathcal{E}} \rangle$ from basic model → variation on triplet closure for turbulent interaction.

i.e. H-M Energy Relation

$$\partial_t \left(\phi^2 + (\nabla \phi)^2 \right) + \nabla \cdot \Gamma_{\mathcal{E}} = \text{stirring}$$

$$\Gamma_{\mathcal{E}} = -\left\langle \phi^2 \left(\nabla (\nabla^2 \phi) \times \hat{z} \right) \right\rangle \rightarrow \langle \tilde{V} \tilde{\mathcal{E}} \rangle$$

Proceed via 2 scale closure. ....

– W.O. Zonal flows

$\Gamma_{\mathcal{E}} \rightarrow 0$ on 3 mode resonance, in W.T.T.

nonlinear broadening $\Delta\omega_k$ required to obtain flux $\Gamma_{\mathcal{E}}$.

– but.... with zonal flows

~ 3 mode resonance 'relaxed'

i.e. 1 with $k_y = 0$

ZF ⟷ non adiabatic

~ energy coupled to zonal flows

⇒ work $(\langle J \cdot E \rangle)$ in energy theorem } → total energy ....

∴ can formulate energy thm:

$$\partial_t \Big(\underbrace{\tilde{\phi}^2 + (\nabla\tilde{\phi})^2}_{\text{drift waves}} + \underbrace{(\nabla\bar{\phi})^2}_{\text{zonal flow energy}} \Big) + \nabla \cdot \Gamma_\varepsilon = 0$$

$$\Gamma_\varepsilon = -\Big\langle \tilde{\phi}^2 (\nabla \nabla^2 \bar{\phi} \times \hat{z}) + \bar{\phi}\tilde{\phi}\, \nabla(\nabla^2 \tilde{\phi}) \times \hat{z} \Big\rangle$$

Lots algebra:

$\Gamma_\varepsilon \neq 0$, and in W.T.T.:

$$\Gamma_\varepsilon \sim \underbrace{\gamma}_{\text{Z.F. damping!}} \langle k^2 \rangle \underbrace{\bar{\mathcal{E}}}_{\text{Z.F. energy}} \underbrace{\mathcal{E}}_{\text{fluctn. energy}}$$

– irreversibility....

- Hasegawa - Wakatani $\left\{ \begin{array}{l} \text{minimal} \\ \text{non-trivial} \\ \text{case} \end{array} \right.$

$$(\partial_t + \underline{v}\cdot\underline{\nabla})\nabla^2\phi = \alpha(\phi - n) \qquad \alpha = \frac{k_\parallel^2 v_{th}^2}{\omega_* \nu}$$

$$(\partial_t + v\cdot\nabla)\, n + \partial_y \phi = \alpha(\phi - n)$$

- $\alpha > 1$ → H-M case (adiabatic)

- $\alpha < 1$ → hydro limit $\qquad \tilde{n} \neq \tilde{\phi}$

$$\left\{ \begin{array}{l} \mathcal{E} = \underbrace{\tilde{n}^2}_{\text{internal}} + \underbrace{(\nabla\tilde{\phi})^2}_{\text{kinetic}} + \underbrace{(\nabla\bar{\phi})^2}_{\text{Z.F.}} \\ \Gamma_{\mathcal{E}} \text{ predominantly } \langle \tilde{\underline{v}}\, \tilde{n}^2 \rangle \end{array} \right.$$

⇒ spreading "lead" by internal energy – testable

Advertisement: $PV = \ln n_0 + \tilde{n} - \nabla^2\tilde{\phi}$

Conserved, arb. α

$PV \left\{ \begin{array}{l} \text{conservation} \\ \text{flux balance} \end{array} \right.$

→ useful insight into interplay $\left\{ \begin{array}{l} \text{transport} \\ \text{zonal flows} \end{array} \right.$

→ Nonlocality Length: Waltz and Candy PoP '05

Spreading (their version):

rate of global mode formation vs. shearing due Z.F.

$$1/\tau_{Global} \quad \text{vs} \quad \tilde{V}_E'$$

$$\wr$$

$$\frac{V_0}{L} \sim \frac{\rho_i v_{Ti}}{R L} \quad \text{vs} \quad V_E'$$

(calc' Garbet '94)

$$L \cong \rho_* \left(\frac{a}{R}\right) \frac{v_{Ti}}{\tilde{V}_E'}$$ → spreading length

global mode formation ...

vs

shearing ...

Interesting:

- simulations show inhibition of global mode formation by Z.F.

  c.f. Z. Lin's overworked VG

- full ballooning structure dubious...

  ⇒ 'modelets', ala' Connor, Wilson?

  ↳ few, coupled poloidal subharmonics

- $\boxed{\text{but}}$ $\tilde{V}_E \neq \gamma_L$, ala' W&C.

  → Z.F. population adjusts self-consistently — predator-prey, etc.

  ⟺ Z.F. damping crucial to transport regulation ....

→ why not dust off old barrier dynamics models ?

→ evoln $\mathcal{E}$, $\tilde{V}_E'^2$?

with linear propagation

W + C effect

i.e.

$$\partial_t \mathcal{E} + \partial_x (V_d \mathcal{E}) - \partial_x (D_0 \mathcal{E} \partial_x \mathcal{E})$$
$$= \gamma \mathcal{E} - \underline{\alpha V_E'^2 \mathcal{E}} - \beta \mathcal{E}^2$$

$$\partial_t V_E'^2 - \partial_x (D_1 \mathcal{E} \partial_x V_E'^2)$$
$$= \underline{\alpha V_E'^2 \mathcal{E}} - \mu V_E'^2$$

→ consider self-consistent spreading, back-transition dynamics ??

→ explore construction in "modelet" basis ...