Plasmas in Laboratory and Astrophysics
Alfven Wave Turbulence and Heating.

Center for Multi-scale Plasma Dynamics.

Bill Dorland, Maryland.

Alex Schekochihin, Cambridge.

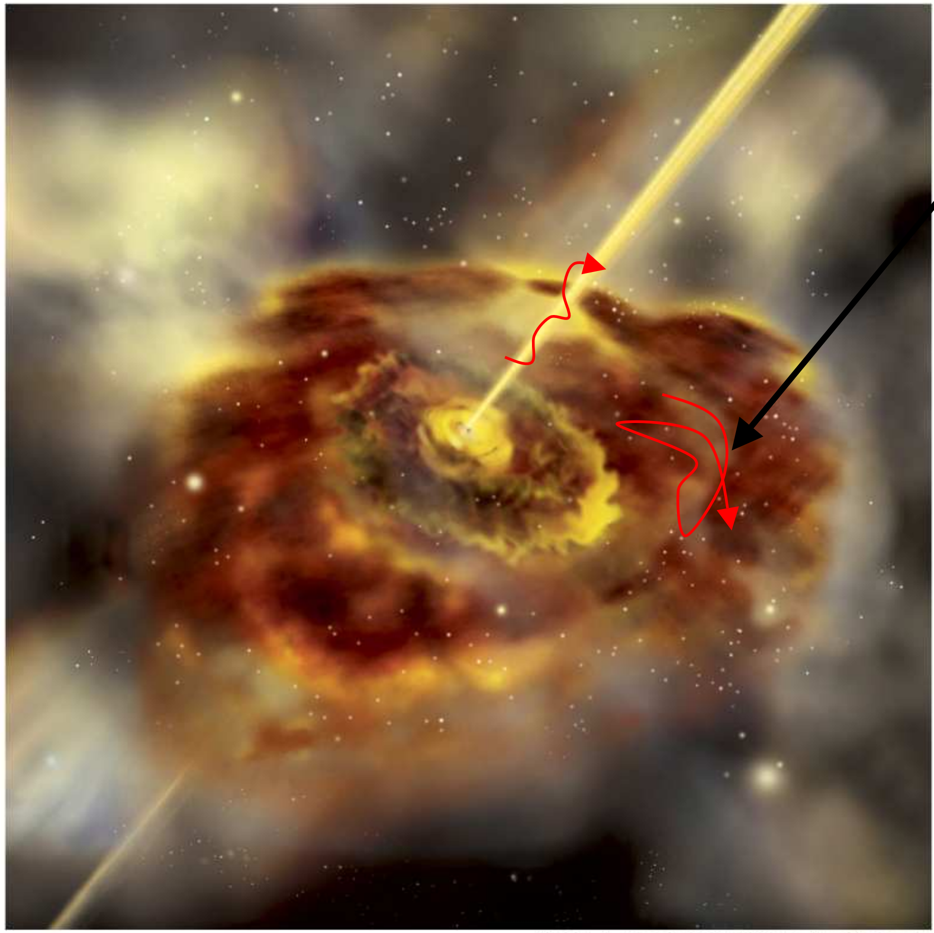
Steve Cowley, UCLA/Imperial College.

Troy Carter, Brian Brugman, Pat Pribyl, UCLA.

Greg Howes, Eliot Quataert, Berkeley.

Greg Hammett, Princeton.

Accretion and MHD.



Credit: Aurore Simonnet, Sonoma State University

The Inner Part of an Active Galactic Nucleus
(Artist's Impression)

ESO PR Photo 18a/03 (19 June 2003) ©European Southern Observatory

Instability $\Rightarrow$Tangled **B** Field.

$\Downarrow$

Momentum transport.

$\Downarrow$

Gravitational energy loss

$\Downarrow$

Turbulent energy

$\Downarrow$

Dissipation, Heat to ??

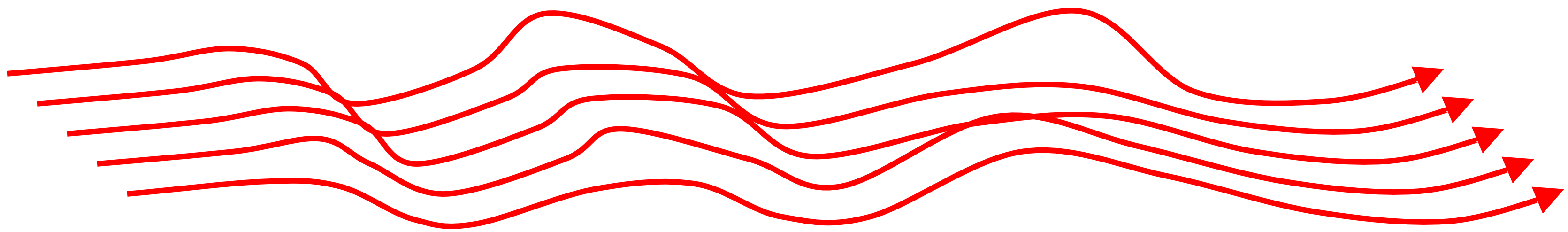
(i) Electrons -- radiation.

(ii) Ions -- swallowed by black hole.

Quataert and Gruzinov.

Small Scales.

Kraichnan argued that the small scale motions will see a large quasi-uniform field. The small scale motions will become Alfven/slow waves traveling along a roughly uniform **B**.



Most of the dissipation happens at small scales -- these arguments suggest that the dissipation is of small amplitude Alfven/slow waves.

Introduction.

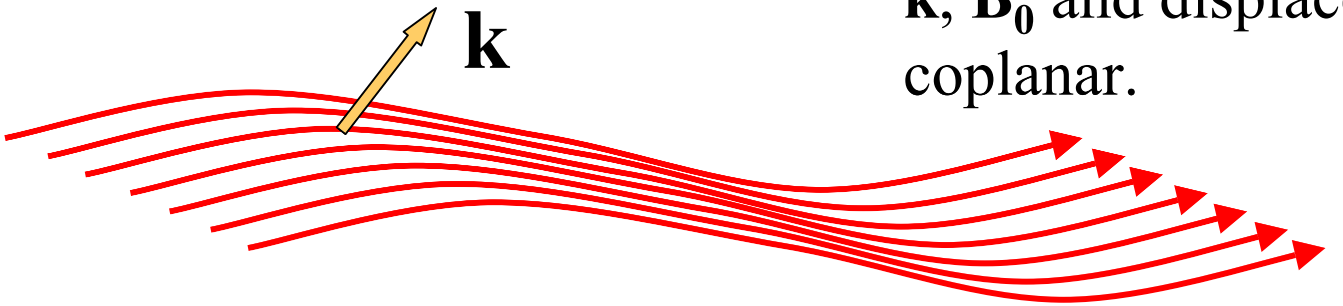
- Need to understand the turbulent cascade of Alfven waves.

  Many applications: astrophysics, fusion, space.
- MHD regime -- Goldreich - Sridhar theory.
- Collisionless regime -- gyrokinetics. Prototypical of collisionless turbulence.
- Heating in gyrokinetics
- Simulations. Bill Dorland, Greg Howes.

MHD Waves. (Review) $k_\perp \gg k_{||}$

Slow Waves $\omega = k_{||}\sqrt{\frac{C_s^2 V_A^2}{C_s^2 + V_A^2}}$

k, $\mathbf{B_0}$ and displacement coplanar.

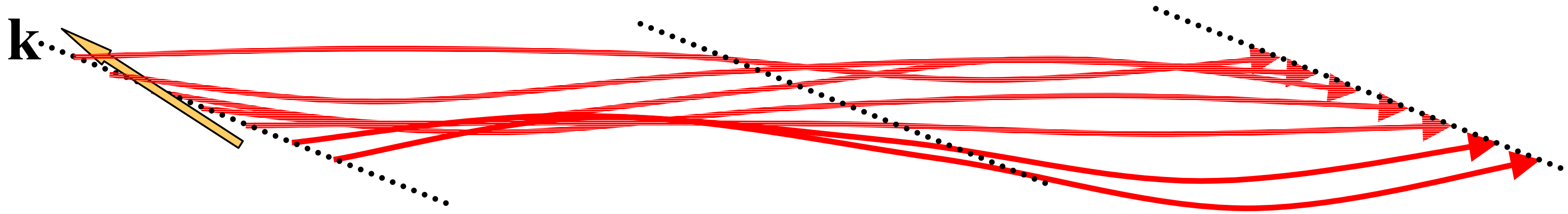


Collision Mean Free path

$\delta B_{||} B_0 + \delta p = 0$ Damped by transit time damping when $k_{||} \lambda_{mfp} \sim 1$.

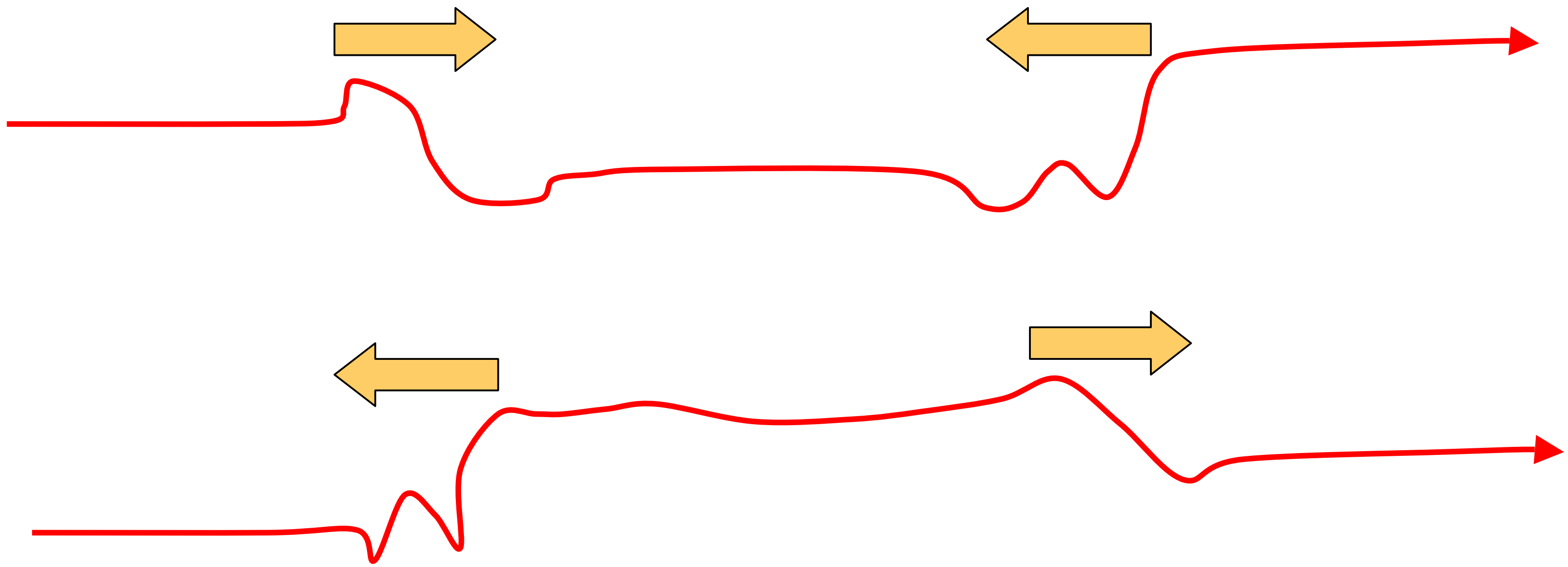
Alfven Waves $\omega = k_{||} V_A$

Displacement perpendicular to $\mathbf{B_0}$ and **k.**



$\delta B_{||} = 0$ No damping until $k_\perp \rho_i \sim 1$ Ion Larmor radius

Two Wave packets - passing through.



Three wave weak interaction for oppositely directed waves gives.

$$\omega_3 = \omega_1 + \omega_2 \;\rightarrow k_{||3} = k_{||1} - k_{||2} \quad but\ also\ \mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$$

Thus $k_{||2} = 0$ and there is no cascade in $k_{||}$

Strong Alfven Cascade.

Goldreich-Sridhar 1995. Komolgorov argument.

Alfven waves stirred at scale l_0 with velocity v_0

Energy flux from scale l to l/2 $$\epsilon = \frac{\rho v_l^2}{2\tau_l}$$ Constant from scale to scale.

Cascade time $$\tau_l = \frac{l}{v_l}$$

Assuming strong interaction

$$v_l = v_0 \left(\frac{l}{l_0}\right)^{1/3}$$

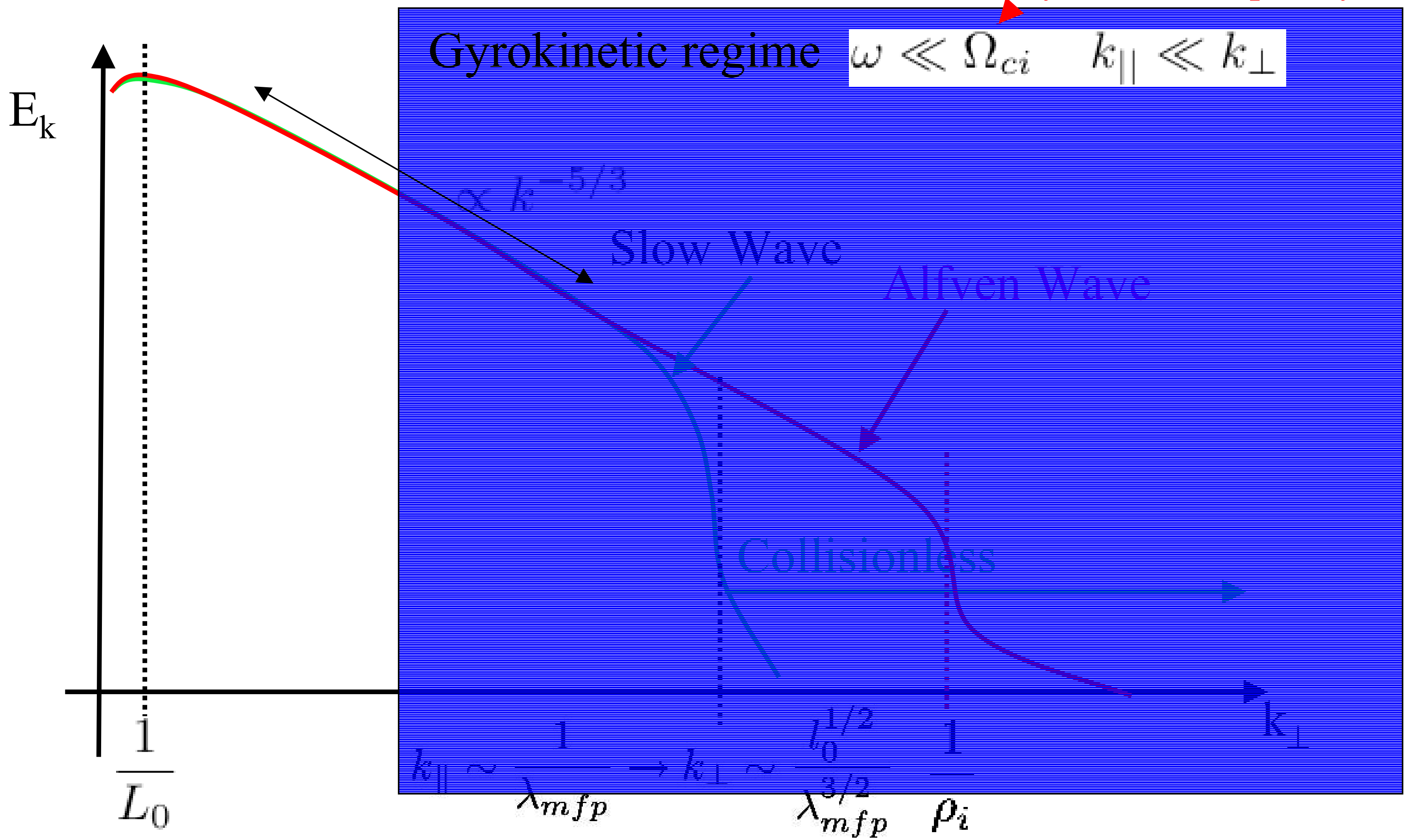
GS argue that the scale l is the perpendicular scale and that the parallel scale is set by "critical balance."
Nonlinear rate = Linear rate

$$\frac{v_l}{l} = k_{||} V_A$$

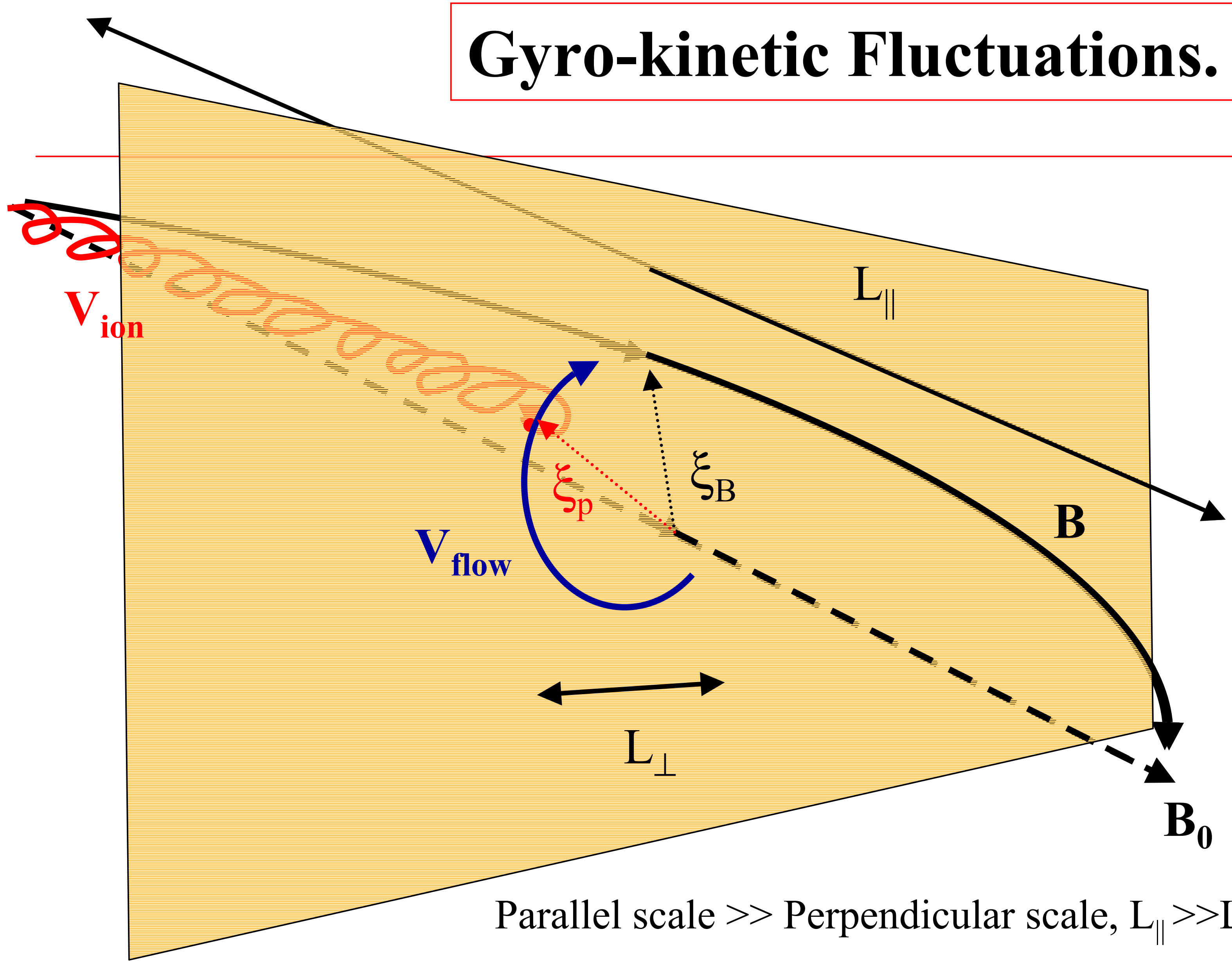
$$k_{||} = \frac{V_0}{V_A} k_\perp^{2/3} l_0^{-1/3} \ll k_\perp$$

Spectrum.

MHD simulations Maron & Goldreich confirm scaling (almost). Cyclotron frequency



Gyro-kinetic Fluctuations.



Parallel scale >> Perpendicular scale, $L_{\|} >> L_{\perp} \sim \rho_i$

$V_{flow} \approx \frac{\mathbf{E} \times \mathbf{B}}{B^2}$ Particle displacement, $\xi_p \sim$ Of Order Field line displacement, $\xi_B \sim O(L_{\perp})$

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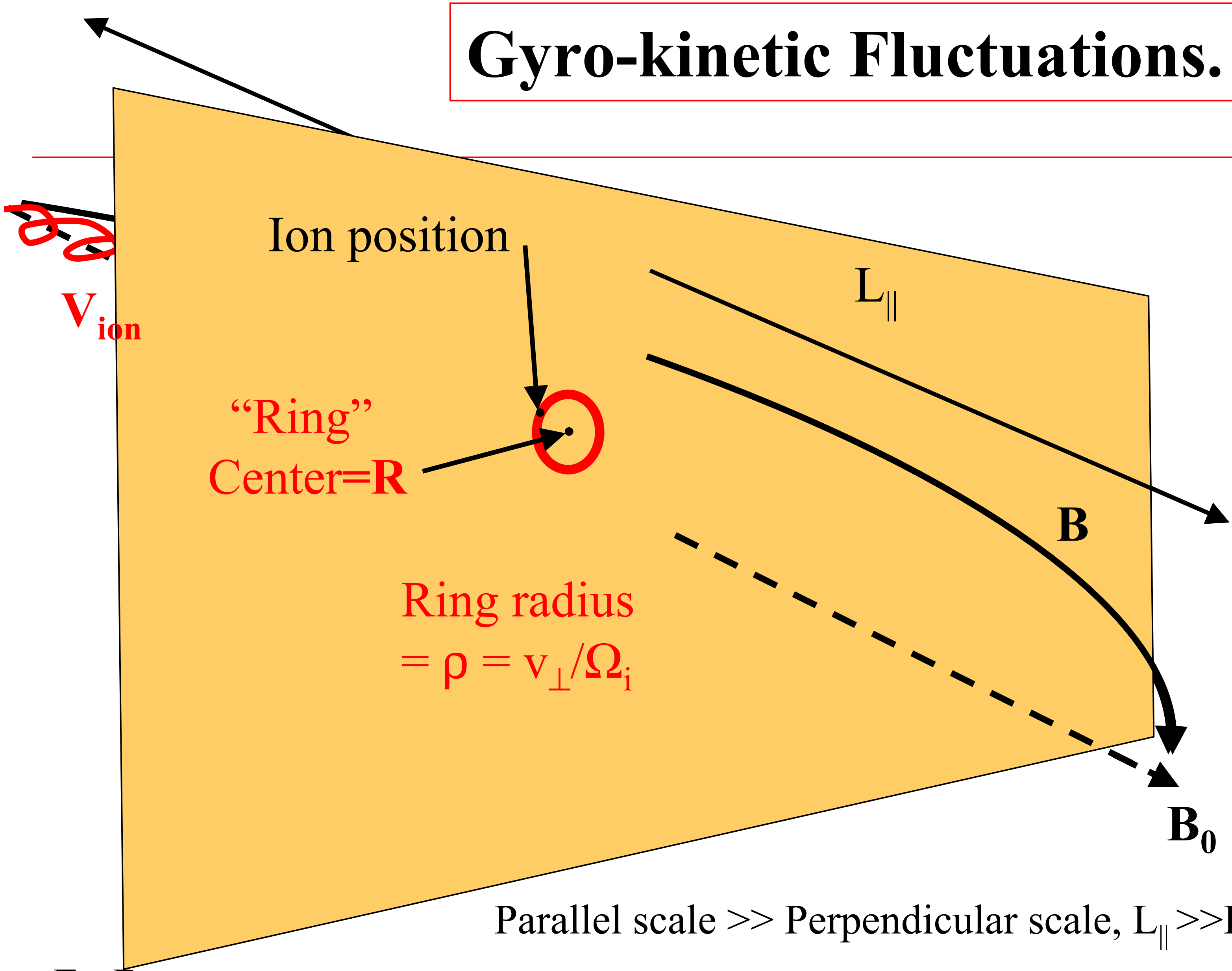

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Gyro-kinetic Equation.

Taylor and Hastie, Frieman and Chen, Dubin

Ion kinetic energy

$$f = F_0(\epsilon, t/t_{heat}) - q\frac{\phi(\mathbf{r},t)}{T_0}F_0 + h(\mathbf{R},\mu,\epsilon,t).$$

Maxwellian evolving on slow heating timescale

Shift in energy Due to potential

Perturbed Distribution Of "rings"

Collisions

Evolution of the Rings

$$\frac{\partial h}{\partial t} + v_{\parallel}\frac{\partial h}{\partial z} + [\langle\chi\rangle, h] - C(h) = -q\frac{\partial F_0}{\partial \epsilon}\frac{\partial\langle\chi\rangle}{\partial t}$$

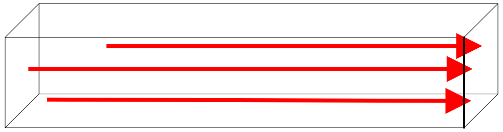
5D!

$$[\langle\chi\rangle, h] = \frac{1}{B_0}(\nabla\langle\chi\rangle \times \nabla h)\cdot\mathbf{z} = \mathbf{v}_{drift}\cdot\nabla h$$

$$\chi = \phi - \mathbf{v}\cdot\mathbf{A} \qquad <\chi>= Ring\ average\ \chi$$

+ Maxwell's equations.

Box Gyro-kinetic Simulation MHD regime -- *Bill Dorland.*



Waves launched by “numerical antenna” driven by stochastic oscillator.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

$$\beta = 8 = 800\%$$

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Gyro-kinetic heating.

$$\frac{3}{2} n_{0s} \frac{dT_{0s}}{dt} = \int \frac{d^3\mathbf{R}}{L_z L_\perp^2} \int d^3\mathbf{v} \overline{(q \frac{\partial \langle \chi \rangle}{\partial t} h)} \quad + \quad n_{0s} \nu_\epsilon^{sr} (T_r - T_s)$$

Time averaged q**E.v**
Work done on particles

Collisional
energy exchange

Numerically it is hard to compute the heating this way because energy sloshes in and out of particles. However we can obtain this term from from the GK equation by multiplying by h/F_0 and integrating.

$$\frac{d}{dt} \int \frac{d^3\mathbf{r}}{V} \int d^3\mathbf{v} \frac{T}{2F_0} h^2 - \int \frac{d^3\mathbf{r}}{V} \int d^3\mathbf{v} \frac{T_{0s}}{F_{0s}} h C(h) = \int \frac{d^3\mathbf{R}}{V} \int d^3\mathbf{v} (q \frac{\partial \langle \chi \rangle}{\partial t} h$$

- Entropy in fluctuations

Heating and Entropy Production

Averaging over the fast times we get:

$$\frac{3}{2} n_{0s} \frac{dT_{0s}}{dt} = - \int \frac{d^3\mathbf{r}}{L_z L_\perp^2} \int d^3\mathbf{v} \frac{T}{F_0} \overline{[h_s C_s(h_s)]} \quad + \quad n_{0s} \nu_\epsilon^{sr} (T_r - T_s)$$

This term is positive and much easier to average, heating always comes from collisions even in collisionless regime! It is not hard to show from Boltzmann's H theorem that this is the entropy production.

We normally have small collisionality - how does the dissipation by collisions work?

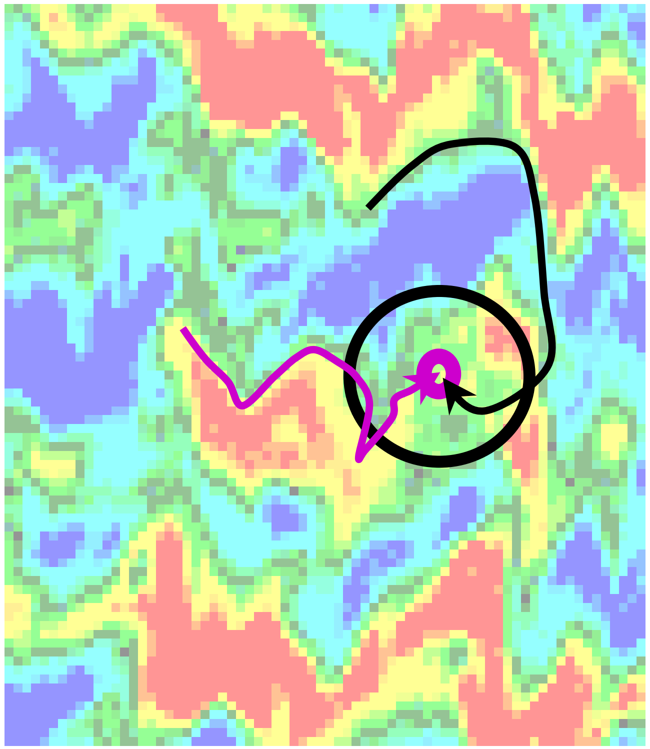
Some related work on this by Landau 1946 and Hu and Krommes 1996.

Mixing in Phase Space.

$$\frac{\partial h}{\partial t} + v_{\|}\frac{\partial h}{\partial z} + \underbrace{[\langle\chi\rangle, h]}_{\mathbf{v}_{drift}\cdot\nabla h} - C(h) = -q\frac{\partial F_0}{\partial \epsilon}\frac{\partial\langle\chi\rangle}{\partial t}$$

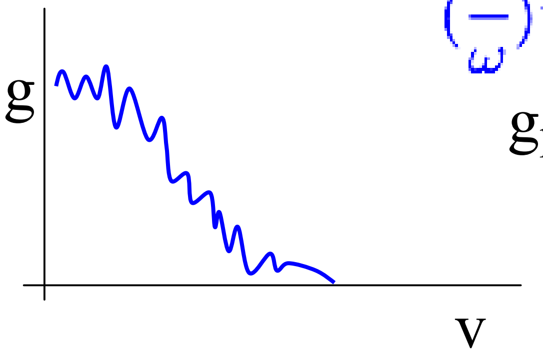
$$< \frac{d\epsilon}{dt} >$$

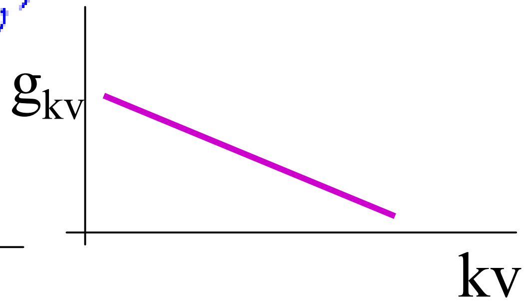
Smooth source in v space.



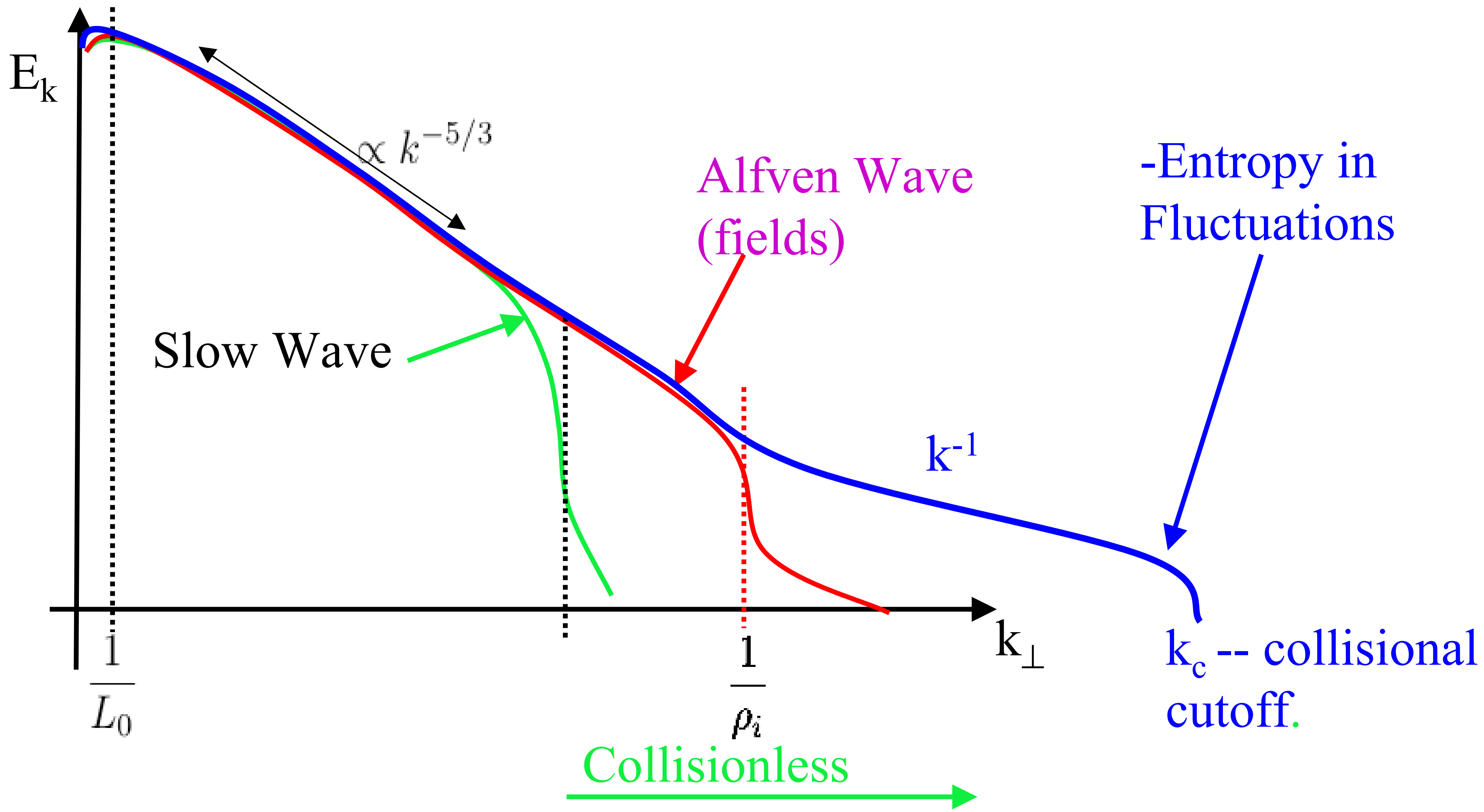
Different rings drift at different velocities mixing the distribution h. Small velocity scales act like a passive scalar in Fluid turbulence. Cascade in both real **and** velocity space. Velocity space cascade terminated at collisional scales. Velocity space inertial scale range:

$$(\frac{\nu}{\omega})^{1/2} \ll \delta v \ll 1$$





Spectrum.



E_k
$\propto k^{-5/3}$
Alfven Wave (fields)
-Entropy in Fluctuations
Slow Wave
k^{-1}
$k_\perp$
$\frac{1}{L_0}$
$\frac{1}{\rho_i}$
k_c -- collisional cutoff.
Collisionless

2 point. Closure Calculation.

Alex Schekochihin and SCC.

$$S_k(U) = < h_{-\mathbf{k}}(v_\perp) h_{\mathbf{k}}(v'_\perp) >, \qquad U = \frac{v_\perp - v'_\perp}{v_{thi}}$$

$$\frac{\partial S_k(U)}{\partial t} = \gamma \frac{\partial}{\partial k}\left(k^2 \frac{\partial S_k(U)}{\partial k} - k S_k(U)\right) - k^2(\nu \rho_i^2 + DU^2) S_k(U) + \nu \frac{\partial^2 S_k(U)}{\partial U^2}$$

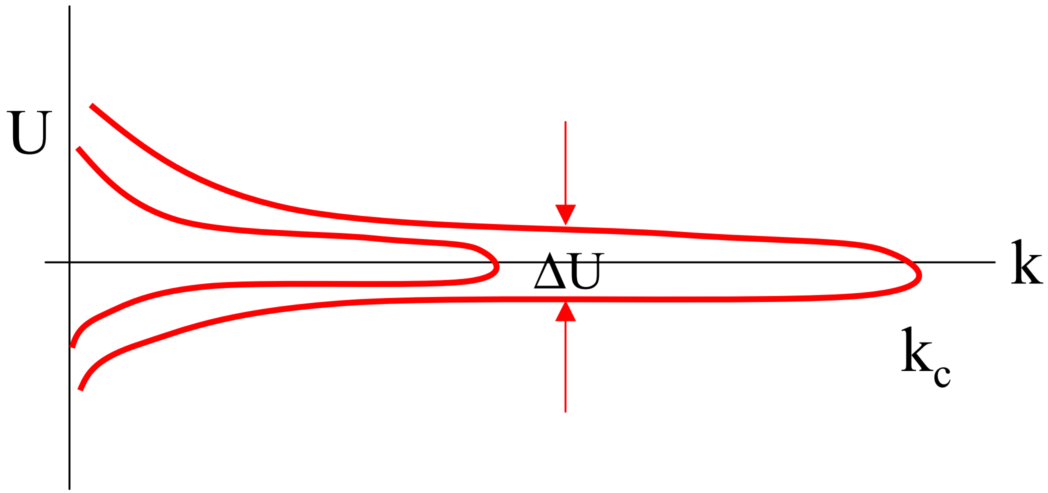
γ-- eddy turnover rate of last eddy

γD -- relative diffusion of gyro-centers.

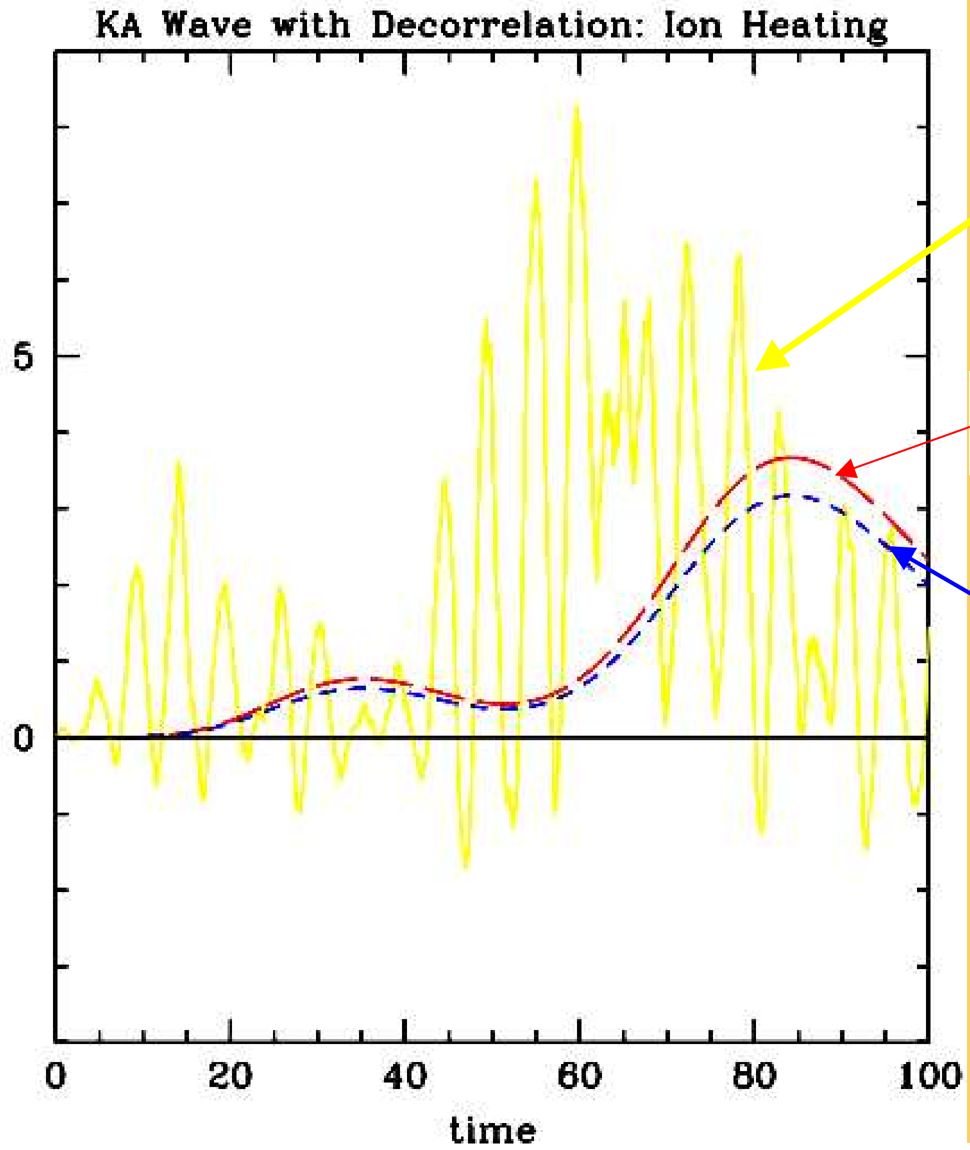
ν-- collisionality

$k_c\, \rho_i = (\gamma/\nu)^{1/2} >> 1$ defines cut-off in k

$\Delta U = (\nu/\gamma)^{1/2} << 1$ collisional Velocity scale.



Heating Three Ways in GS2.



$$\int \frac{d^3\mathbf{R}}{V} \int d^3\mathbf{v} (q \frac{\partial < \xi >}{\partial t} h)$$

Raw **E.v**

$$\int \frac{d^3\mathbf{R}}{V} \int d^3\mathbf{v} (q \frac{\partial \langle \chi \rangle}{\partial t} h) - \frac{d}{dt} \int \frac{d^3\mathbf{r}}{V} \int d^3\mathbf{v} \frac{T}{2F_0} h^2$$

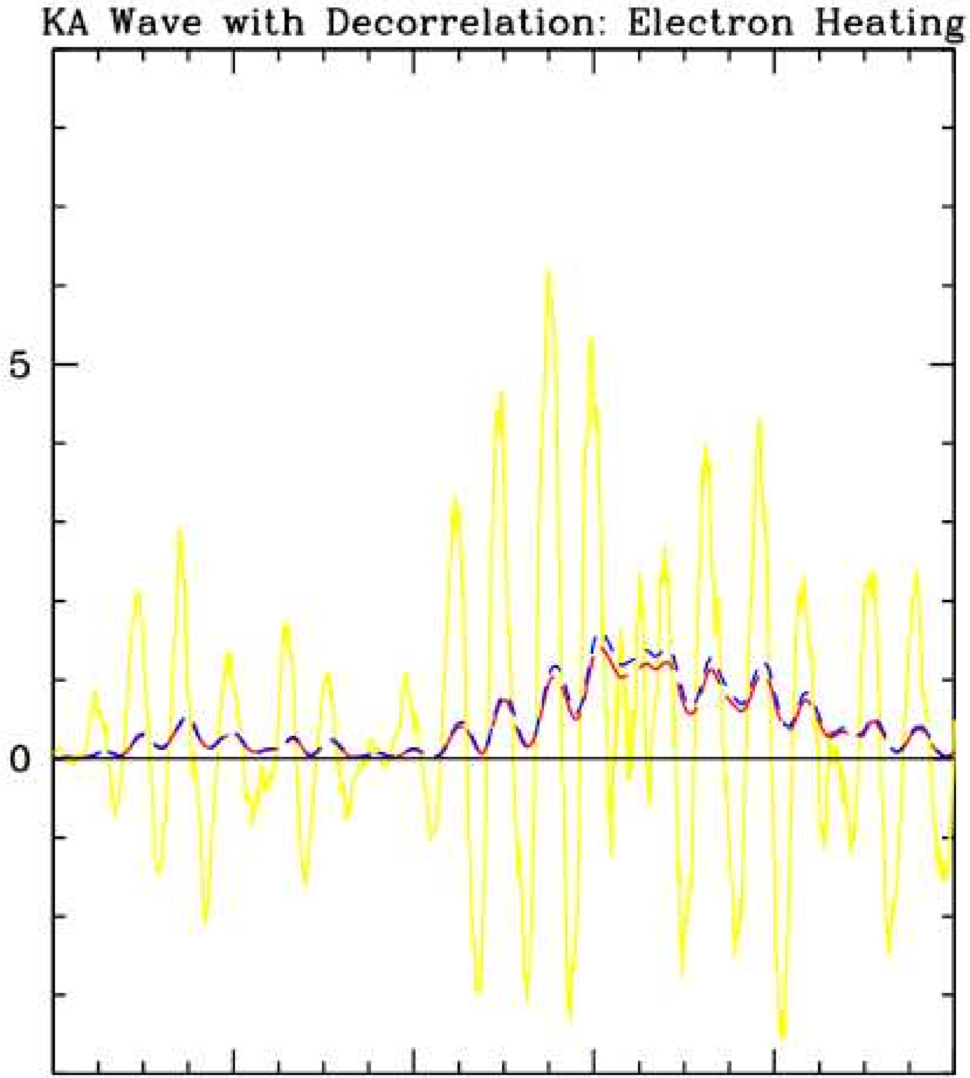
Raw **E.v** + sloshing, mostly large v scale

$$-\int \frac{d^3\mathbf{r}}{V} \int d^3\mathbf{v} \frac{T_0}{F_0} hC(h)$$

Entropy production -- small v scale dominates

Red and blue should be the same. Difference is due to lack of resolution in v space

Electrons too.



Simulations

- How much velocity space resolution is needed? Continuum simulations have typically 20×20. PIC typically has much less. Similar issue with real space resolution. We are doing scaling studies to look at this issue.
- Current simulations are doing the kinetic regime and calculating the ion and electron heating. $Q_e=Q_e(T_e/T_i, \beta)$, $Q_i=Q_i(T_e/T_i, \beta)$ Bill Dorland, Greg Howes.
- Need to understand the cascade in this regime and examine the dynamics.

CONCLUSIONS

- Well defined problem that needs a solution.
- Excellent use of fusion codes - well adapted to the problem.
- Experimental study on LAPD will yield further results this year.