Some challenges to the theory of *astrophysical* dynamos

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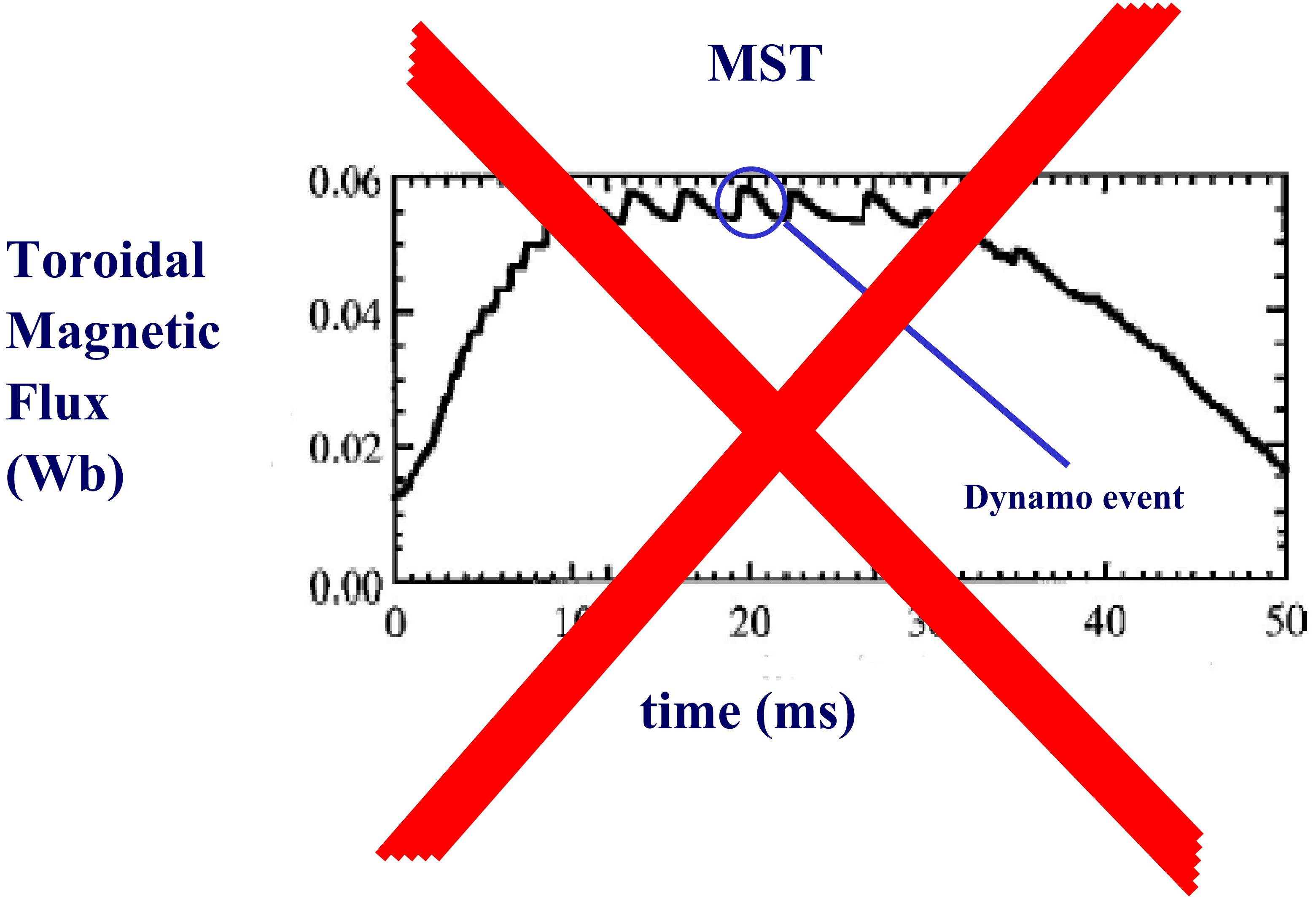
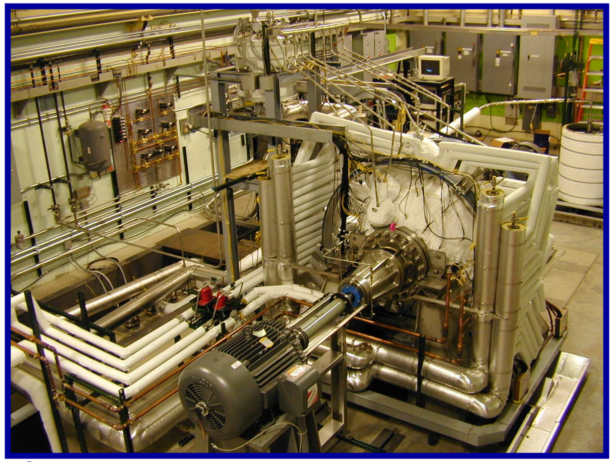
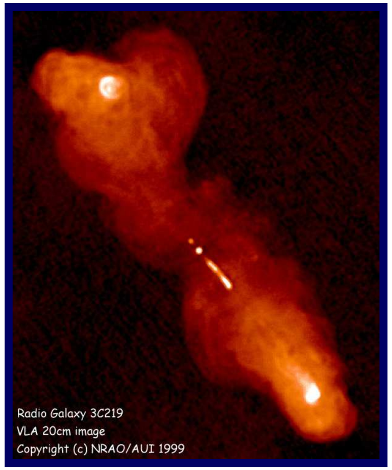

MST
Toroidal
Magnetic
Flux
(Wb)
0.06
0.04
0.02
0.00
0
10
20
30
40
50
Dynamo event
time (ms)

Observations

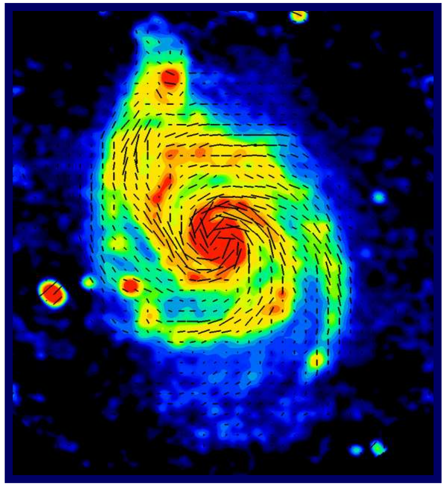
- **Magnetic fields are ubiquitous**
- **Dynamo action invoked to explain origin and maintenance (WMAP ➔ hardly any field at recombination)**
- **Often** $|\langle \mathbf{B} \rangle|^2 \approx U^2$



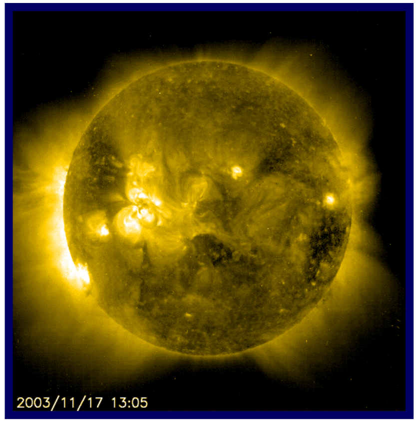
2 m



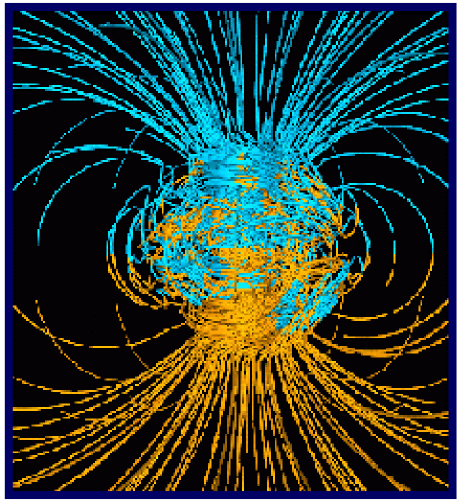
300 Kp



10^{20} m (3 Kp)



700,000 Km



6400 Km

Historical considerations

- **1919** — **Dynamo action introduced (Larmor)**
- **30's – 50's** — **Anti-dynamo theorems (Cowling; Zel'dovich)**
- **50's - 60's** — **Averaging is introduced → Formulation of Mean Field Electrodynamics (Parker 1955; Braginskii 1964; Steenbeck , Krause & Radler 1966)**
- **90's – now** — **Large scale computing. MHD equations can be solved directly (everybody with a big computer)**

Mean Field Theory

- **Evolution equations for the Mean field (homogeneous, isotropic case)**

$$\partial_t \langle \mathbf{B} \rangle = \alpha \nabla \times \langle \mathbf{B} \rangle + \beta \nabla^2 \langle \mathbf{B} \rangle$$

- **Transport coefficients determined by velocity and *Rm***
  - **α mean induction—requires lack of reflectional symmetry (helicity)**
  - **β turbulent diffusivity**

- **Many assumption needed**
  - **Linear relation between $\langle \mathbf{u} \times \mathbf{b} \rangle$ and $\langle \mathbf{B} \rangle$**
  - **Separations of scales**

- **FOS (quasi-linear) approximation**
  - **Short correlation time**
  - $Rm << 1$

- **Assumed that fluctuations not self-excited**

Troubles

In order for MFT to work:

- **fluctuations must be controlled by smoothing procedure (averages)**
- **system must be strongly irreversible**

When R_m << 1 irreversibility provided by diffusion

When R_m >> 1 , problems arise:

- **development of long memory → loss of irreversibility**
- **unbounded growth of fluctuations**

Two examples

- **Exactly solvable kinematic model (Kazantsev-Kraichnan)**
  - **lots of assumptions**
  - **can be treated analytically**

- **Nonlinear rotating convection**
  - **fewer assumptions**
  - **solved numerically**

Solvable models

- Model for random passive advection (Kazentsev 1968; Kraichnan 1968)
- Velocity: zero mean, stationary, homogeneous, isotropic, incompressible, Gaussian and delta-correlated in time
- Exact evolution equation for magnetic field correlator

$$\left\langle u_i(\mathbf{x},t)u_j(\mathbf{x}',t')\right\rangle = \kappa_{ij}(|\mathbf{x}-\mathbf{x}'|)\delta(t-t')$$

$$\kappa_{ij}(x) = \kappa_N(x)\left(\delta_{ij} - \frac{x_i x_j}{x^2}\right) + \kappa_L(x)\frac{x_i x_j}{x^2} + g\varepsilon_{ijk}x_k$$

$$\nabla\cdot\mathbf{u}=0 \quad \Rightarrow \quad \kappa_N = \kappa_L + (x\,\kappa_L')/2$$

Input: velocity correlator

$$\kappa_L,\ g$$

$$\left\langle B_i(\mathbf{x},t)B_j(\mathbf{x}',t)\right\rangle = H_{ij}(|\mathbf{x}-\mathbf{x}'|)$$

$$H_{ij}(x) = M_N(x)\left(\delta_{ij} - \frac{x_i x_j}{x^2}\right) + M_L(x)\frac{x_i x_j}{x^2} + K\varepsilon_{ijk}x_k$$

$$\nabla\cdot\mathbf{B}=0 \quad \Rightarrow \quad M_N = M_L + (x\,M_L')/2$$

Ouput: magnetic correlator

$$M_L,\ K$$

Solvable models

- **Exact evolution equation**
  - **Non-helical case (C= 0): Kazantsev 1968**
  - **Helical case: Vainshtein & Kichatinov 1986; Kim & Hughes 1997**
  - **Spectral version: Kulsrud & Anderson 1992; Berger& Rosner 1995**
  - **Symmetric form: Boldyrev, Cattaneo & Rosner 2005**

$$\begin{bmatrix} \partial_t W_2 \\ \partial_t W_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{x} E \frac{\sqrt{2}}{x} & \frac{\sqrt{2}}{x^2} C \frac{\partial}{\partial x} x^2 \\ -x^2 \frac{\partial}{\partial x} C \frac{\sqrt{2}}{x^2} & x^2 \frac{\partial}{\partial x} \frac{B}{x^2} \frac{\partial}{\partial x} x^2 \end{bmatrix} \begin{bmatrix} W_2 \\ W_3 \end{bmatrix}$$

$$M_L = \frac{\sqrt{2}}{x^2} W_2, \qquad K = -\frac{1}{\sqrt{2} x^4} \frac{\partial}{\partial x} \left(x^2 W_3 \right)$$

Operator in square brackets is self-adjoint

Solvable models

- **Dynamo growth rate from MFT:** $\lambda_o = \alpha^2 / 2\beta$
- **In this model MFT is <u>exact</u>; with** $\alpha = 2g_o, \quad \beta = 2\eta + \kappa_o \approx \kappa_o$
- **Dynamo growth rate of mean field:** $\lambda_o = g_o^2 / \kappa_o \quad \approx u / \ell$
- **Dynamo growth rate of fluctuations:** $\lambda \approx u(\ell_\eta) / \ell_\eta > u / \ell$
- **Large scale asymptotics of corresponding eigenfunction**

$$M_\lambda, K_\lambda \propto \frac{1}{x} \exp(\lambda t - \kappa_\lambda x) \times \text{oscillatory terms}$$

$$\kappa_\lambda = \left[(\lambda - \lambda_0) / \kappa_o\right]^{1/2}$$

Conclusion:

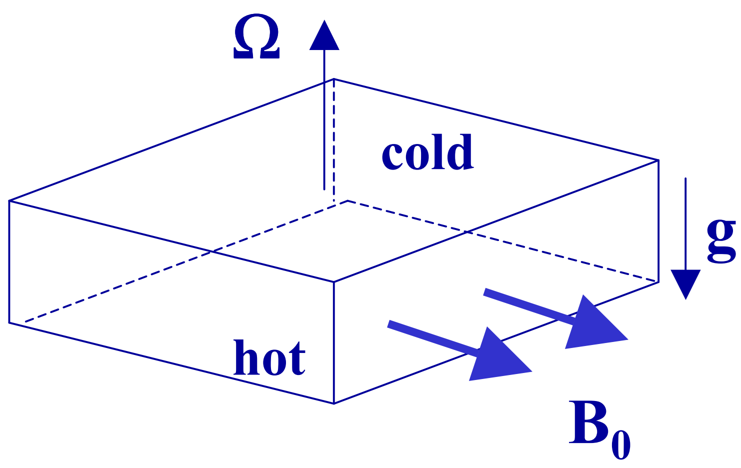
- **For a fixed time *t*, large enough spatial scales exist such that averages of the fluctuations are negligible.**
  - **on these scales the evolution of the average field is described by MFT**

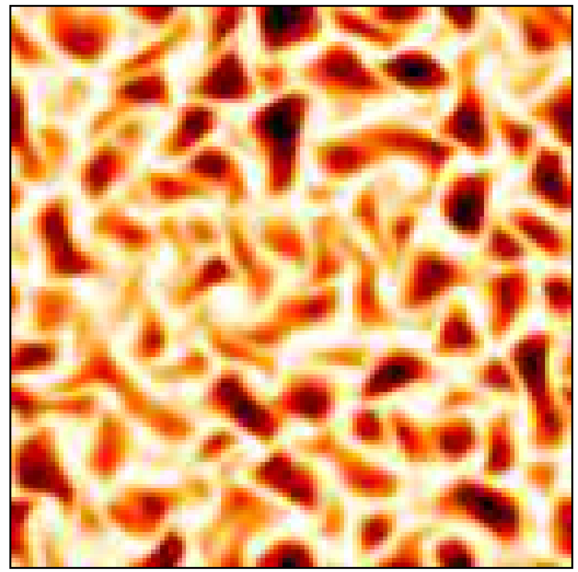
However

- **For any spatial scale *x*, contributions from mean field to the correlator at those scales quickly becomes subdominant**

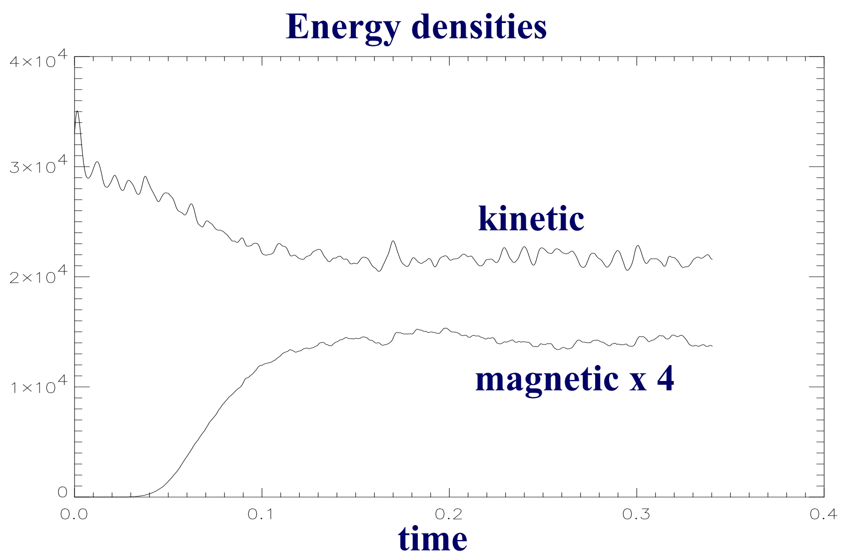
Fluctuations eventually take over on any scale

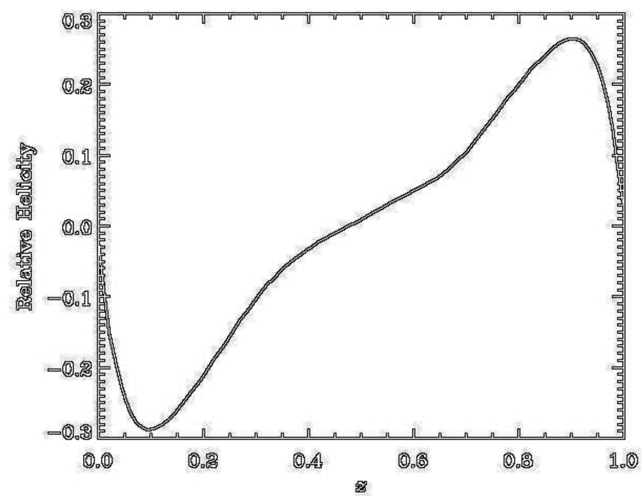
Convectively driven dynamos with rotation





Cattaneo & Hughes

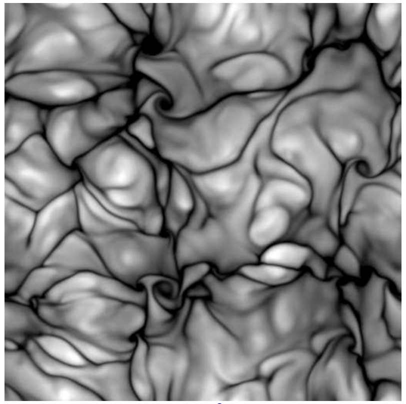




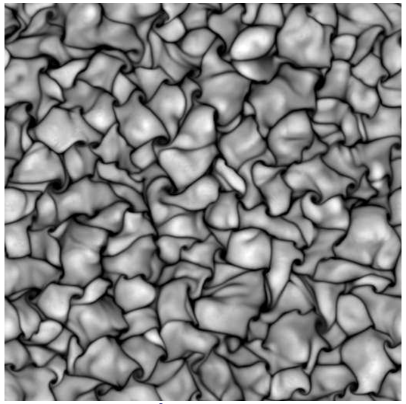
Rotating convection

- Turbulent convection with near-unit Rossby number
- System has strong small-scale fluctuations
- No evidence for mean field generation

Cattaneo & Hughes

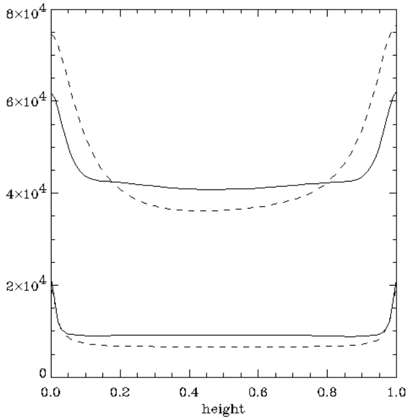


Non rotating

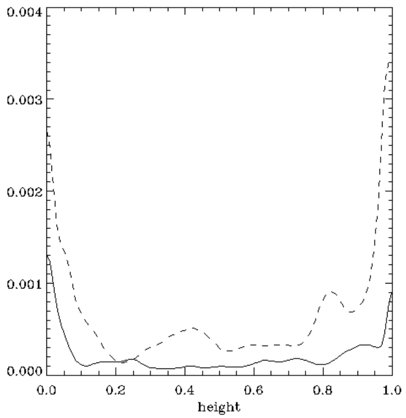


Rotating

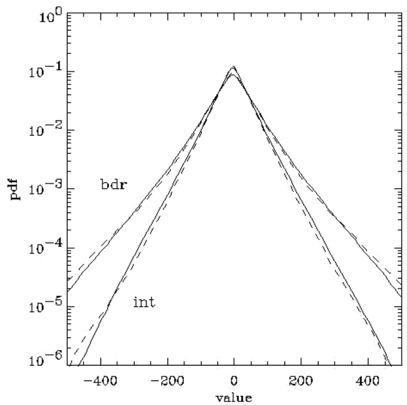
Energy: hor. average



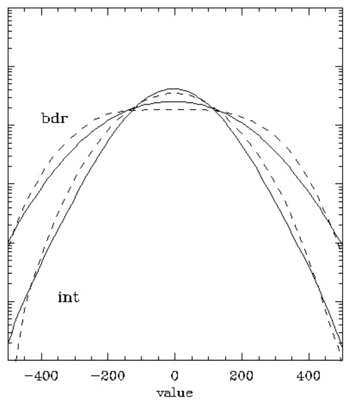
Energy ratio



Magnetic field



Velocity



What is going on?

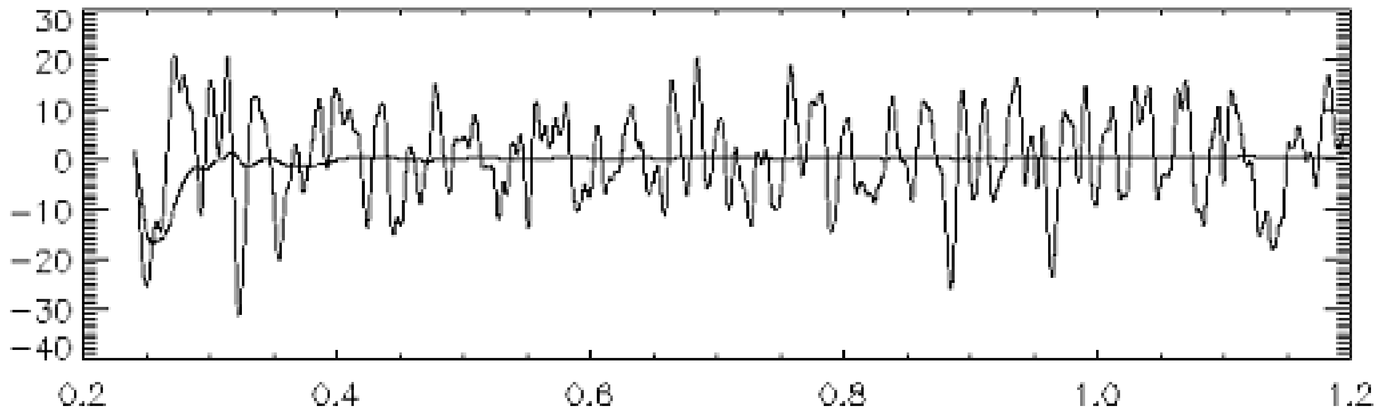
- System has helicity, yet no mean field
- Consider two possibilities:
  - Nonlinear saturation of turbulent α-effect (Cattaneo & Vainshtein 1991; Kulsrud & Anderson 1992; Gruzinov & Diamond 1994)
  - α-effect is "collisional" and not turbulent

Averages and α-effect

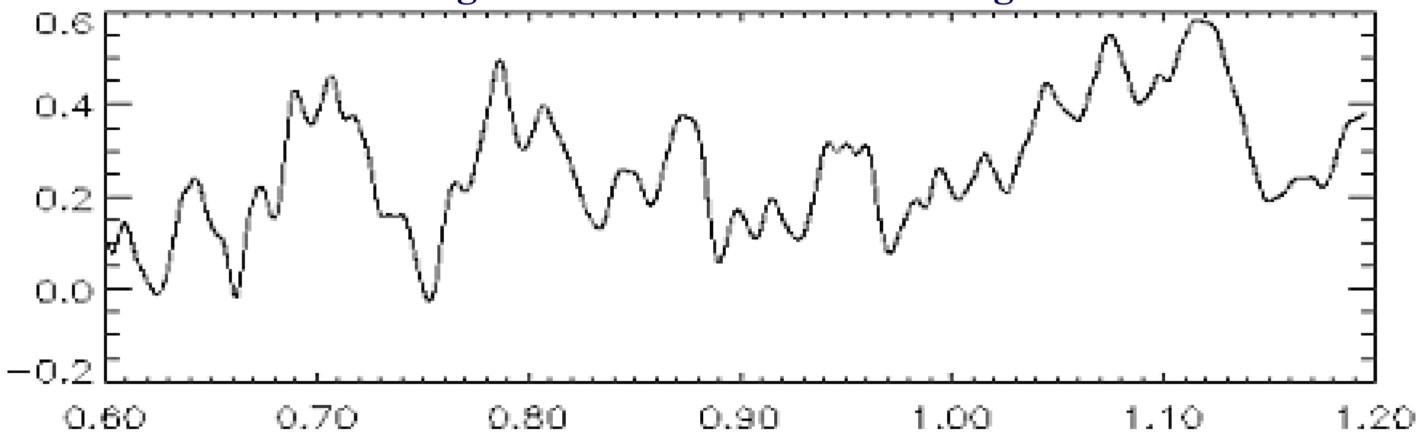
Introduce (external) uniform mean field. Compute below dynamo threshold.

Calculate average emf. Extremely slow convergence.

emf: volume average

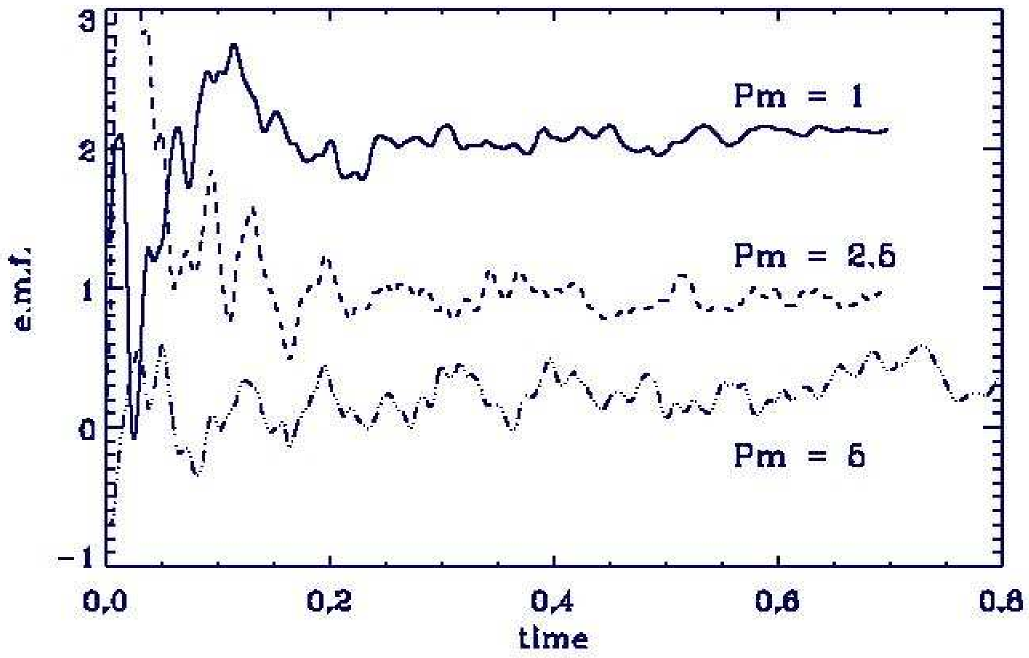


emf: volume average and cumulative time average



Cattaneo & Hughes

Convectively driven dynamos



The α-effect here is inversely proportional to *Pm* (i.e. proportional to η).

It is therefore *not* turbulent but collisional

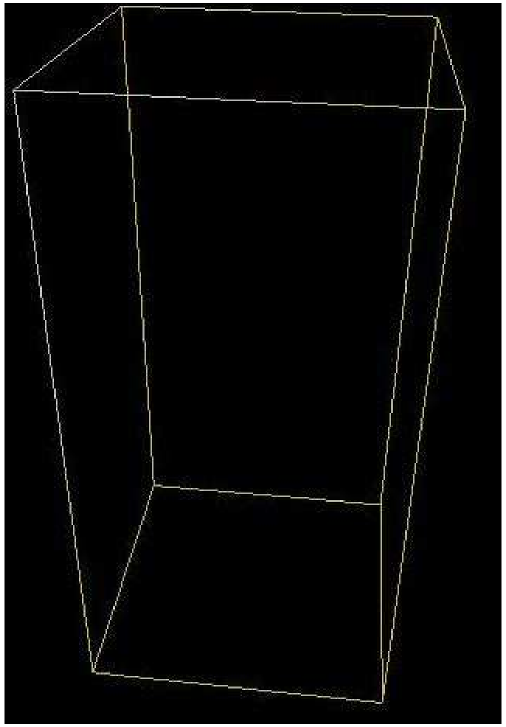
- **Convergence requires huge sample size.**
  - **Divergence with decreasing η?**
- **Small-scale dynamo is turbulent but α-effects is not!!**

… and now

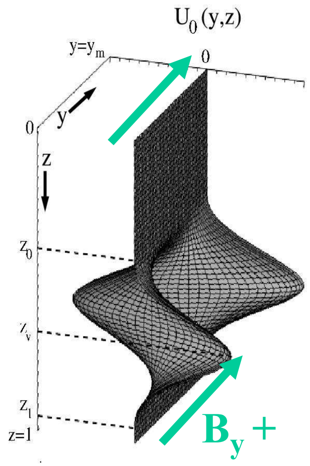
for something completely different

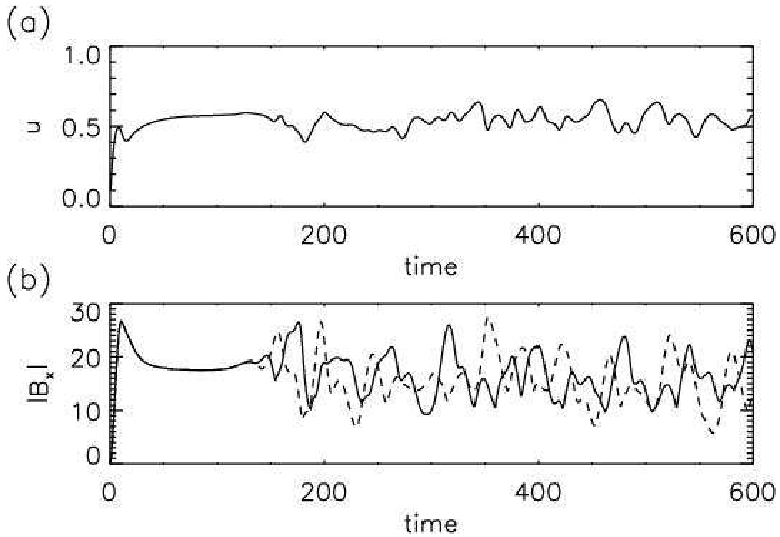
Something completely different

- Interaction between localized velocity shear and weak background poloidal field generates intense toroidal magnetic structures
- Magnetic buoyancy leads to complex spatio-temporal beahviour



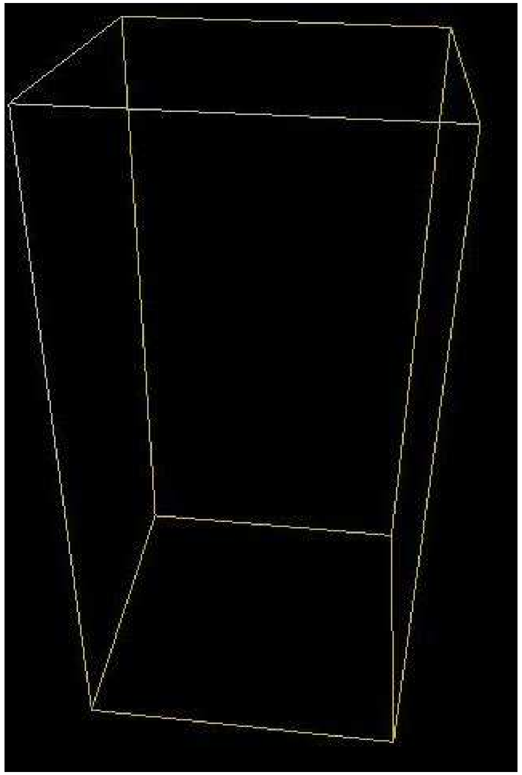
Cline, Brummell & Cattaneo

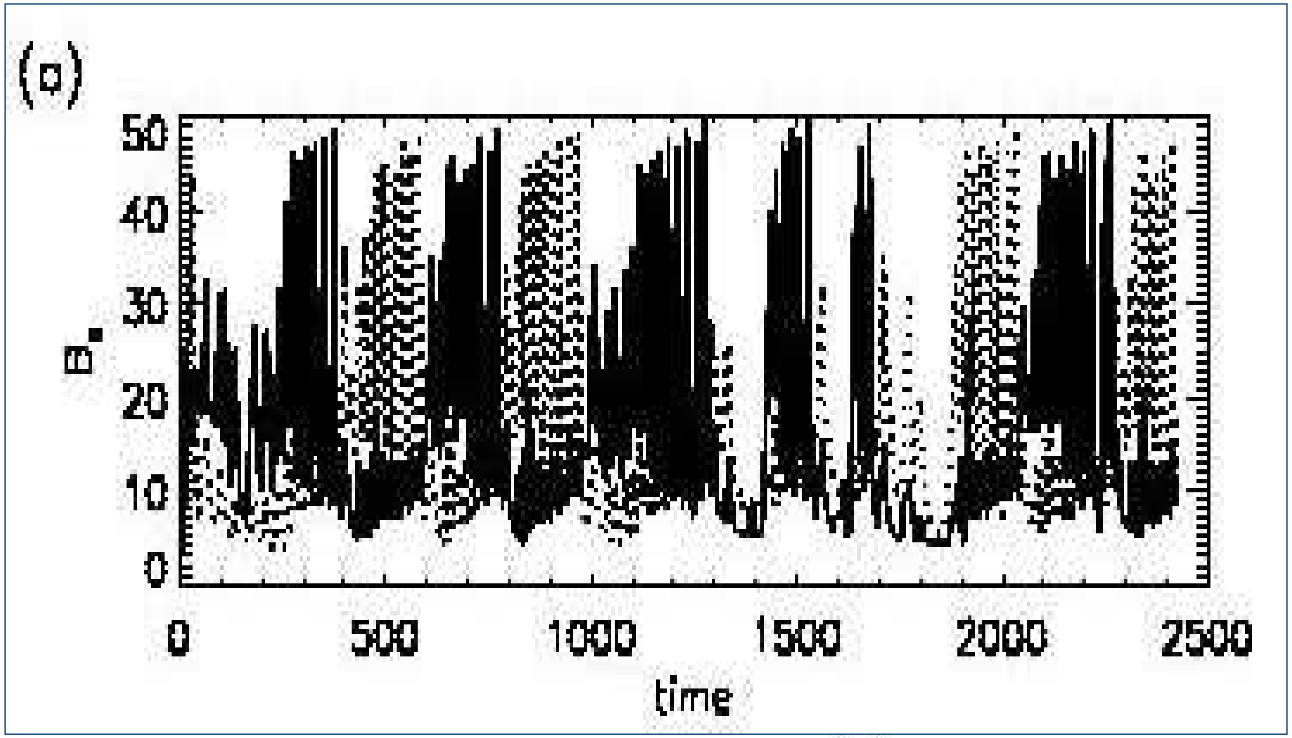




Shear driven dynamo

- Slight modification of shear profile leads to sustained dynamo action
- System exhibits cyclic behaviour, reversals, even episodes of reduced activity





Conclusion

- **In turbulent dynamos behaviour dominated by fluctuations**
- **Averages not well defined for realistic sample sizes**

- **In realistic cases large-scale field generation possibly driven by non-universal mechanisms**
  - **Shear**
  - **Large scale motions**
  - **Boundary effects**
  - **Etc. etc.**

The end

Kazantsev model

$$\langle u_i(\mathbf{x},t)u_j(\mathbf{x}',t')\rangle = K_{ij}(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Velocity correlation function

$$K_{ij}(\mathbf{r}) = K_N(r)\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) + K_L(r)\frac{r_i r_j}{r^2}$$

Isotropy + reflectional symmetry

$$\nabla\cdot\mathbf{u} = 0 \quad \Rightarrow \quad K_N = K_L + (rK_L')/2$$

Solenoidality

$$\langle B_i(\mathbf{x},t)B_j(\mathbf{x}',t)\rangle = H_{ij}(\mathbf{x}-\mathbf{x}',t)$$

Magnetic correlation function

$$\partial_t H_L = KH_L'' + \left(\frac{4}{r} + K'\right)H_L' + \left(\frac{4}{r}K' + K''\right)H_L$$

Kazantsev equation

where $K(r) = 2\eta + K_L(0) - K_L(r)$

Renormalized correlator

$$H_L = \psi(r,t)r^{-2}K(r)^{-1/2}$$

Funny transformation

$$V(r) = \frac{2}{r^2}K(r) - \frac{1}{2}K''(r) - \frac{2}{r}K'(r) - \frac{(K'(r))^2}{4K(r)}$$

Funny potential

$$\partial_t\psi = K(r)\psi'' - V(r)\psi$$

Sort of Schrödinger equation