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GLOBAL TRANSPORT OF ENERGETIC PARTICLES IN PRESENCE OF MULTIPLE UNSTABLE MODES

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INTRODUCTION

- **Species of interest:** Alpha particles in burning plasmas
  NBI-produced fast ions
  ICRH-produced fast ions
  Others…

- **Initial fear:** Alfvén eigenmodes (TAEs) with global spatial structure may cause global losses of fast particles

- **Second thought:** Only resonant particles can be affected by low-amplitude modes

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PHYSICS INGREDIENTS

- Resonant wave-particle interaction
- Continuous injection of energetic particles
- Collisional relaxation of the particle distribution
- Discrete spectrum of unstable waves
- Background damping of linear modes

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TRANSPORT MECHANISMS

- **Neoclassical:** Large excursions of resonant particles (banana orbits) + collisional mixing
- **Convective:** Locking in resonance + collisional drag
  BGK modes with frequency chirping
- **Quasilinear :** Phase-space diffusion over a set of overlapped resonances

Important Issue: Individual resonances are narrow. How can they affect every particle in phase space?

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NEAR-THRESHOLD NONLINEAR REGIMES

- **Why study the nonlinear response near the threshold?**
  - Typically, macroscopic plasma parameters evolve slowly compared to the instability growth time scale
  - Perturbation technique is adequate near the instability threshold

- **Single-mode case:**
  - Identification of the soft and hard nonlinear regimes is crucial to determining whether an unstable system will remain at marginal stability
  - Bifurcations at single-mode saturation can be analyzed
  - The formation of long-lived coherent nonlinear structure is possible

- **Multi-mode case:**
  - Multi-mode scenarios with marginal stability (and possibly transport barriers) are interesting
  - Resonance overlap can cause global diffusion

WAVE-PARTICLE LAGRANGIAN

- **Perturbed guiding center Lagrangian:**

$$L = \;_{\text{particles}}\; P\;\dot{}\; + P\;\dot{}\;\; H\left(P\;;P\;;\;\right) + \;_{\text{modes}}\;\dot{}\;A^2$$

$$+2\,\mathrm{Re}\;_{\text{particles}}\;_{\text{modes}}\;_{\text{sidebands }l}\; AV_l\left(P\;;P\;;\;\right)\exp(\;i\quad i\;\;t + in\;\; + il\;\;)$$

- **Dynamical variables:**
  - $P\;,\quad,P\;,$ **are the action-angle variables for the particle unperturbed motion**
  - A **is the mode amplitude**
  - α **is the mode phase**

- **Matrix element $V_l\left(P_\vartheta;P_\varphi;\mu\right)$ is a given function, determined by the linear mode structure**
- **Mode energy: $W = \;\;A^2$**

PARTICLE LAGRANGIAN

- **Guiding center Lagrangian (Littlejohn)**

$$L = \frac{1}{2}\frac{e}{c}B_0 r^2 \dot{\ } + \dot{\ } \quad M u_{\|}(R + r\cos \quad) \quad \frac{e}{c}B_0 \int_0^r \frac{r}{q}dr \quad \frac{1}{2}M u_{\|}^2 \quad B_0(1 \quad \frac{r}{R}\cos \quad) \quad e \quad \dot{\ } + u_{\|}\left(\mathbf{b}_0 \quad - \right)$$

- **Dynamical variables:** $r, \quad , \quad , u_{\|}$

- **For low-frequency perturbations ($\ll \quad _*$), change in particle energy is negligible compared to the change in toroidal angular momentum:**

$$u_{\|} = const; \qquad = \ _0 + u_{\|} t / R$$

- **Reduced guiding center Lagrangian:**

$$L = \frac{e}{c}B_0 - \frac{r^2}{2} \ \dot{\ } + \quad u_{\|}^2 + \frac{B_0}{M} \quad \frac{r}{R \quad _B}\cos \qquad \frac{u_{\|}}{R} \int_0^r \frac{r}{q}dr \qquad e \quad \dot{\ } + u_{\|}\left(\mathbf{b}_0 \qquad \right)$$

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REDUCTION TO BUMP-ON-TAIL PROBLEM

- **Action-angle variables for unperturbed motion:**

$$r = \quad + \quad \cos$$

$$= \qquad 2 \text{—} \sin \qquad\qquad u_{\parallel}^{2} + \frac{B_0}{M} \frac{q(\quad)}{u_{\parallel} - \ _{B}}$$

- **Transformed Lagrangian:**

$$L = \frac{e}{c} B_0 \quad \frac{\ ^{2}}{2} \dot{\ } \quad \frac{u_{\parallel}}{R} \ _{0} \frac{r}{q} dr \quad e \quad \dot{\ } + u_{\parallel} \left(\mathbf{b}_0 \quad - \right)$$

- **Resonance condition:** $\omega - n \dfrac{u_{\parallel}}{R} - l \dfrac{u_{\parallel}}{Rq(\rho)} = 0$

- **Lagrangian for 1-D electrostatic bump-on-tail problem:**

$$L = \ _{\text{particles}} \quad p\dot{x} \quad \frac{p^2}{2m} + \ _{\text{modes}} \quad \dot{\ } _{k} A_k^{\ 2} + 2\,\text{Re} \ _{\text{particles}} \ _{\text{modes}} \frac{e}{k} \sqrt{2 \quad _{k}} A_k \exp(\quad i \quad _{k} \quad i \quad _{k} t + ikx)$$

MULTI-MODE FORMALISM

- **Electric field representation**

$$E(x;t) = \frac{1}{2} \sum_{l,>0} \left\{ E_l(t) \exp \; i\left(k_l x \quad {}_p t\right) + E_l^*(t) \exp \quad i\left(k_l x \quad {}_p t\right) \right\}$$

- **Distribution function**

$$f(x;v;t) = f_0(v;t) + \sum_{l,n>0} \left\{ f_{l,n}(v;t) \exp \; in\left(k_l x \quad {}_p t\right) + f_{l,n}^*(v;t) \exp \quad in\left(k_l x \quad {}_p t\right) \right\}$$

- **Wave equation :**

$$\frac{dE_l}{dt} = \quad {}_d E_l \quad 4 \quad e \quad f_{l,1}(v;t) v dv$$

- **Kinetic equation:**

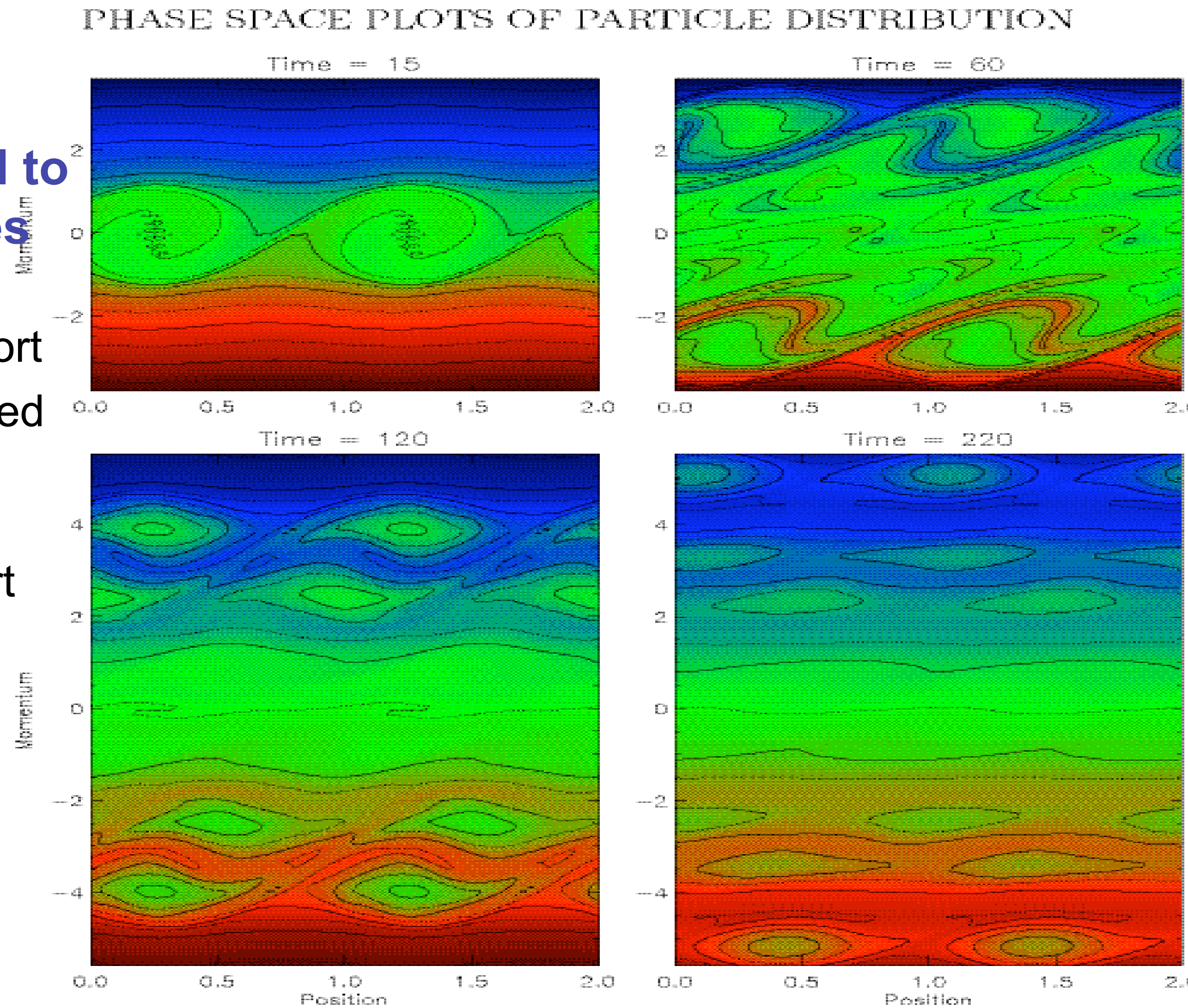
$$\frac{\partial f_0}{\partial t} + \frac{e}{2m} \sum_l \left(E_l \frac{\partial f_{l,1}^*}{\partial v} + E_l^* \frac{\partial f_{l,1}}{\partial v} \right) = St\left(f_0\right)$$

$$\frac{\partial f_{l,n}}{\partial t} + in(k_l v - \omega_p) f_{l,n} + \frac{e}{2m} E_l \frac{\partial f_{l,n-1}}{\partial v} + \frac{e}{2m} E_l^* \frac{\partial f_{l,n+1}}{\partial v} = St\left(f_{l,n}\right)$$

CONVECTIVE AND DIFFUSIVE TRANSPORT

CONVECTIVE TRANSPORT IN PHASE SPACE

- **Single-mode instability can lead to coherent structures**
  - Can cause convective transport
  - Single mode: limited extent
  - Multiple modes: extended transport ("avalanche")



N. Petviashvili, et al., Phys. Lett. A (1998)

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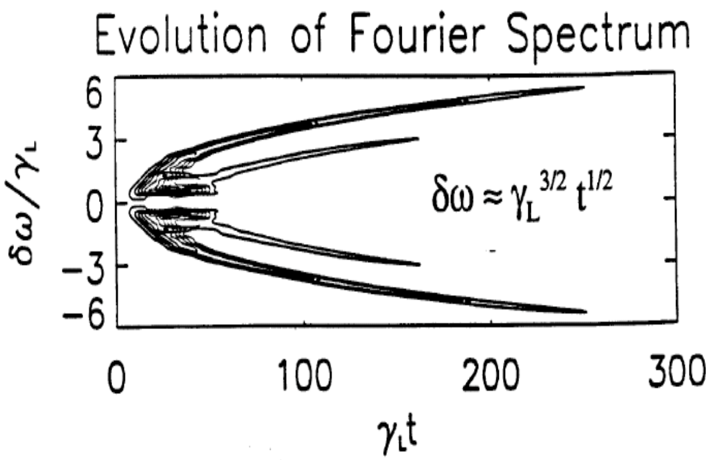
DETERMINATION OF INTERNAL FIELDS BY FREQUENCY SWEEPING OBSERVATION

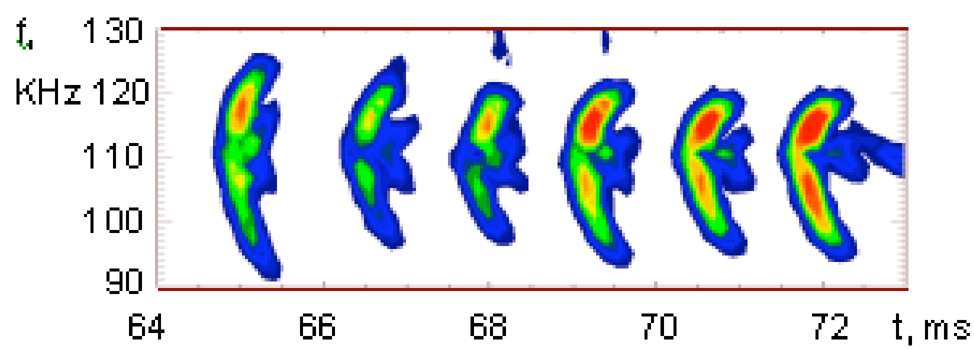
IFS numerical simulation
Petviashvili [Phys. Lett. (1998)]



L linear growth without dissipation; for spontaneous hole formation; L d.

b =(ekE/m)$^{1/2}$ 0.5 L

TAE modes in MAST



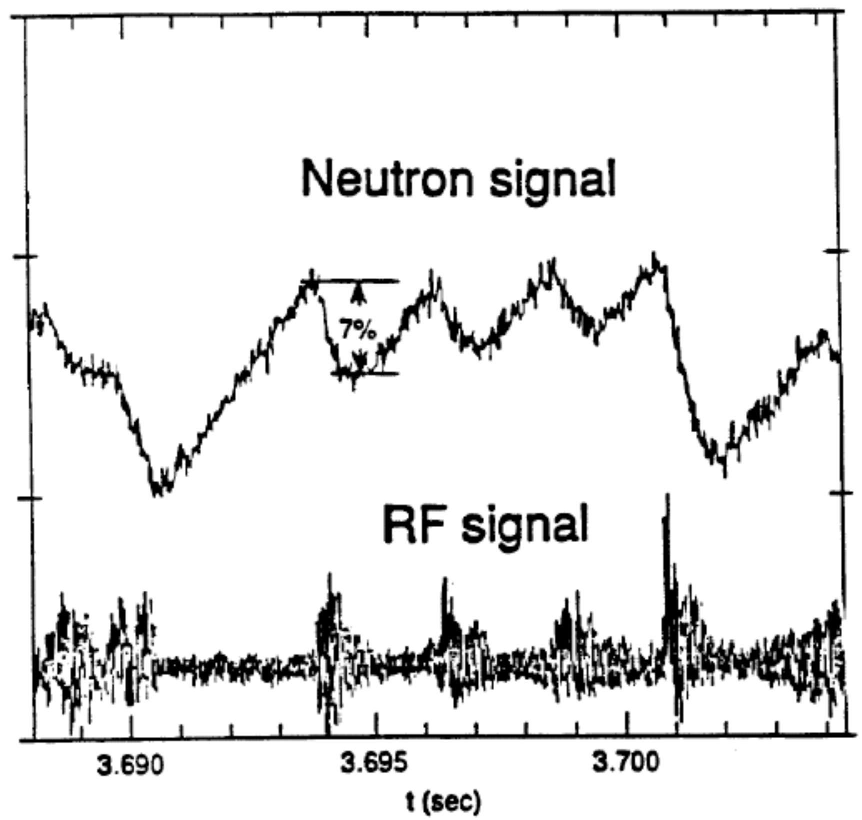
(Culham Laboratory, U. K. courtesy of Mikhail Gryaznevich)

With geometry and energetic particle distribution known internal perturbed fields can be inferred

S. Pinches et al., Plasma Phys. and Cont. Fusion (2004)
H. Berk et al., IAEA (2004) TH/5-2Ra

INTERMITTENT LOSSES OF FAST IONS

- **Experiments show both benign and deleterious effects**
- **Rapid losses in early TAE experiments:**
  - Wong (TFTR)
  - Heidbrink (DIII-D)
- **Simulation of rapid loss:**
  - Todo, Berk, and Breizman, Phys. Plasmas (2003)
  - Multiple modes (n=1, 2, 3)



K.L. Wong et al., PRL (1991)

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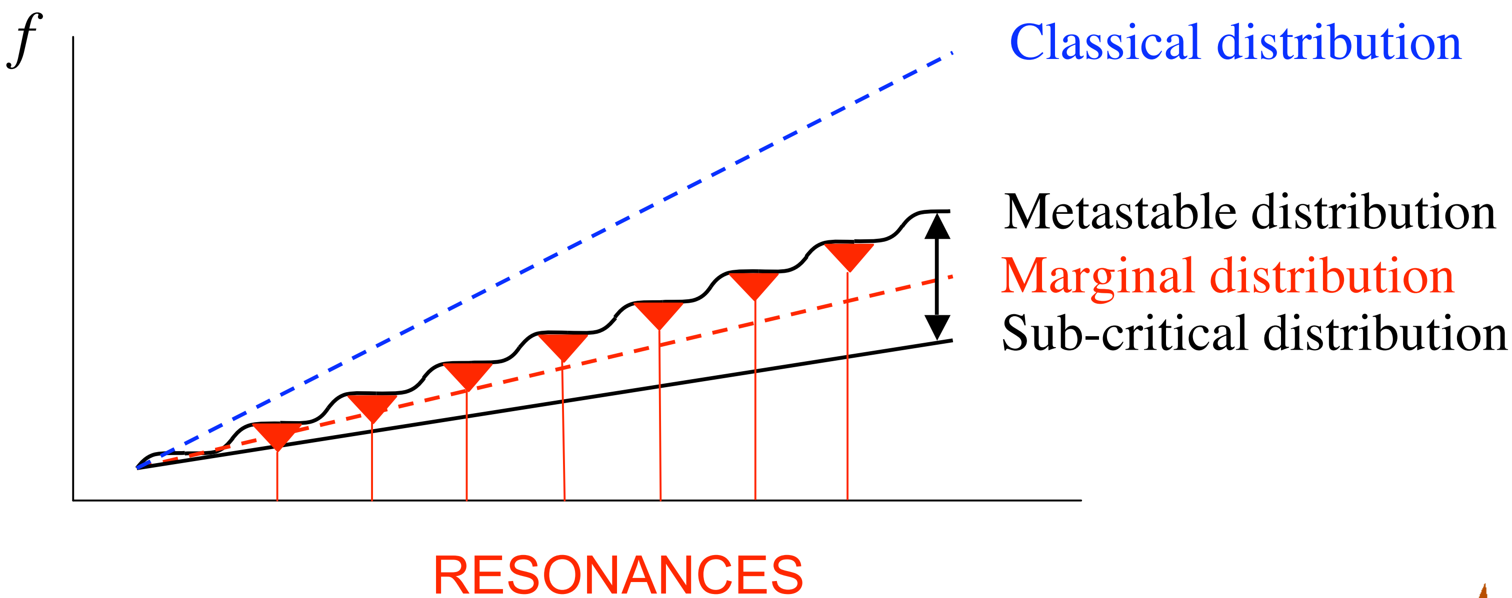
INTERMITTENT QUASILINEAR DIFFUSION

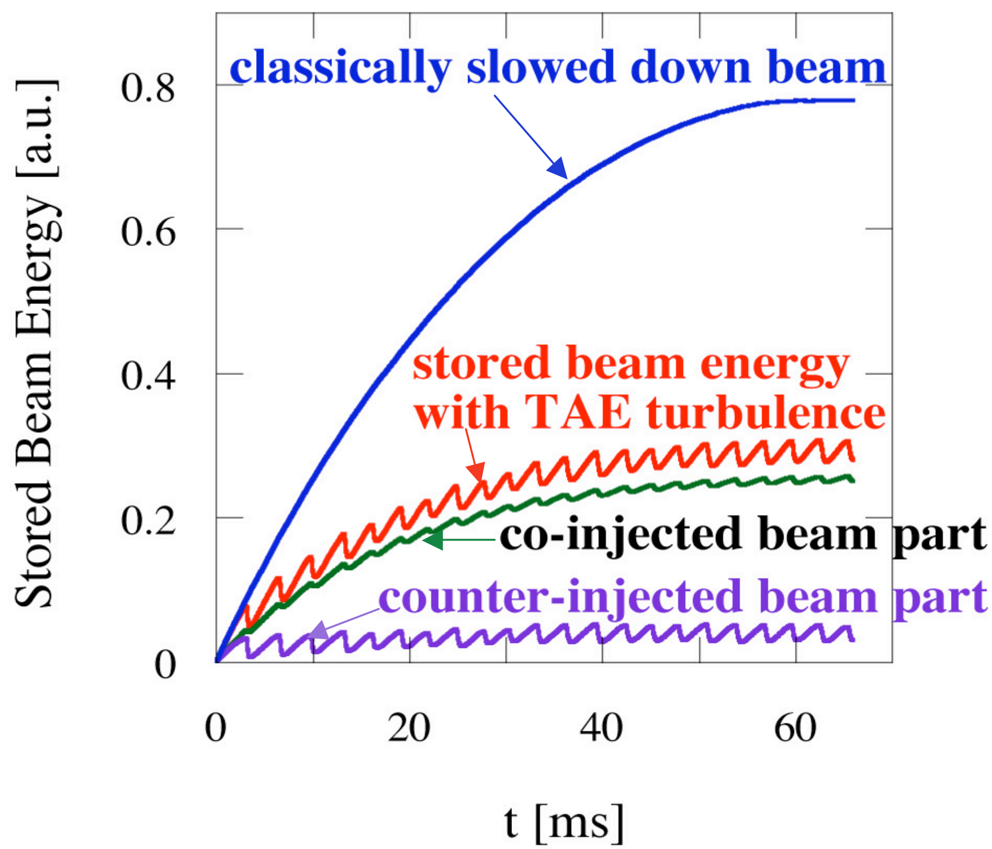
A weak source (with insufficient power to overlap the resonances) is unable to maintain steady quasilinear diffusion

Bursts occur near the marginally stable case



SIMULATION OF INTERMITTENT LOSSES

- **Simulations reproduce NBI beam ion loss in TFTR**
- **Synchronized TAE bursts:**
  - At 2.9 ms time intervals (cf. 2.2 ms in experiments)
  - Beam energy 10% modulation per burst (cf. 7% in experiment)
- **TAE activity reduces stored beam energy wrt to that for classical slowing-down ions**
  - 40% for co-injected ions
  - Larger reduction (by 88%) for counter-injected beam ions (due to orbit position wrt limiter)



Y. Todo et al., Phys Plasmas (2003)

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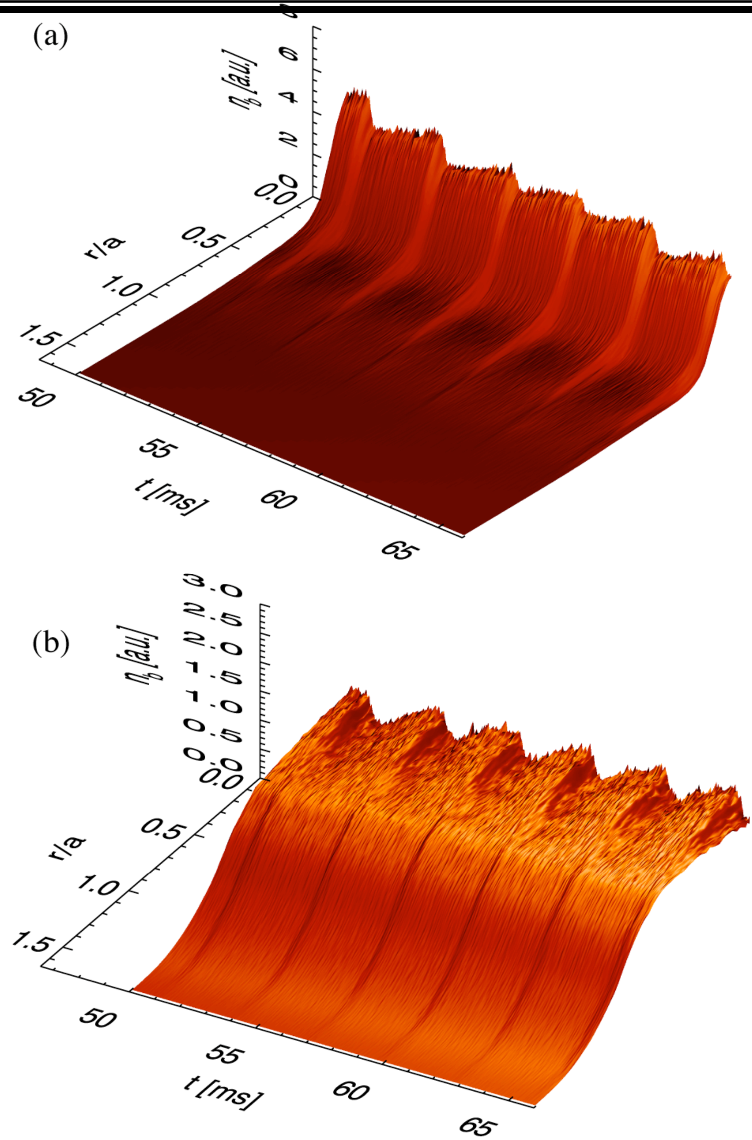
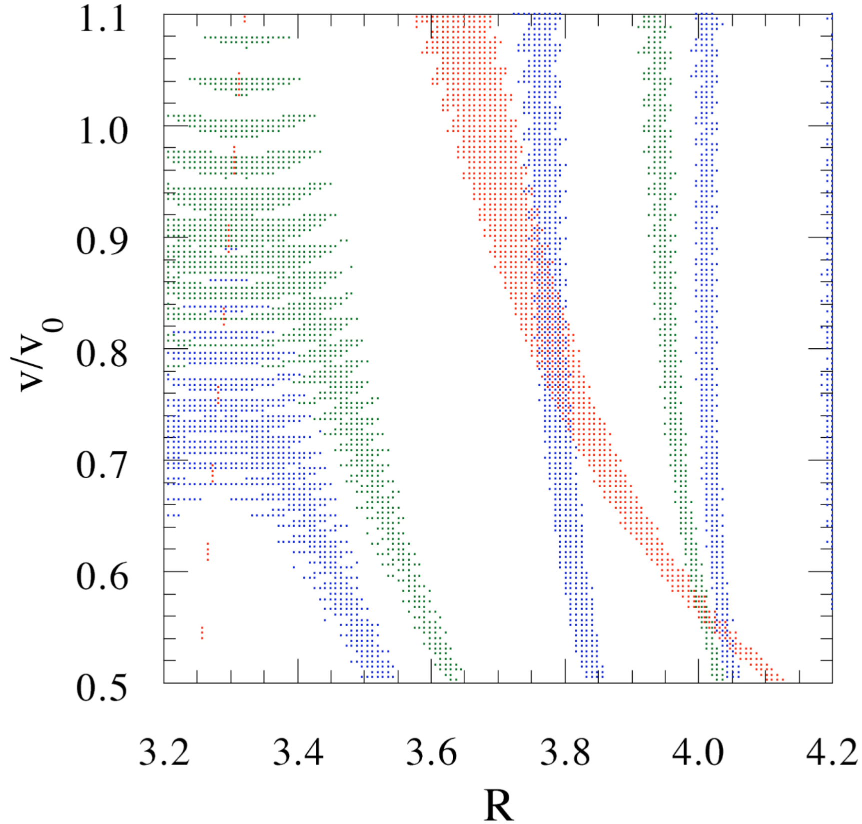
TEMPORAL RELAXATION OF RADIAL PROFILE



- **Counter-injected beam ions:**
  - Confined only near plasma axis
- **Co-injected beam ions:**
  - Well confined
  - Pressure gradient periodically collapses at criticality
  - Large pressure gradient is sustained toward plasma edge

Y. Todo et al., PoP (2003)

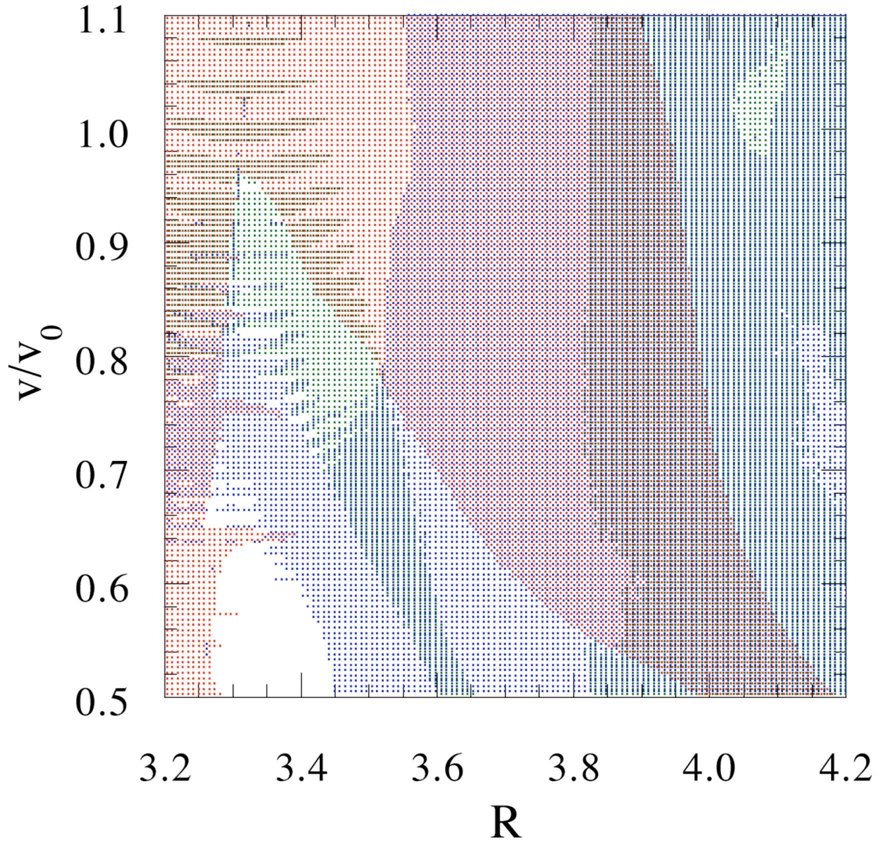
PHASE SPACE RESONANCES



For low amplitude modes:

B/B = 1.5 X 10^{-4}

n=1, n=2, n=3



At mode saturation:

B/B = 1.5 X 10^{-2}

n=1, n=2, n=3

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ISSUES IN MODELING DIFFUSIVE TRANSPORT

- **Reconciliation of mode saturation levels with experimental data**
  - Simulations (Y. Todo) reproduce experimental behavior for repetition rate and accumulation level
  - However, saturation amplitude appears to be larger than exp'tal measurements
- **Edge effects in fast particle transport**
  - Sufficient to suppress modes locally near the edge
  - Need better description of edge plasma parameters
- **Transport barriers for marginally stable profiles**
- **Resonance overlap in 3D**
  - Different behavior: (1) strong beam anisotropy, (3) fewer resonances

FISHBONES

(example of hard nonlinear response)

FISHBONE ONSET

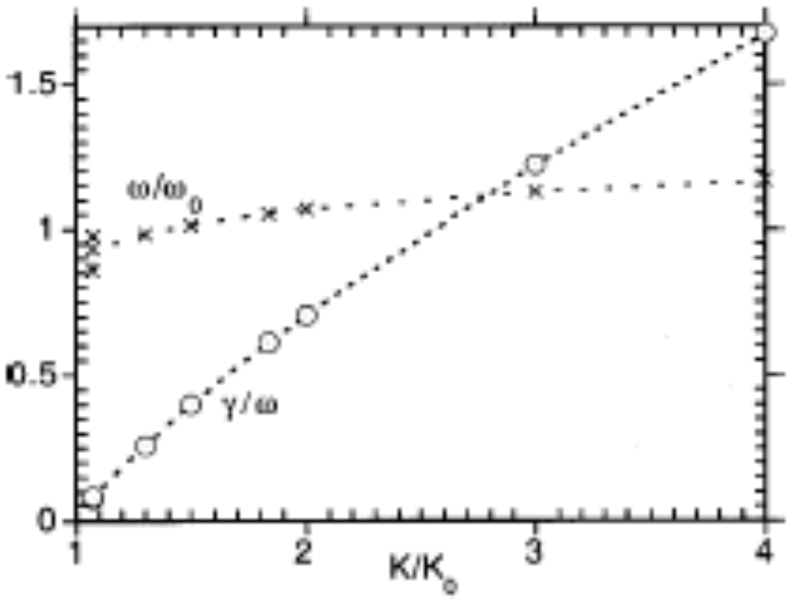
- **Linear responses from kinetic (wave-particle) and fluid (continuum) resonances are in balance at instability threshold; however, their nonlinear responses differ significantly.**

- **Questions:**
  - Which resonance produces the dominant nonlinear response?
  - Is this resonance stabilizing or destabilizing?

- **Approach:**
  - Analyze the nonlinear regime near the instability threshold
  - Perform hybrid kinetic-MHD simulations to study strong nonlinearity

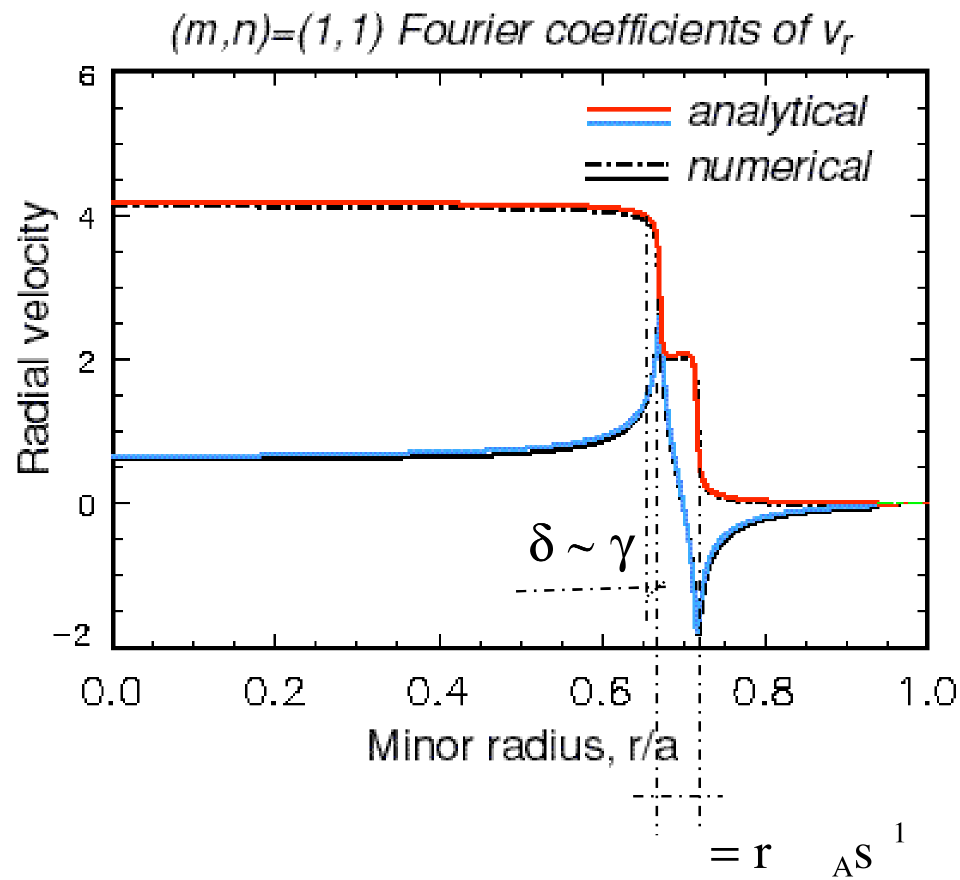
LINEAR NEAR-THRESHOLD MODE

Frequency and growth rate



Energetic ion drive

Double resonance layer at =± (r)



A. Odblom et al., Phys Plasmas (2002)

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EARLY NONLINEAR DYNAMICS

- **Weak MHD nonlinearity of the q = 1 surface destabilizes fishbone perturbations**
  - Near threshold, fluid nonlinearity dominates over kinetic nonlinearity
  - As mode grows, q profile is flattened locally (near q=1) continuum damping reduced explosive growth is triggered

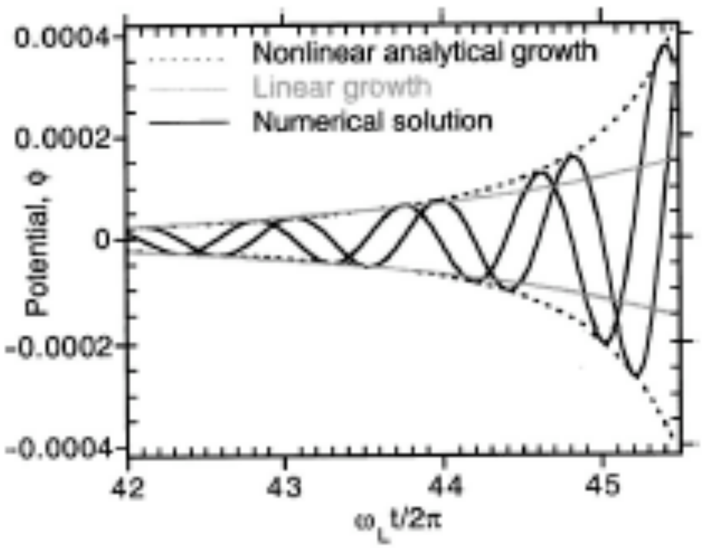

FIG. 4. The $m=1$ amplitude evolution in the numerical solution (solid black), and as predicted by the explosive analytical envelope (dashed), in contrast to a linear growth (solid gray), for a mode near the instability threshold, $\gamma/\omega\sim 8\%$, with $m=0-8$, $\eta=0=\nu$, $T/\tau_A=40$.

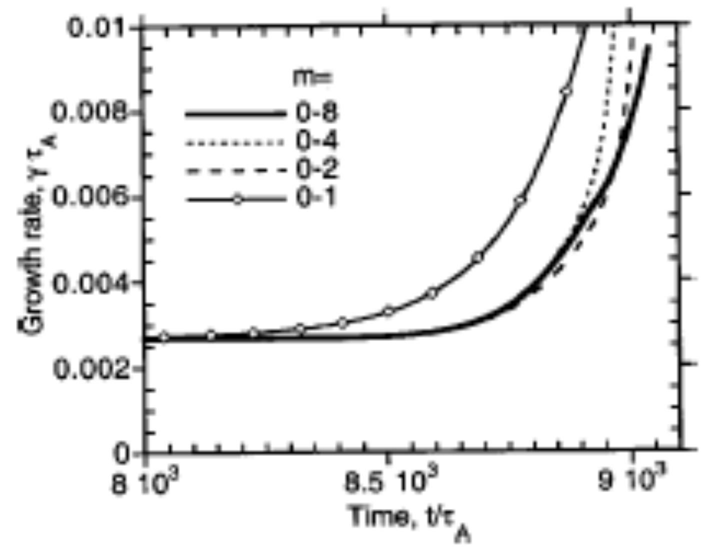

FIG. 5. The nonlinear growth rate evolution of the $m=1$ radial velocity amplitude, $\gamma=\partial \ln|V_r|/\partial t$, for a mode near the instability threshold, $\gamma/\omega \sim 8\%$, and its dependence on the numerical spectral resolution: $m=0-8$ (thick solid), $m=0-4$ (short dashed), $m=0-2$ (long dashed), $m=0-1$ (solid with markers). Here, $T/\tau_A=40$.

UNEXPLAINED FISHBONE FEATURES

- **Transition from explosive growth to slowly growing MHD structure (i.e., island near q=1 surface)**
- **Modification of fast particle distribution**
- **Mode saturation and decay**
- **Quantitative simulation of frequency sweeping**
  - Frequency change during explosive phase suggests mode will slow down and saturate
- **Burst repetition rate in presence of injection**
  - Need to include sources/sinks/collisions

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