Dynamics of transport barrier relaxations in tokamak edge plasmas

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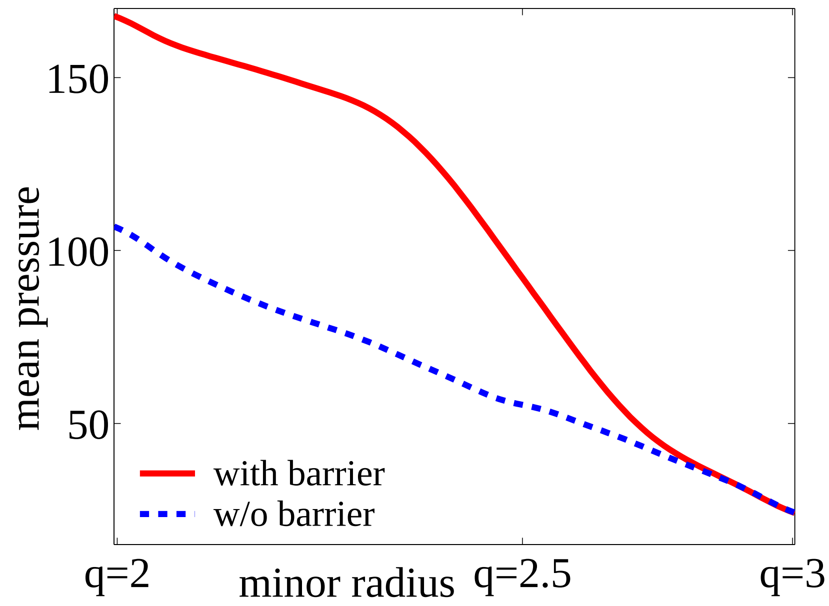
Introduction

- The operational regime of future fusion reactors is characterized by
  - an edge transport barrier,
  - relaxation oscillations of the barrier (Edge Localized Modes, ELMs).



- Explanations for relaxations are usually based on MHD instability,
  - analysis of linear stability properties, no dynamics.
- Most existing dynamical models are phenomenological,
  - not based on 1st principles, i.e. turbulence simulations.
- Frequency, crash time and energy release are central issues.

Outline

- Overview of existing reduced dynamical models for transport barrier relaxations.
- 3D fluid turbulence simulations.
- Subsequent reduced 1D model.
- Systematic reduction $\rightarrow$ 0D model.

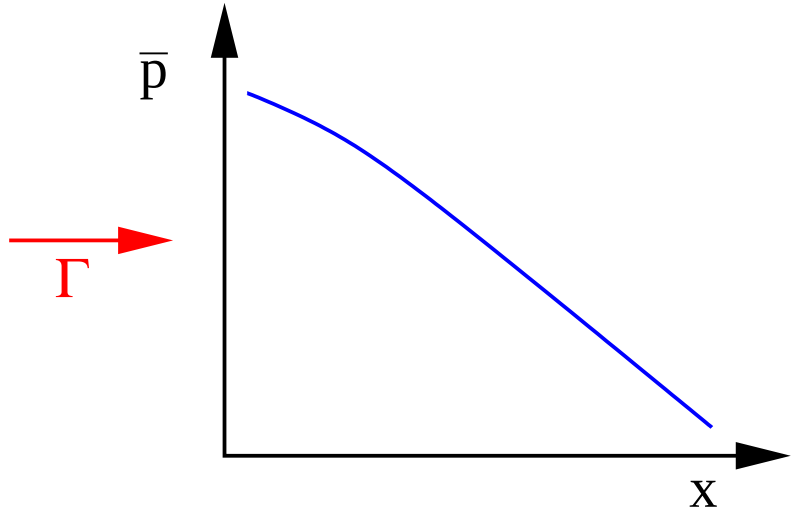
Reduced models for barrier dynamics: not straightforward

- simple model (1D):

  transport eqn. coupled to instability amplitude eqn. at plasma edge

$$\begin{aligned} \partial_t \bar{p} &= -\partial_x \left(\chi_0 \bar{\pi} + \chi_1 |\xi|^2 \bar{\pi} - \Gamma \right) \\ \partial_t \xi &= \gamma_0 (\bar{\pi} - \alpha_c) \xi + \nu_0 \partial_x^2 \xi \end{aligned}$$

$$\bar{\pi} = -\partial_x \bar{p}$$



$\bar{p}$: pressure profile, ξ: perturbation ampl.,

Γ: incoming energy flux, x: minor radius

- no oscillations, stable fixed point, robust property

Possible modifications to obtain oscillations or relaxations

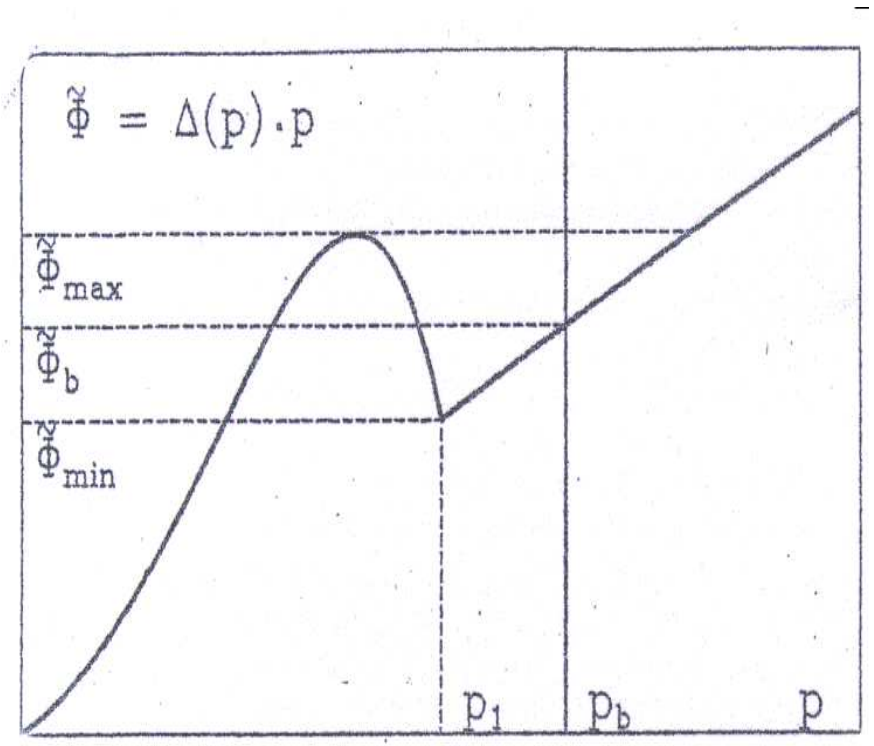
- Introduction of S–curve
  - for dependency of flux vs gradient (due to ExB shear flow),
  - in dynamical eqn. for perturbation amplitude (explosive instability),
  - in dynamical eqn. for ExB shear flow (multiple states: L–H).
- Introduction of characteristics of ideal MHD eigenmodes
  - vanishing growth rate below threshold,
  - radial shape of global modes.

S–curve for flux vs gradient produces relaxations

- Introduce ambient turbulent flux Γ_{turb} due to drift waves, etc.
- $\tilde{\Phi}(\overline{\pi}) = \Gamma_{\text{turb}}(\overline{\pi}) + \chi_0 \overline{\pi}$: S-curve due to turb. stabilization by ExB shear flow.

$$\begin{aligned} \partial_t \overline{p} &= -\partial_x \left[\tilde{\Phi}(\overline{\pi}) + \chi_1 |\xi|^2 \overline{\pi} - \Gamma \right] \\ \partial_t \xi &= \gamma_0 (\overline{\pi} - \alpha_c) \xi + \nu_0 \partial_x^2 \xi \end{aligned}$$

$$\overline{\pi} = -\partial_x \overline{p}$$



$p \equiv \overline{\pi}$

Lebedev, Diamond, PoP 95

- Relaxations, frequency ↗ with power.
- More sophisticated models available.

Explosive instability

- Account for non-linear terms in amplitude equation:
  first is destabilizing, second is stabilizing.

$$\begin{aligned} \partial_t \bar{p} &= -\partial_x \left(\chi_0 \bar{\pi} + \chi_1 |\xi|^2 \bar{\pi} - \Gamma \right) \\ \partial_t \xi &= \gamma_0 \left(\bar{\pi} - \alpha_c \right) \xi + \mu \xi^2 - \nu \xi^3 \end{aligned}$$

$$\bar{\pi} = -\partial_x \bar{p}$$

- Dynamics close to Van der Pol oscillations. Cowley, Wilson, PPCF 03

Multiple states for shear flow

- L–H transition: multiple states for ExB shear flow $\bar{u}(\bar{\pi})$,
  and: effective flux $\chi_{\text{eff}}(\bar{u})\,\bar{\pi}$ depends on shear flow.

$$
\begin{aligned}
\partial_t \bar{p} &= -\partial_x \left[\chi_{\text{eff}}(\bar{u})\,\bar{\pi} - \Gamma\right] \\
\partial_t \bar{u} &= -(\bar{\pi} - \alpha_c) - \mu_1 \bar{u}^3 + \mu_2 \bar{u} + \nu \partial_x^2 \bar{u}
\end{aligned}
$$

$$\bar{\pi} = -\partial_x \bar{p}$$

- Ginzburg–Landau type, limit cycle oscillations.
- No perturbation amplitude, more appropriate for “dithering”.
- Generalization to ELMs available. Itoh, Itoh, PRL 91, PRL 95

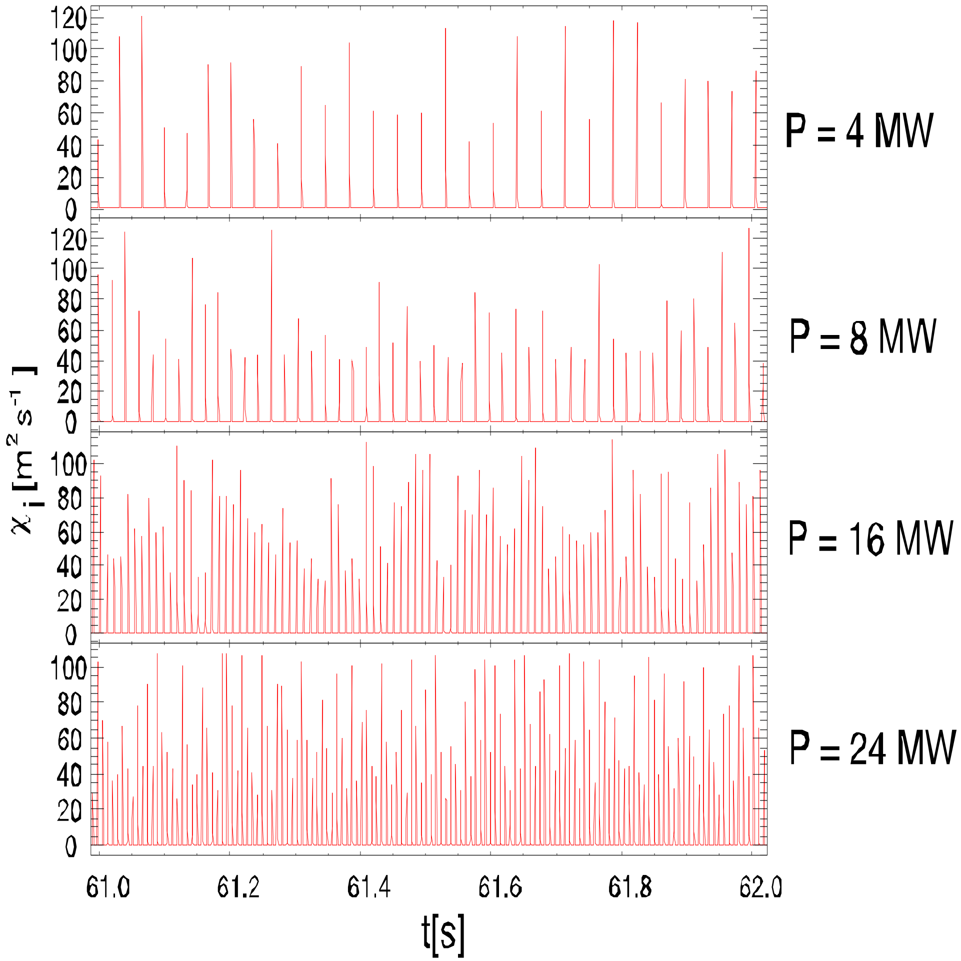
Linear ideal MHD instability model

- Linear ideal MHD eigenmodes:
  - growth rate ≈ 0 below threshold,
  - global mode structure.
- Modeled by
  - Heaviside funct. H on growth rate,
  - Gaussian shape G in eff. diffusivity

$$\partial_t \xi = \gamma_0 (\bar{\pi} - \alpha_c) H (\bar{\pi} - \alpha_c) \xi - \nu (\xi - \xi_0)$$

\+ transp. code with $\chi_{\text{eff}} \propto |\xi|^2 G(x)$

- Relaxations, frequency ↗ with power.
- More sophisticated models (peeling).



Lönnroth, Parail, PPCF 04

Bécoulet, Huysmans, EPS 03

State of the art

- Most existing models are phenomenological.
- Difficult to reproduce relaxations with frequency ↘ with power.
- Turbulence simulations of relaxations exist,
  based on turbulent ExB flow generation, no barrier.
- Need for 1st principles based model, i.e. 3D turbulence simulations,
  reproducing i) transport barrier ii) complete relaxation cycle.

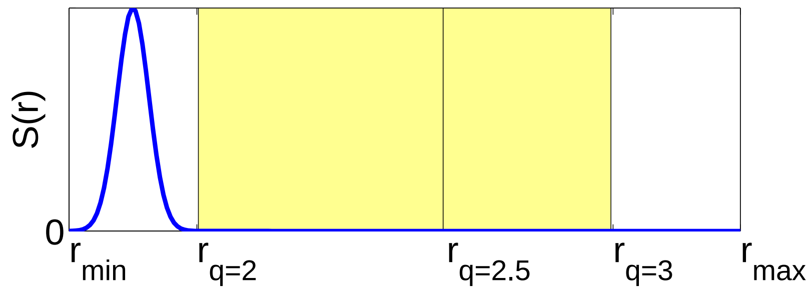
3D edge turbulence simulations with transport barrier

- turbulence model: resistive ball. modes,
  reduced MHD equations

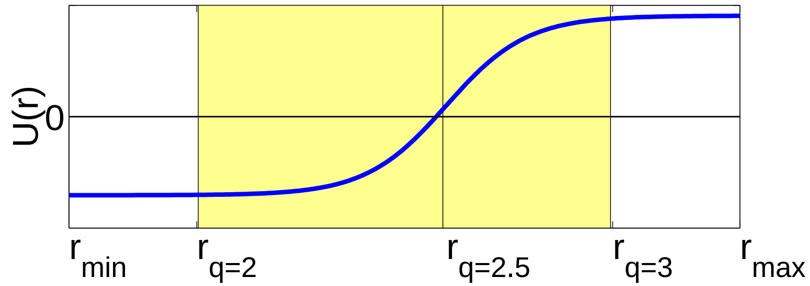
$$\partial_t \nabla_\perp^2 \phi + \left\{ \phi, \nabla_\perp^2 \phi \right\} = -\nabla_{||}^2 \phi - \mathbf{G} p + \nu \nabla_\perp^4 \phi$$

$$\partial_t p + \{\phi, p\} = \delta_c \mathbf{G} \phi + \chi_{||} \nabla_{||}^2 p + \chi_\perp \nabla_\perp^2 p + S$$

- 3D toroidal geometry at plasma edge
- driven by incoming flux $\Gamma_{\text{in}} = \int_{r_{\text{min}}}^{r} S \, \mathrm{d}r'$,
  press. profile evolves self-consistently
- barrier generated by imposed flow U,
  locally sheared, $\omega_{E\,\text{ext}} = (\partial_r U)_{\text{max}}$

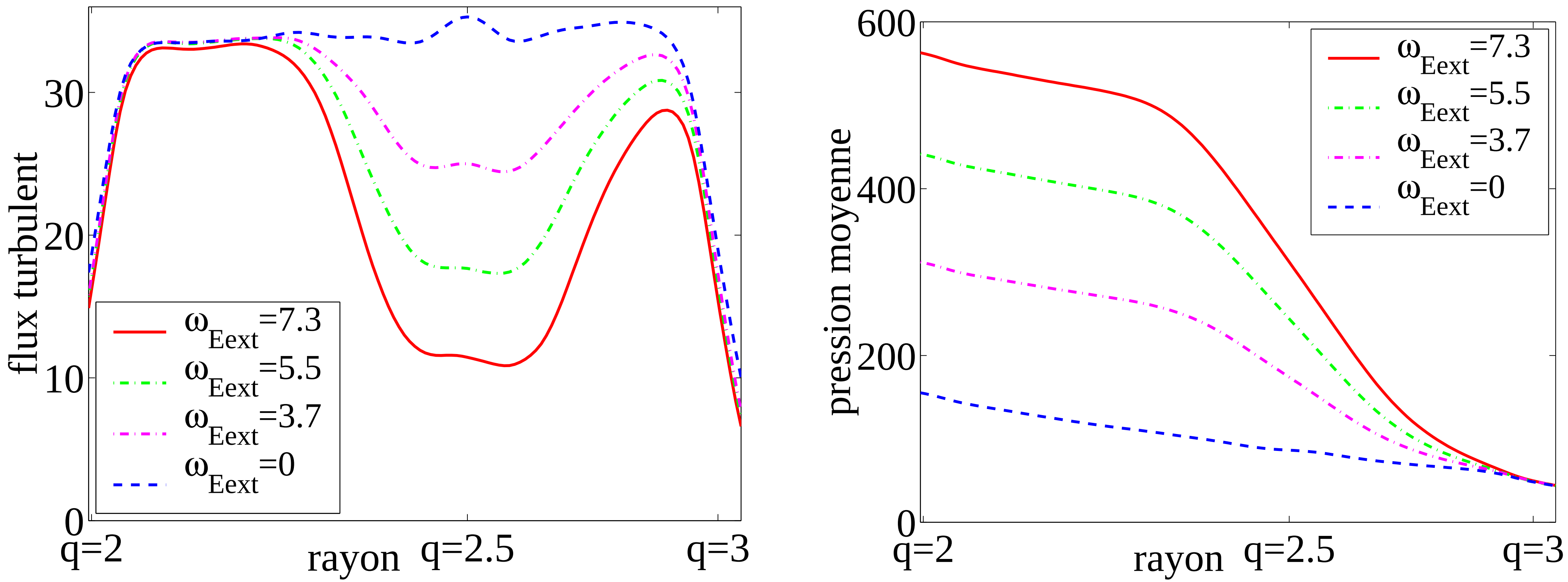


radial profile of source S



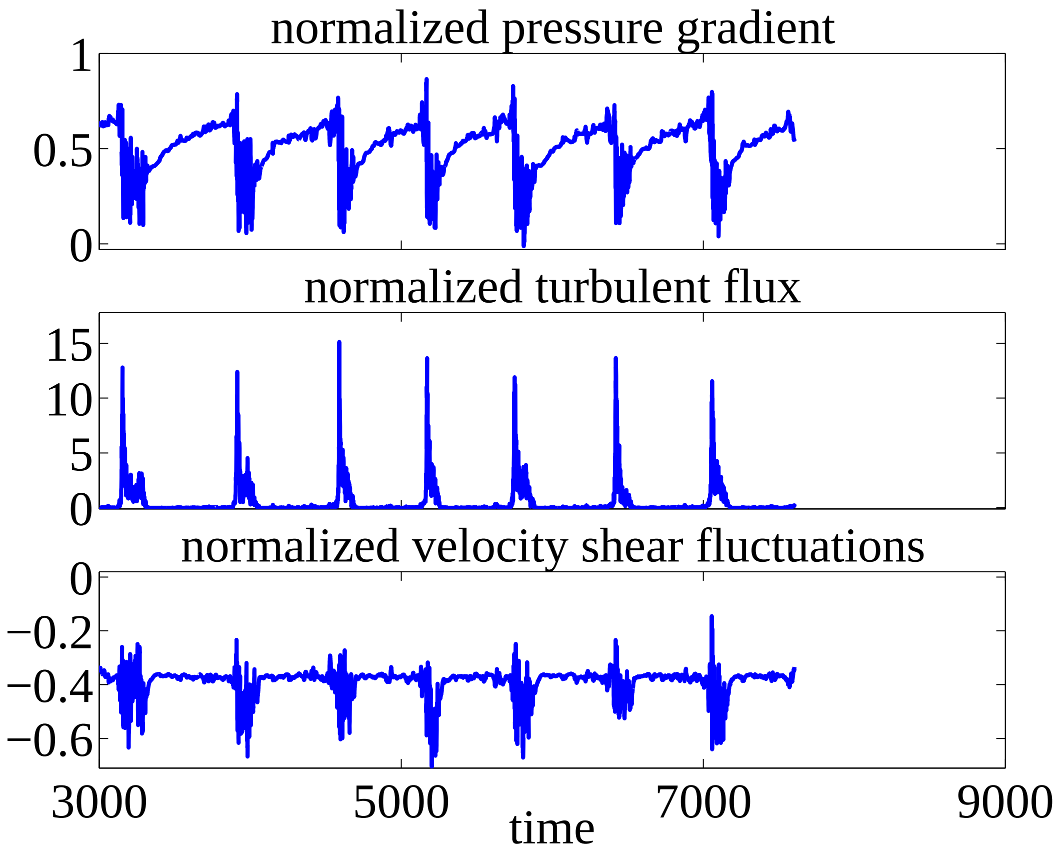
radial profile of imposed flow U

Strong local ExB shear $\rightarrow$ formation of barrier



turbulent flux profile | time averaged | pressure profile

Barrier relaxation oscillations appear



1. $|\partial_x \bar{p}| \,/\, (\Gamma_{in}/\chi_\perp)$

2. $\Gamma_{turb}/\Gamma_{in}$

3. $(\omega_E - \omega_{E\,\mathrm{ext}})\,/\,\omega_{E\,\mathrm{ext}}$

- all evaluated at barrier center

- observed in a range of Γ_{in}, $\omega_{E\,\mathrm{ext}}$
- robust property

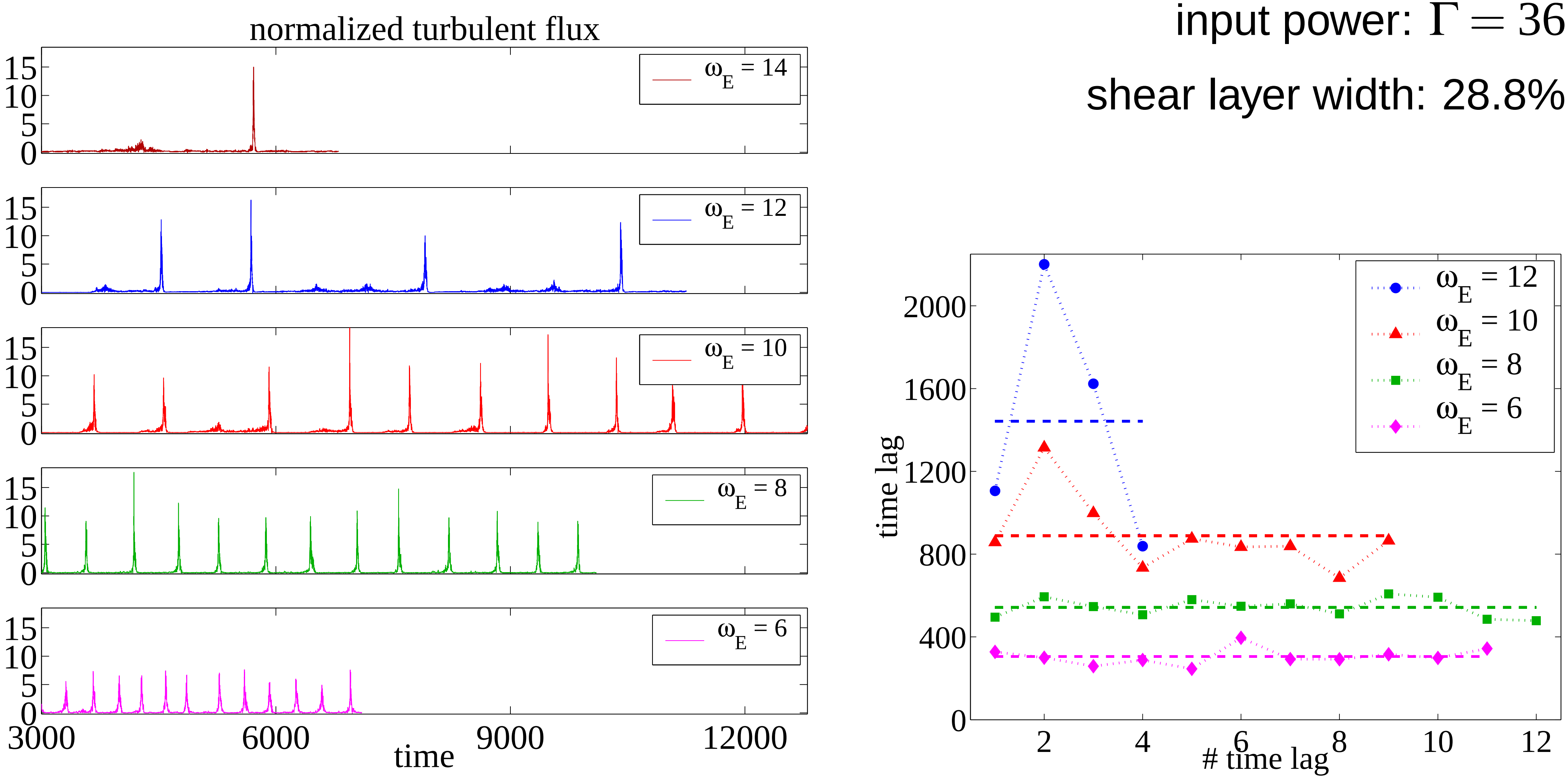
scenario:

turb. state $\rightarrow$ relaxations $\rightarrow$ quiescent st.
$\Longrightarrow \omega_E$
no barrier $\rightarrow$ barrier

Fixed input power: frequency decreases with shear

input power: $\Gamma = 36$

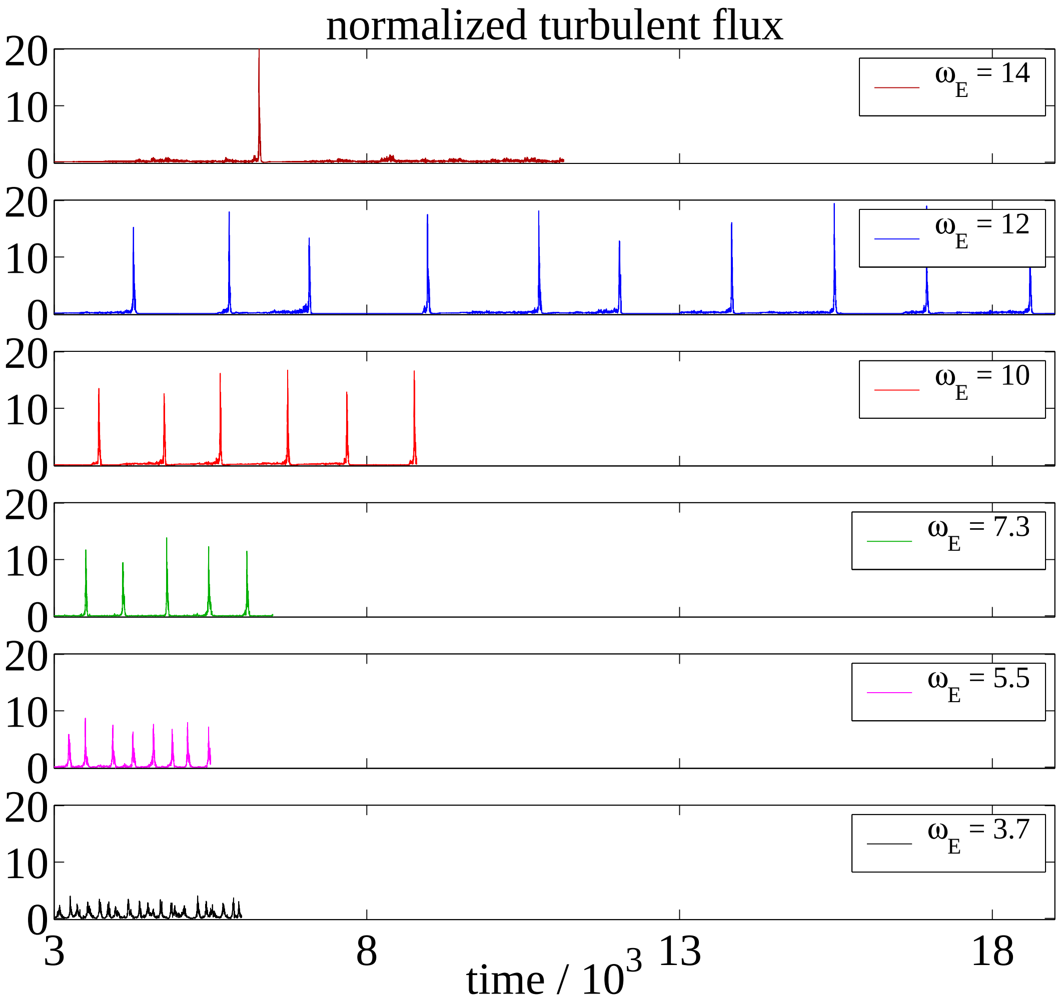
shear layer width: 28.8%

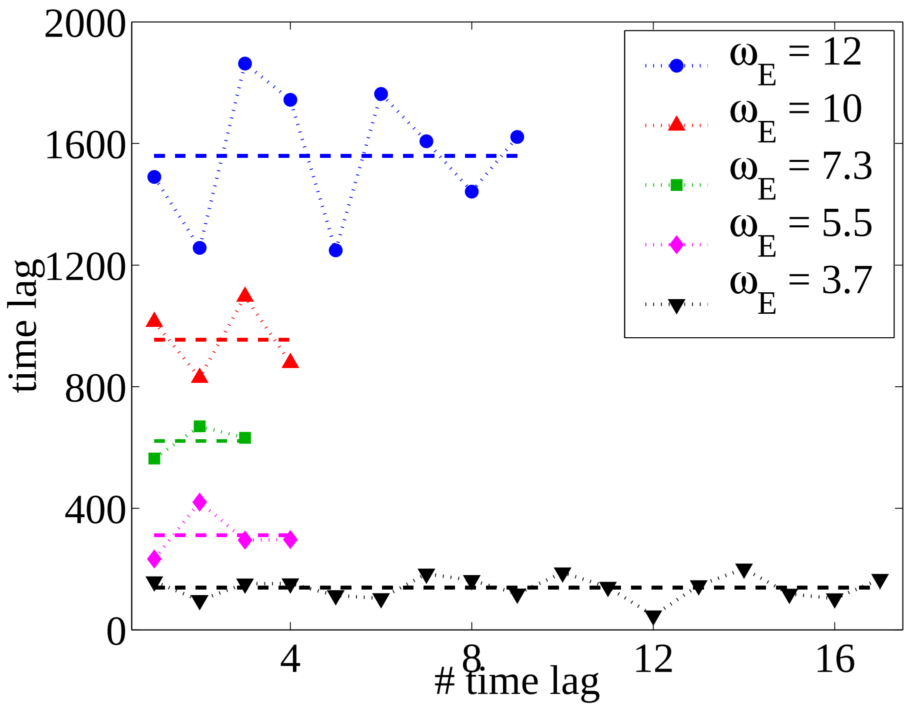


Fixed input power: frequency decreases with shear

input power $\Gamma = 36$

shear layer width: 34.4%

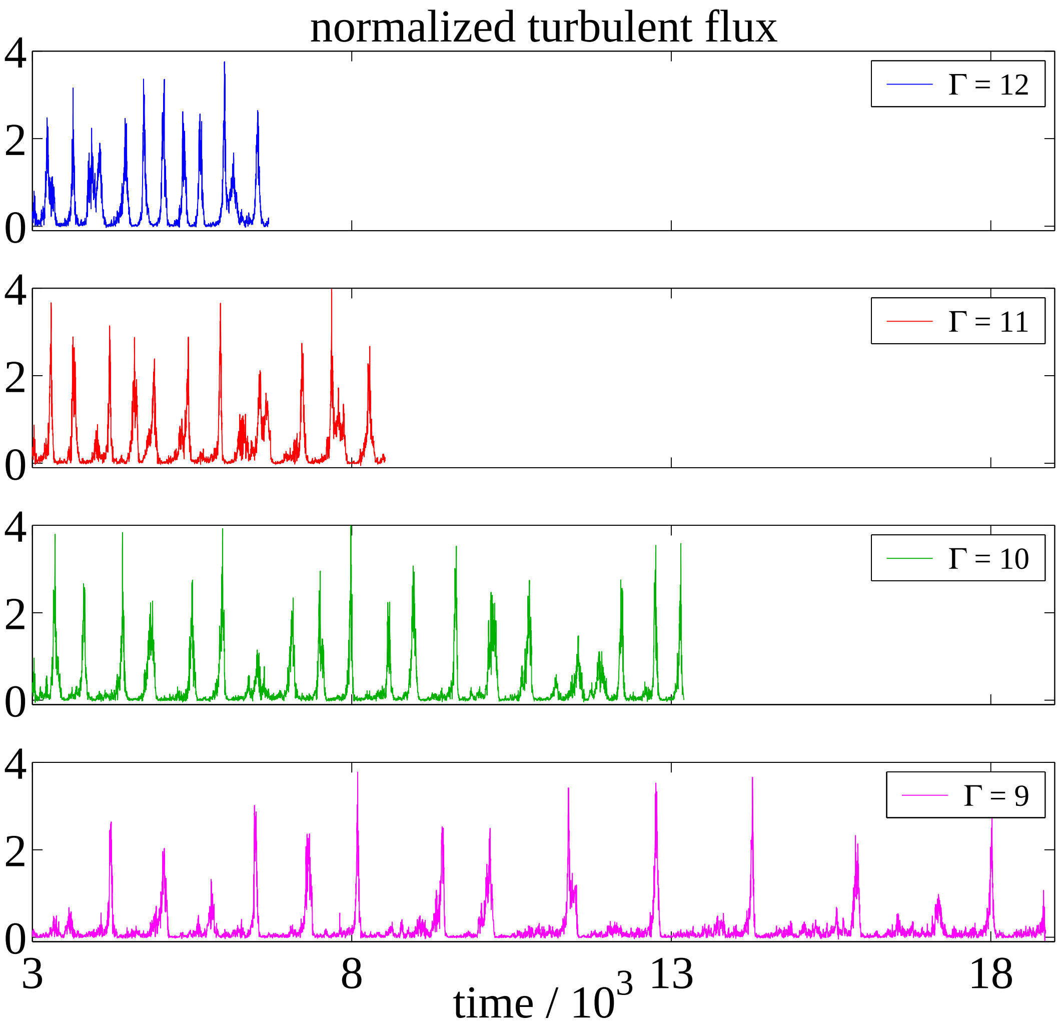


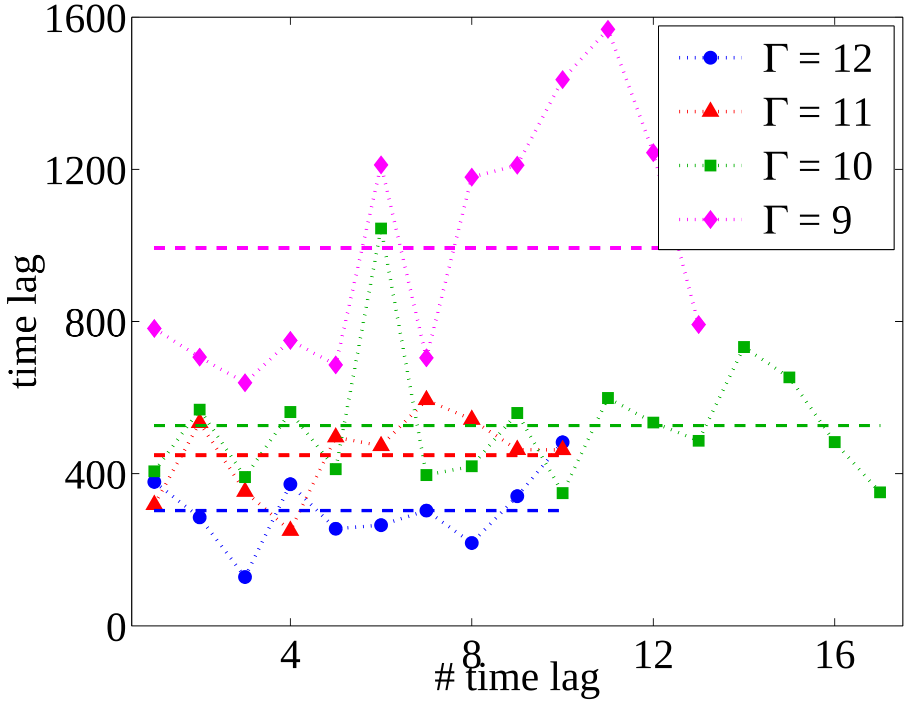


Fixed flow shear: frequency increases with power

flow shear: $\omega_E = 2$

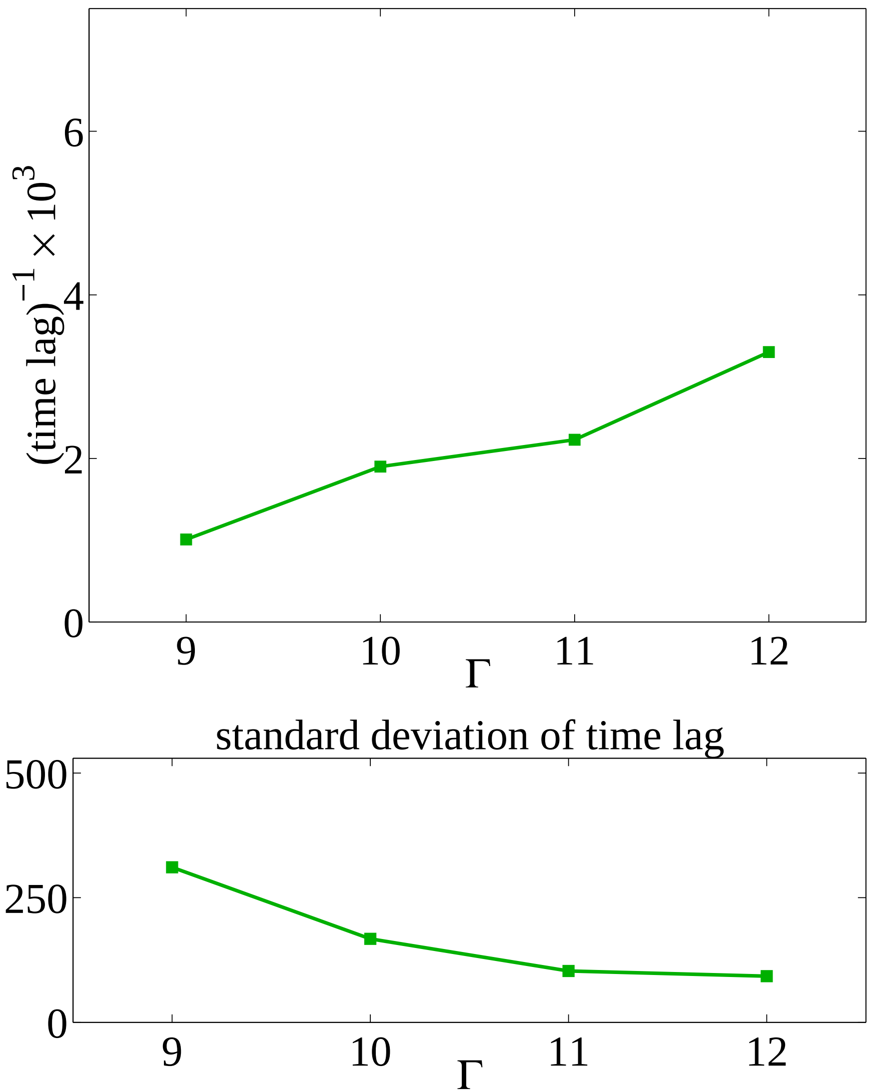
shear layer width: 34.4%



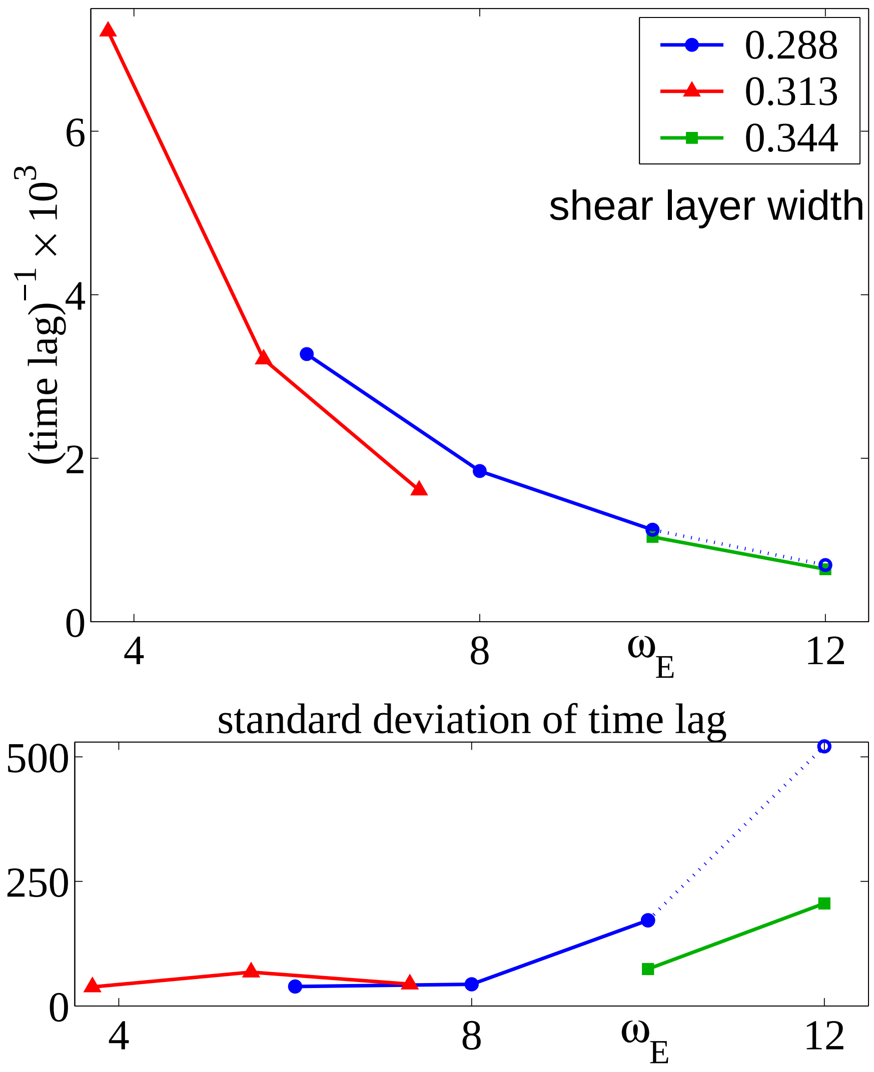


Frequency dependence: two opposite trends

freq. ↗ with Γ_{in} for ω_E fixed ($\omega_E = 2$)

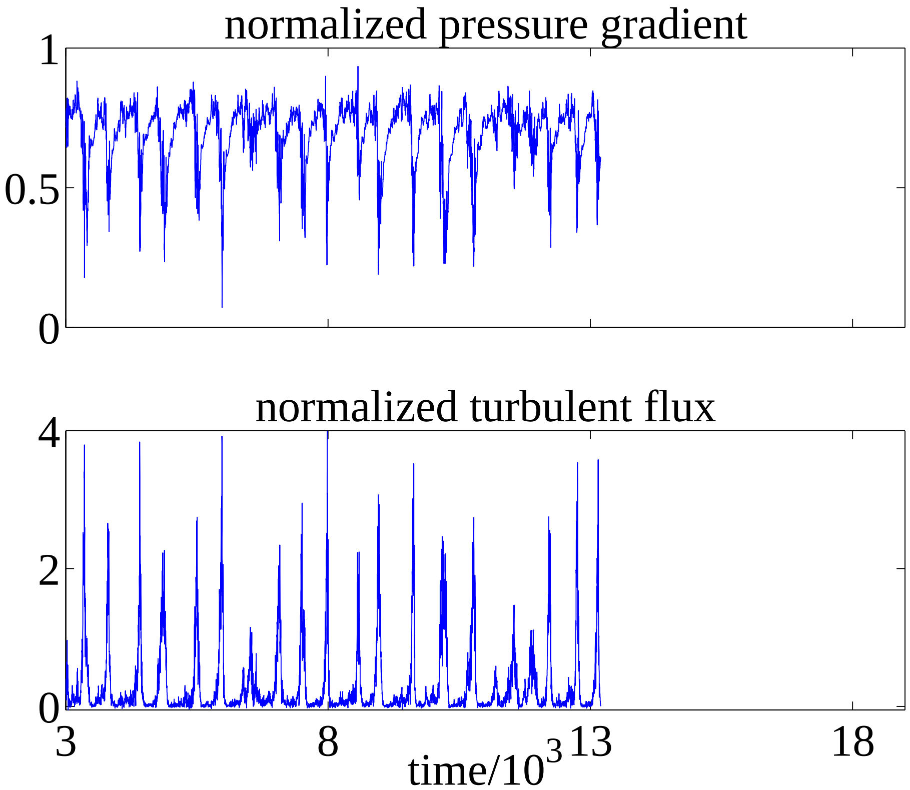


freq. ↘ with ω_E for Γ_{in} fixed ($\Gamma_{\text{in}} = 36$)

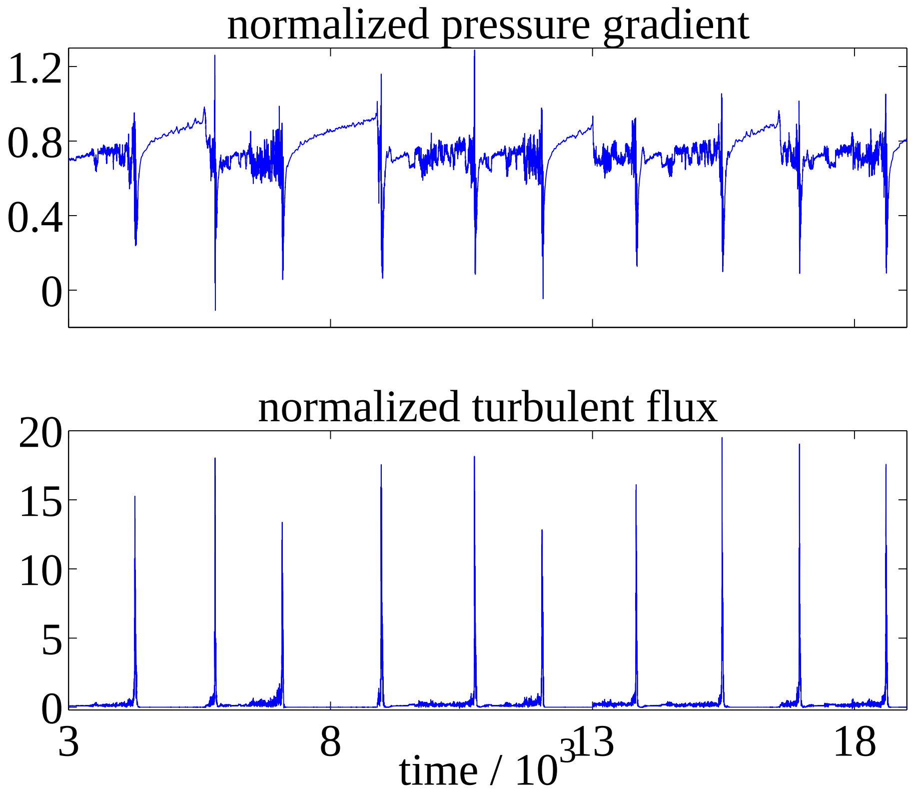


Frequency dependence: two opposite trends

if ω_E increases fast enough with Γ_{in} $\rightarrow$ frequency decreases with Γ_{in}.



$\Gamma_{\text{in}} = 10\,, \quad \omega_E = 2$



$\Gamma_{\text{in}} = 36\,, \quad \omega_E = 12$

Possible mechanisms excluded

- Relaxations persist even if ExB shear flow is frozen

  $\rightarrow$ mechanism $\neq$ turbulent shear flow generation.

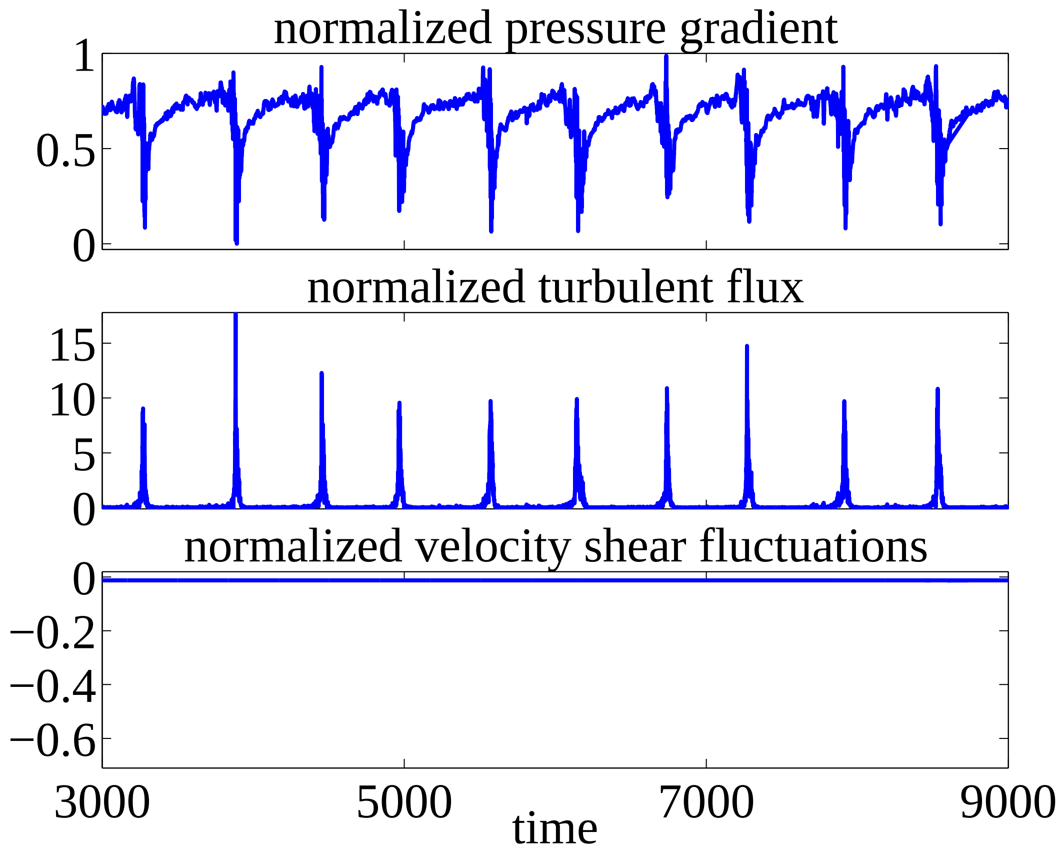
- No significant variation of modes localized outside barrier

  $\rightarrow$ mechanism $\neq$ toroidal mode coupling.

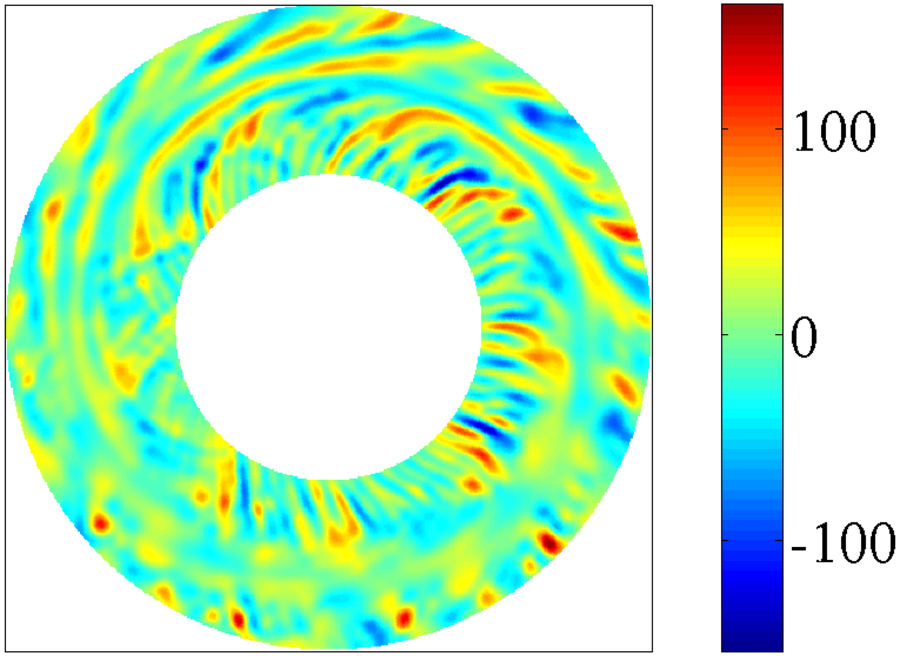
- All fluctuations die out when suppressing curvature

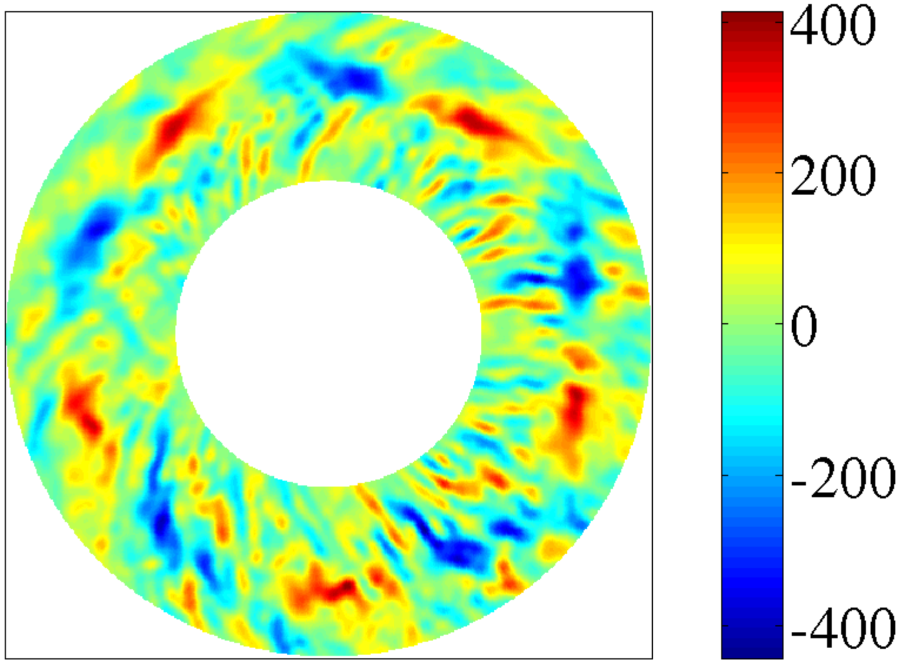
  $\rightarrow$ Kelvin–Helmholtz stable.

$\Gamma_{\text{in}} = 36\,, \quad \omega_E = 8$



Relaxation governed by mode at barrier center





snapshots of potential fluctuations

between relaxations

during relaxation

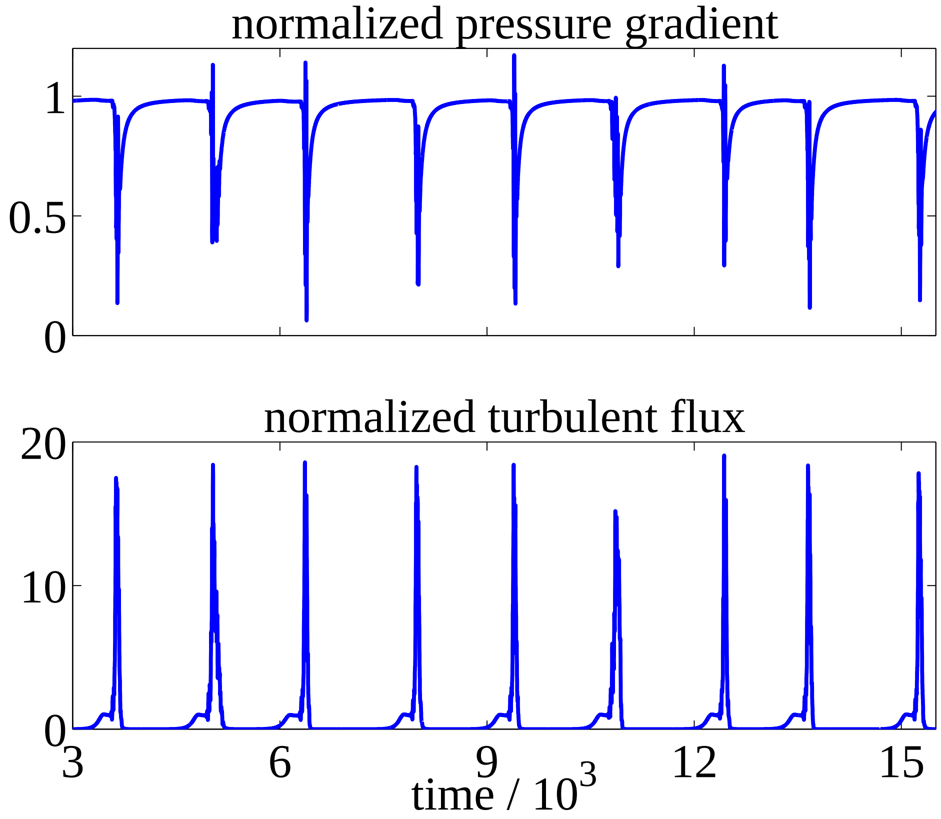
1D model for central mode amplitude $\tilde{p}(x,t)$ & profile $\overline{p}(x,t)$

$$\partial_t \overline{p} = -2\gamma_0 \partial_x |\tilde{p}|^2 + \chi_\perp \partial_x^2 \overline{p} + S$$
$$\partial_t \tilde{p} = \gamma_0 (-\partial_x \overline{p} - \alpha_0)\, \tilde{p} - i\omega_E' x \tilde{p} - \chi_{||}' x^2 \tilde{p} + \chi_\perp \partial_x^2 \tilde{p}$$

$x = r - r_0$: radial dist. from barrier center

m: poloidal wavenumber of central mode

- reproduces relaxation oscillations
- ExB shear $\omega_E' = \omega_E m / r_0$
  $\rightarrow$ nonlinear short-term dynamics.
- Not described by linear modes
  (long-term dynamics).



Description by linear modes is not appropriate

For $\partial_x \bar{p} = -\alpha$, evolution equation for $\tilde{p}$ is linear:

$$\partial_t \tilde{p} = \gamma_0 (\alpha - \alpha_0)\, \tilde{p} - i\omega'_E x \tilde{p} - \chi'_{\parallel} x^2 \tilde{p} + \chi_{\perp} \partial_x^2 \tilde{p}$$

- most unstable eigenmode:

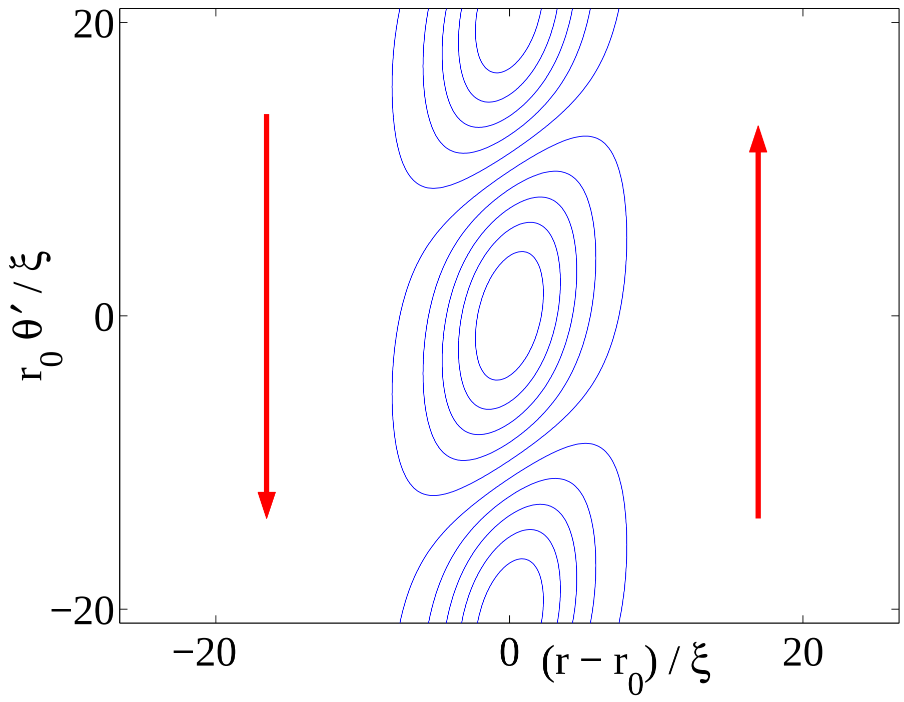
$$\tilde{p} e^{im\theta} \sim \exp\left(-\frac{x^2}{2\sigma^2} + im\theta - \frac{i\omega'_E x}{2\sqrt{\chi'_{\parallel}\chi_{\perp}}} \right)$$

with $\sigma^2 = \sqrt{\chi_{\perp}/\chi'_{\parallel}}$

- growth rate:

$$\gamma = \gamma_0 (\alpha - \alpha_0) - \frac{\omega'^2_E}{4\gamma'_{\parallel}} - \sqrt{\chi_{\perp}\chi'_{\parallel}}$$

- when $i\omega'_E x$ term replaced by shift of instability threshold $\rightarrow$ no oscillations



Time delay in stabilization by ExB shear flow

- Short term dynamics of initial pulse

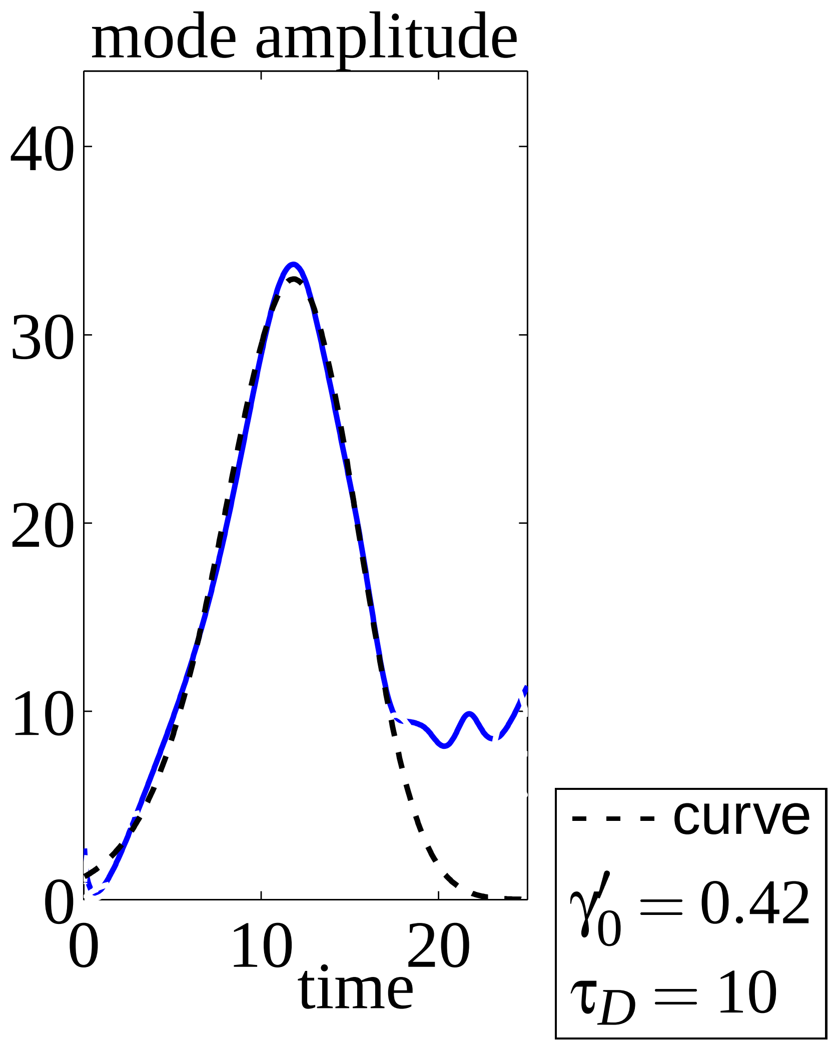
$$\tilde{p}(x,t=0) \propto \delta(x)$$

with $-\partial_x \bar{p} = \alpha$ and $\chi'_{\|}$ term neglected.

- Solution : $$\tilde{p} \propto \exp\left[\gamma'_0 t - t^3/(3\tau_D^3)\right]$$

$$\gamma'_0 = \gamma_0(\alpha - \alpha_0), \quad \tau_D = \left(\tfrac{1}{4}\chi_\perp \omega'^2_E\right)^{-1/3}.$$

- Transient growth before stabilization.
- τ_D large for small $\chi_\perp$ (barrier) and low m.
- Clearly observed in simulations (— curve).



0D model reproduces oscillations
Radial mode structure $\neq$ linear mode

- From 1D model $\rightarrow$ relevant radial structures: $\tilde{p}_R(x)$, $\tilde{p}_I(x)$, $\overline{p}_0(x)$
  [linear mode in case $\partial_x \overline{p} = const$: $\tilde{p}_{\text{lin}} = \tilde{p}_R + \mathrm{i}\tilde{p}_I$]

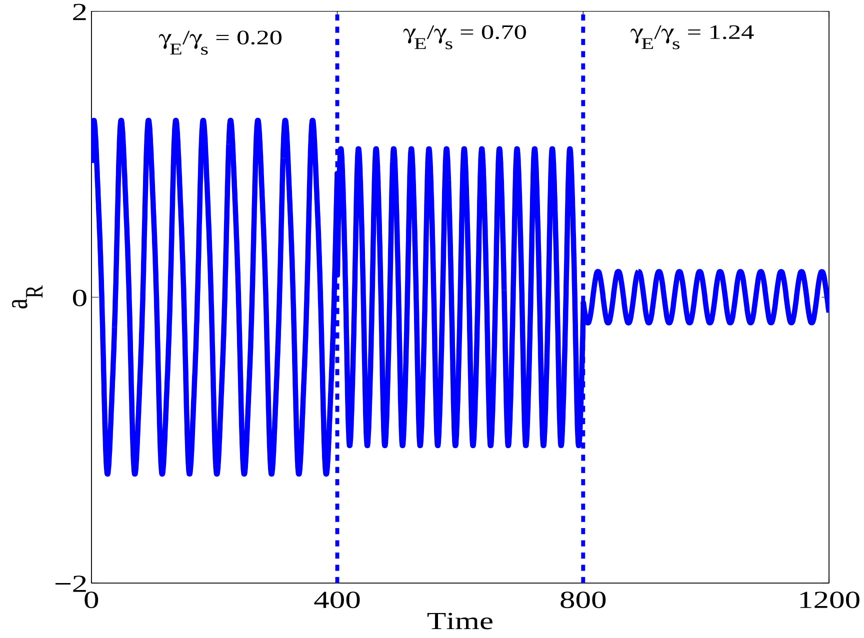
- Projection: $\tilde{p}(x,t) = a_R p_R + \mathrm{i} a_I p_I \,, \quad \overline{p}(x,t) = -\alpha x + a_0 \overline{p}_0$

- Amplitude equations:

$$\begin{aligned} \dot{a}_R &= (\Gamma - \delta_1 a_0)\, a_R + \Omega_1 \,(a_R - a_I) \\ \dot{a}_I &= (\Gamma - \delta_2 a_0)\, a_I - \Omega_2 \,(a_I - a_R) \\ \dot{a}_0 &= -\gamma_0 a_0 + 2\delta_1 a_R^2 + 2\delta_2 a_I^2 \end{aligned}$$

$\Gamma = \gamma_0\,(\alpha - \alpha_0) - \gamma_s - \gamma_E \qquad \gamma_s^2 = \chi'_{\|}\chi_{\perp}$

$\Omega_1 \approx \Omega_2 \approx 2\gamma_E \mathrm{e}^{-\gamma_E/\gamma_s} \qquad \gamma_E = {\omega'_E}^2/(4\chi'_{\|})$



- same frequency dependence on ExB shear as in 3D simulations

Conclusions

- 3D nonlinear turbulence simulations based on 1st principles show
  - onset of transport barrier,
  - barrier relaxation oscillations.
- Mechanism based on effective time delay for stabilization by ExB shear, no obvious S–curve, no global mode.
- Mean features are reminiscent of type III ELMs:
  - frequency dependence,
  - resistive ballooning mode model (low temperature plasma),
  - sensitivity to shear flow.

turb. state $\rightarrow$ relaxations $\rightarrow$ quiescent st.

$\Longrightarrow \omega_E$

no barrier $\rightarrow$ barrier