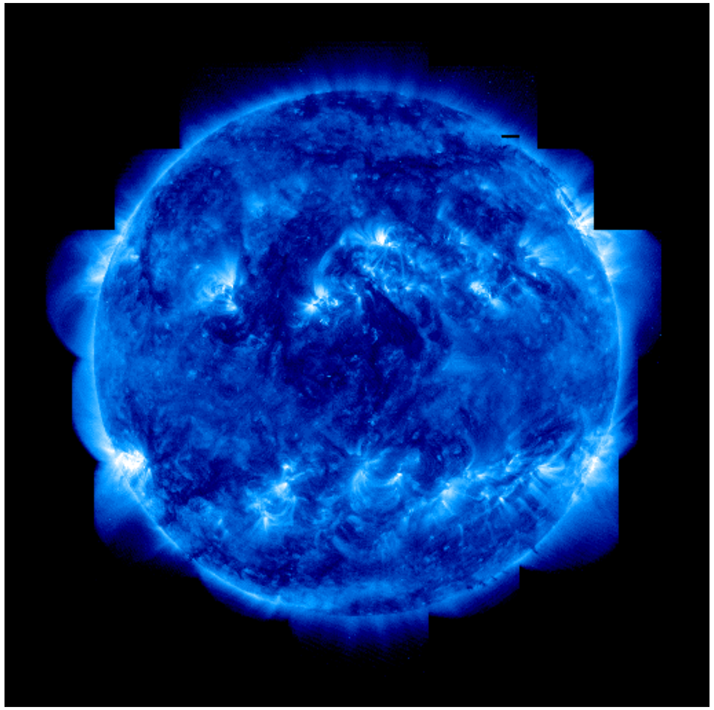
The solar tachocline



Steve Tobias (Applied Maths, Leeds)

Acknowledgements: Nic Brummell, Fausto Cattaneo, Pat Diamond, David Hughes

Tobias S.M. (2005) in "Fluid Dynamics and Dynamos in Astrophysics and Geophysics" eds A.M. Soward et al.

Book: eds D.W. Hughes, R. Rosner, N.O. Weiss

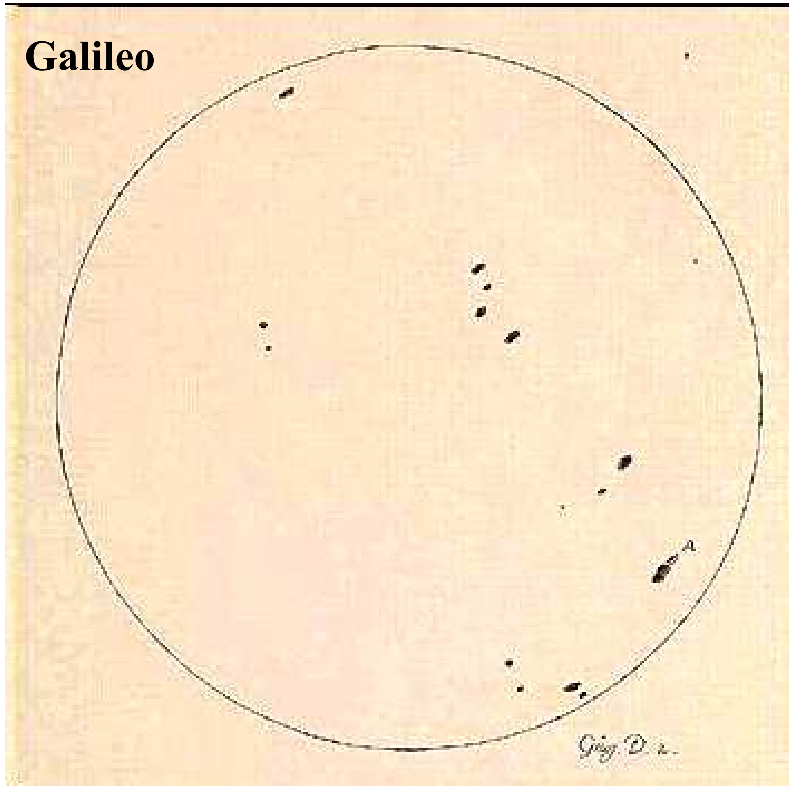
Physics…

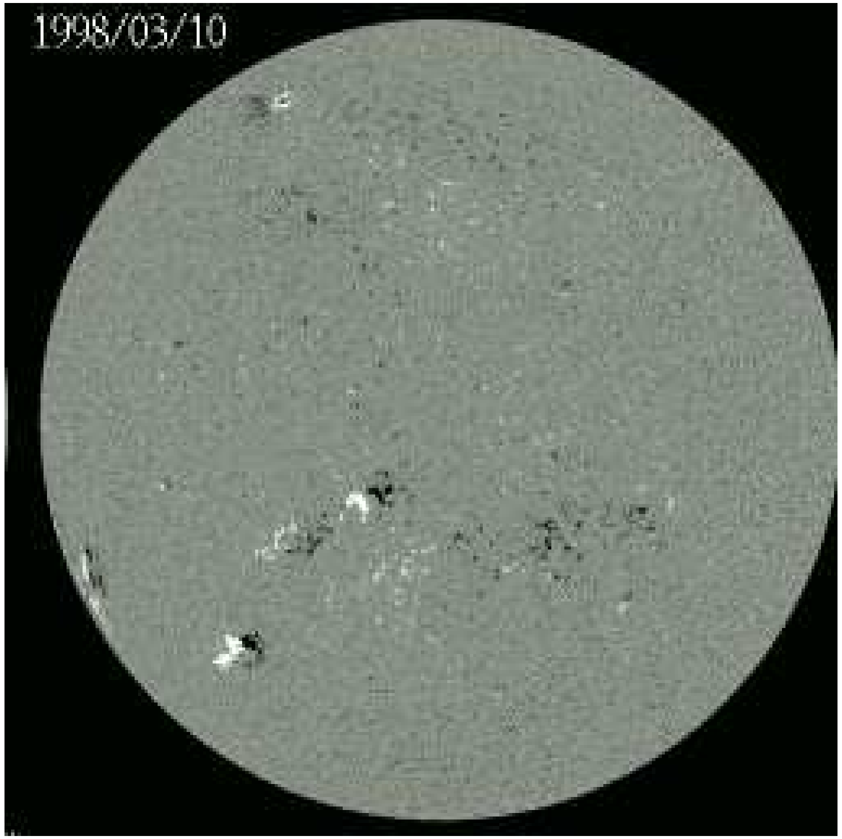
- Transport
  - angular momentum transport, turbulence
  - barriers to transport
    - Shear
    - Magnetic fields
- Competition/interaction between instabilities
  - Differential rotation/fingering
  - Magnetic Buoyancy
  - Dynamo action
- Turbulence in the linearly stable zone
  - Penetrative convection
- Zonal flows and turbulence
- **NOTE TURBULENCE BOTH SPREADS AND PREVENTS SPREADING TIMESCALES**

The Solar tachocline: Contents

- Observations: solar and stellar magnetic fields, internal structure of the Sun
- Why is there a tachocline?
- Stability of the tachocline
  - Differential rotation/joint instabilities
  - Magnetic buoyancy instability
- Tachocline and the solar dynamo
  - Interface dynamos
- Role of Turbulence
  - β-plane MHD turbulence

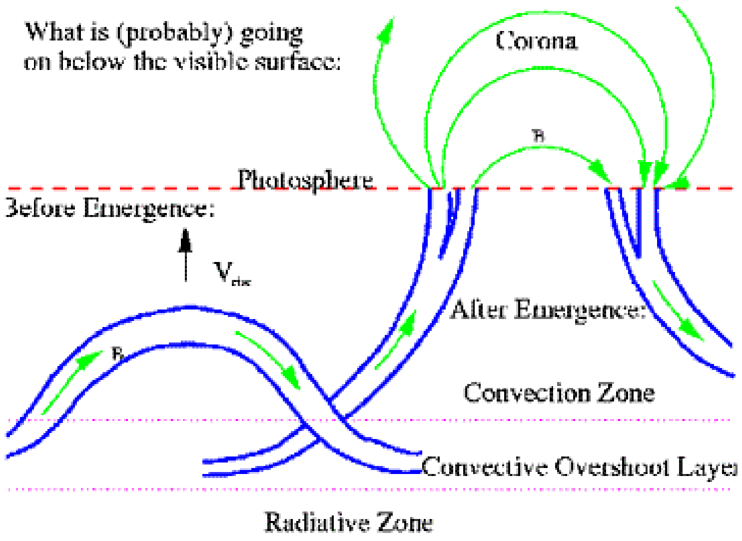
Observations: Solar





Magnetogram of solar surface shows radial component of the Sun's magnetic field.
Active regions: Sunspot pairs and sunspot groups.
Strong magnetic fields seen in an equatorial band (within 30^o of equator). Rotate with sun differentially.

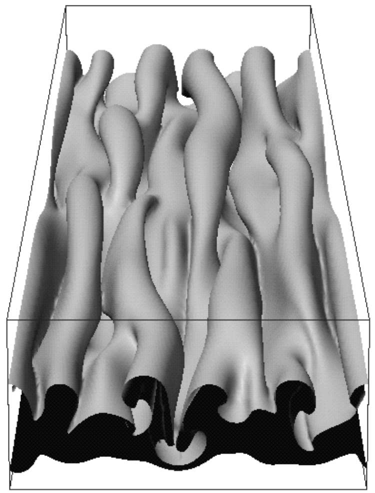
Observations Solar (a bit of theory)



Sunspot pairs are believed to be formed by the instability of a magnetic field generated deep within the Sun.

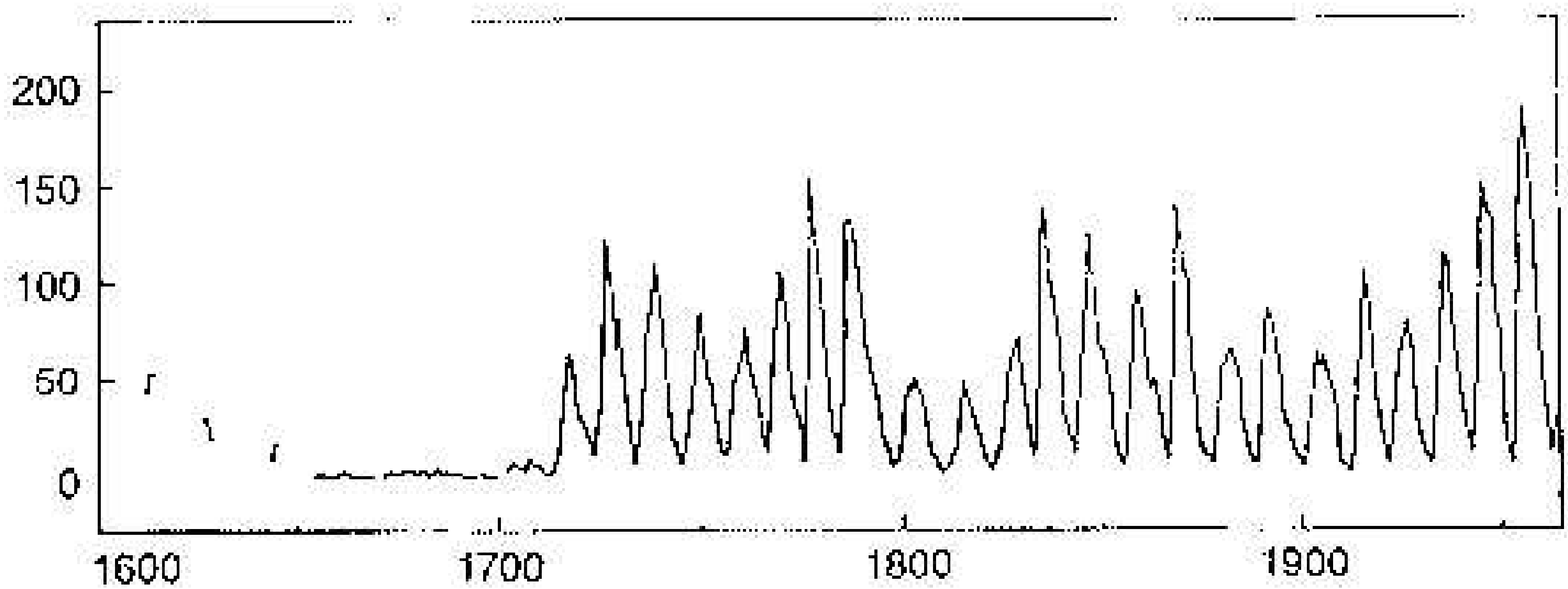
Flux tube rises and breaks through the solar surface forming active regions.

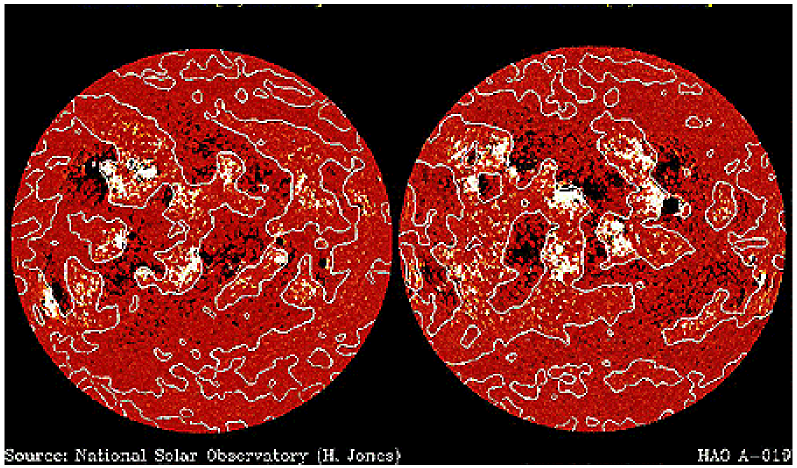
This instability is known as
Magnetic Buoyancy
(just one of the competing instabilities



Wissink et al (2000)

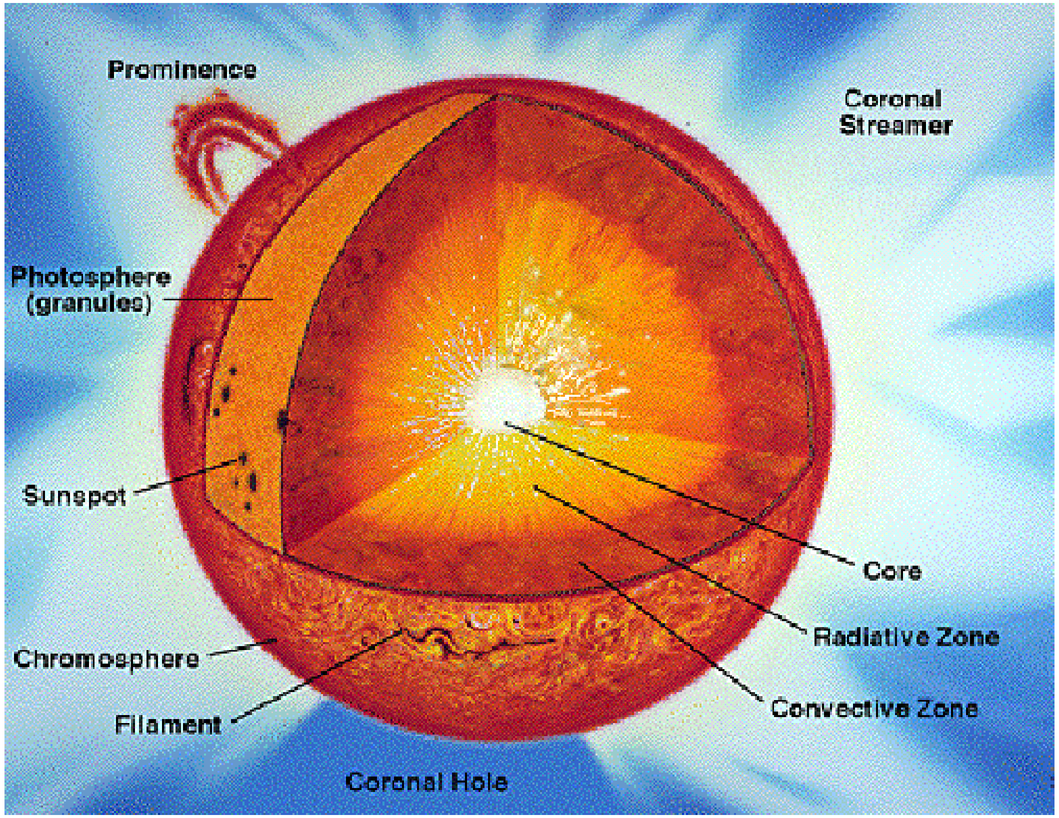
Observations: Solar





Polarity of sunspots opposite in each hemisphere
(Hale's polarity law).
DIPOLAR MAGNETIC FIELD
Polarity of magnetic field reverses every 11 years.
22 year magnetic cycle.

Solar Structure



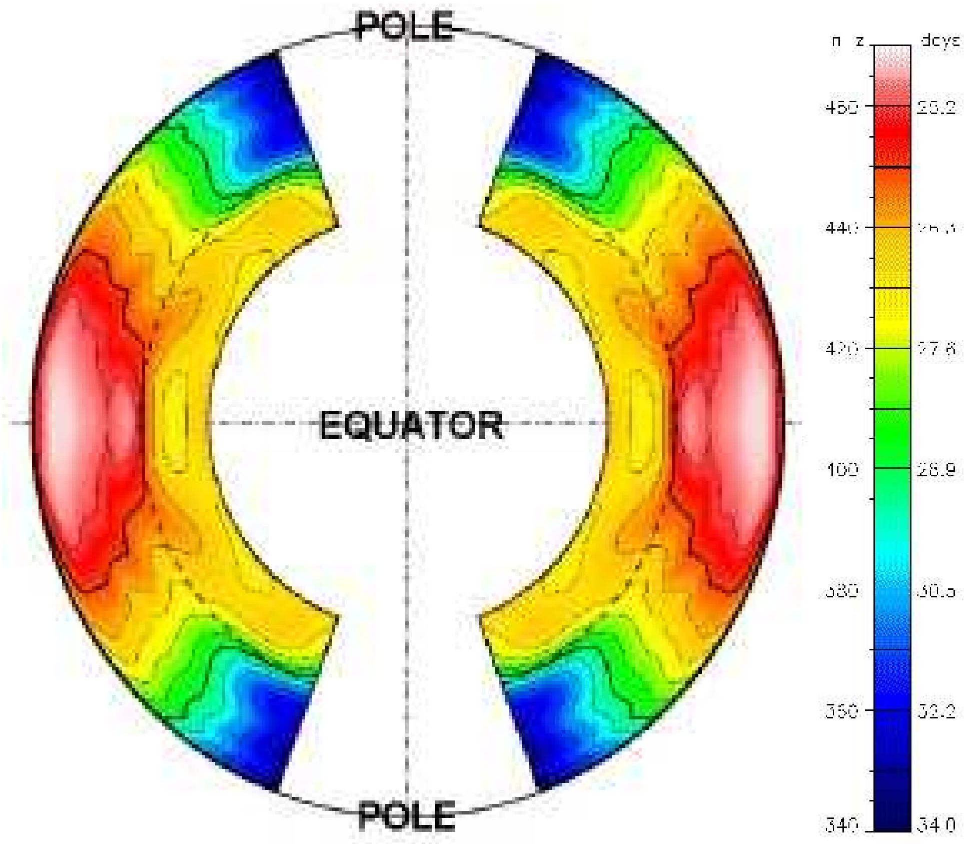
Solar Interior

1. Core
2. Radiative Interior
3. (Tachocline)
4. Convection Zone

Visible Sun

1. Photosphere
2. Chromosphere
3. Transition Region
4. Corona
5. (Solar Wind)

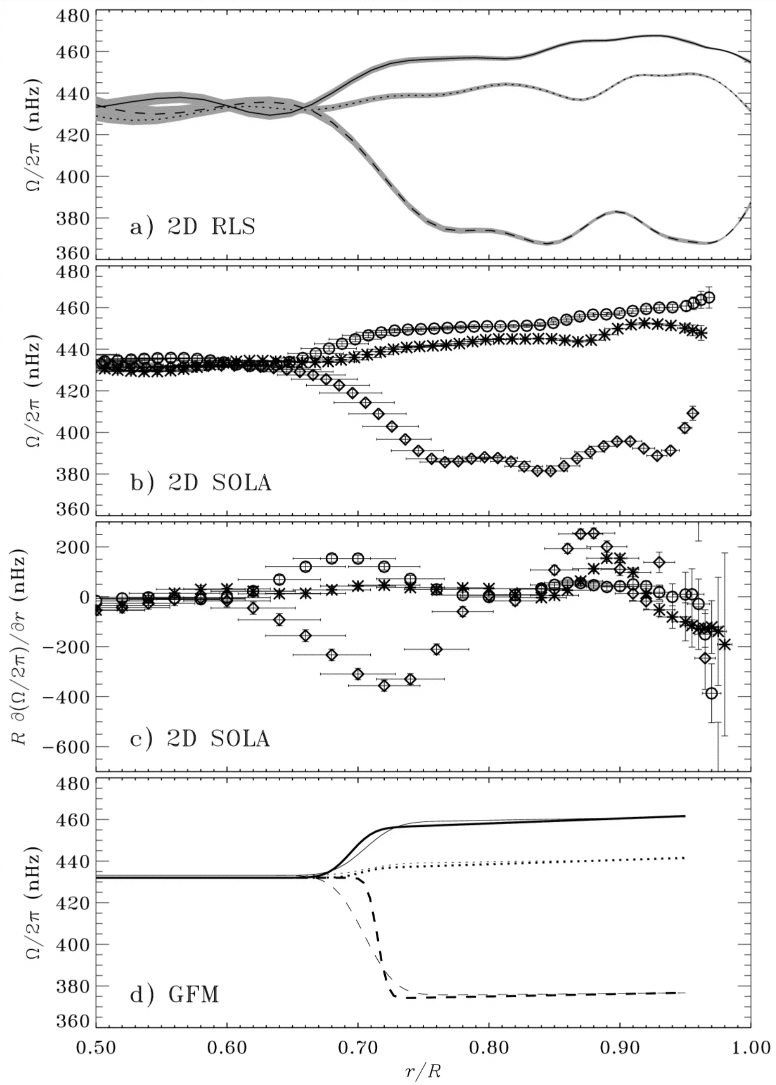
Helioseismology: The discovery of the tachocline



Schou et al (1998)

- Helioseismology shows the internal structure of the Sun.
- Surface Differential Rotation is maintained throughout the Convection zone
- Solid body rotation in the radiative interior
- Thin matching zone of shear known as the tachocline at the base of the solar convection zone (just in the stable region).

Fine detail of tachocline structure



Charbonneau et al (1999)

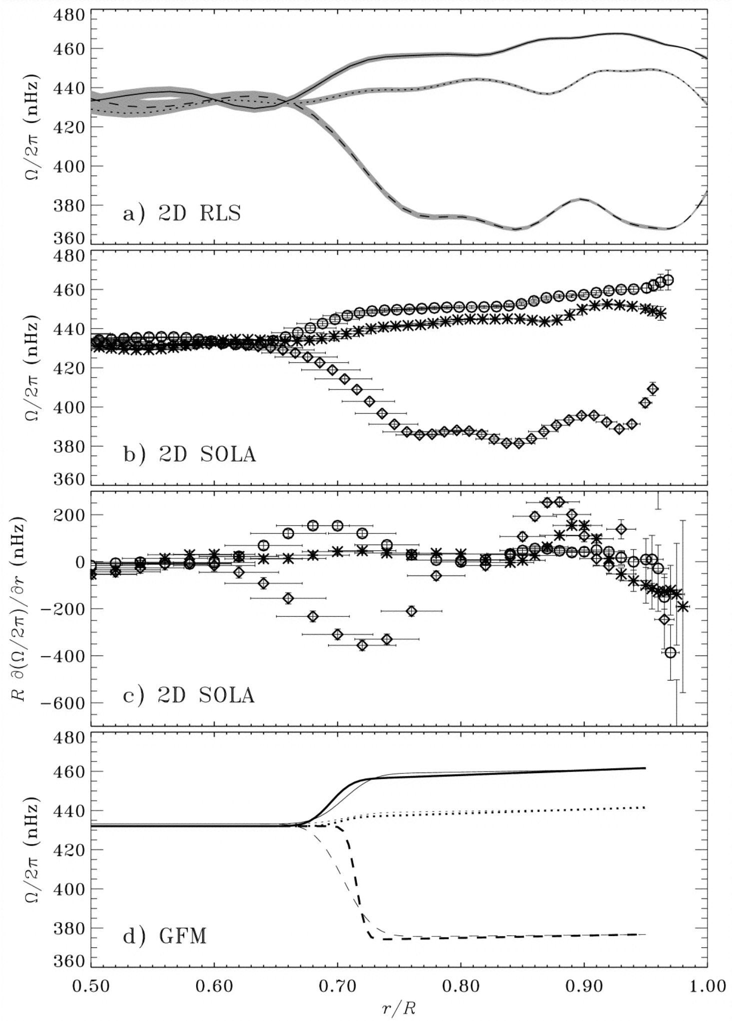
- 30 nHz at equator
- Position
  - $R_c/R_S = 0.693$
- Prolateness
  - R_c/R_S varies by 0.02
- Thickness
  - $R_{tach}/R_S = 0.039$ (no lower bound)
  (Charbonneau et al 1999)
  $R_{tach}/R_S = 0.019$
  (Elliott & Gough 1999)
- Latitudinal dependence

$$\omega = \omega_{eq}(1 - a_2\mu^2 - a_4\mu^4)$$

$$a_2 \approx a_4 \approx 0.15$$

- Oscillation 1.3yr period
  - Howe et al (2000)

Some Physical Parameters



- High density
  - $\rho \sim 0.2$ g/cm^3
- High Temperature
  - T ~ 2.0 x 10^6 K
- P ~ 6 x 10^{12} Pa
- Magnetised
  - Field $10^4 - 10^5$ G
  - $\beta = 10^5$

- Very stably stratified

Why is there a tachocline?

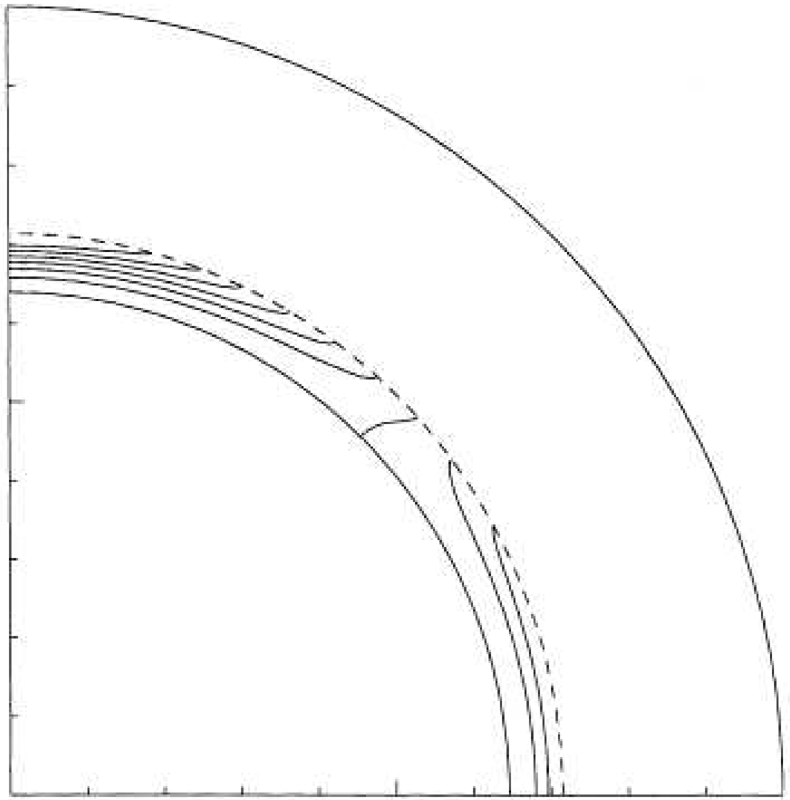
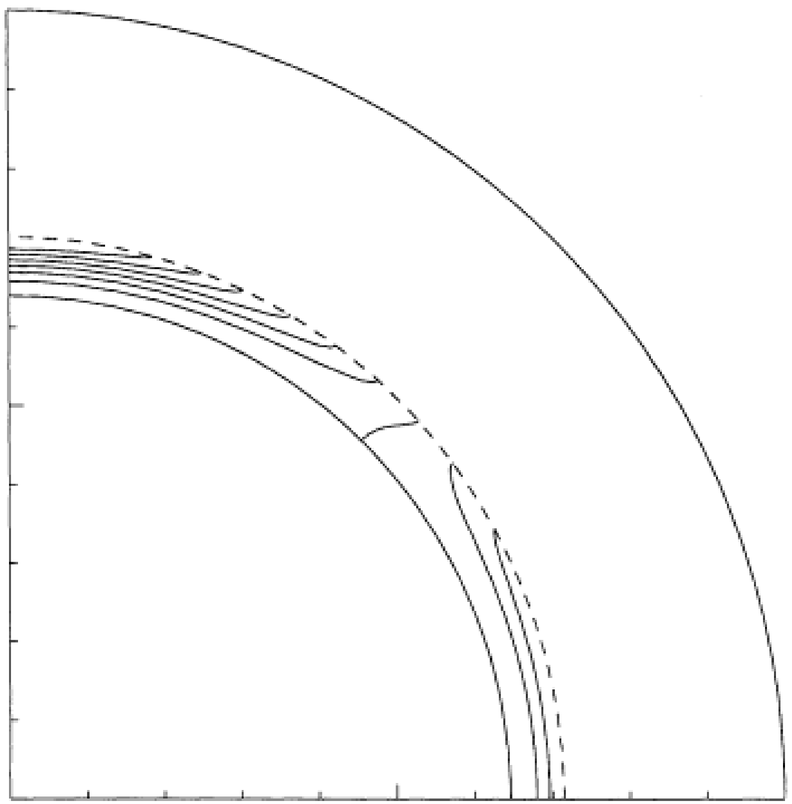


Fig. 3. The turbulent tachocline, whose thickness has been set here arbitrarily to 50,000 km (the actual value depends on the horizontal component v_H of the turbulent viscosity). The continuous lines show the contours of the angular velocity. Below, the interior rotation is nearly uniform, and its angular velocity equals that of the base of the convection zone at the latitude of approximately 42°.

- What is the mechanism that leads to the formation of this thin shear layer?
- Take the differential rotation of the convection zone as given (transport of angular momentum by stresses of turbulence)
- Spiegel & Zahn (1992): a radiation driven meridional circulation transports angular momentum from the convection zone into the interior along cylinders → no tachocline.
- NOTE THIS MERIDIONAL FLOW IS SLOW: $T \sim 10^6$ years
- Stable stratification → 2 dimensional turbulence (if there is any turbulence)
- 2D turbulence → anisotropic turbulent viscosity (drives system towards constant angular velocity) (“TURBULENCE ACTS AS A FRICTION”) prevents tachocline spreading into the interior.
- TURBULENCE (A FAST PROCESS) IS INVOKED TO STOP THE SPREADING OF THE DIFFERENTIAL ROTATION

Why is there a tachocline?



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- Some questions…
- Is there a linear instability leading to turbulence?
- Is this turbulence 2D?
- If this turbulence is 2D does it really act so as to transport angular momentum towards a state of constant differential rotation (i.e. as a friction)
- What (if anything) about the magnetic field?

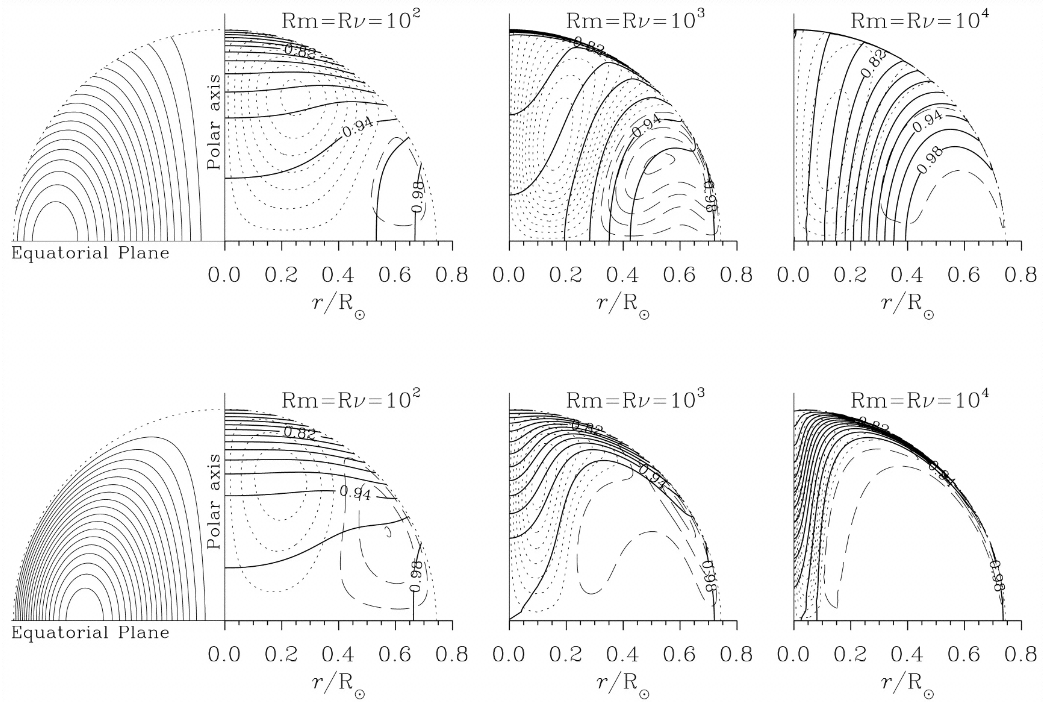
Some objections

- Some answers…
- Is there a linear instability leading to turbulence?
  - See later…
- Is this turbulence 2D?
  - Not necessarily
- If this turbulence is 2D does it really act so as to transport angular momentum towards a state of constant differential rotation?—i.e does the turbulence act as anisotropic viscosity?
  - Analogy with atmospheric models says that (hydrodynamic) 2D turbulence of this type acts so as to mix PV and drive the system away from solid body rotation (Gough & McIntyre 1998, McIntyre 2003)
  - Anti-friction
  - SEE ALSO LATER – β-PLANE MHD DYNAMICS

  (though the addition of a magnetic field can change angular momentum transport – cf MRIs)

  (also atmosphere $\nu/\kappa \sim 0.7$, tachocline $\nu/\kappa \sim 10^{-6}$)

Magnetic models

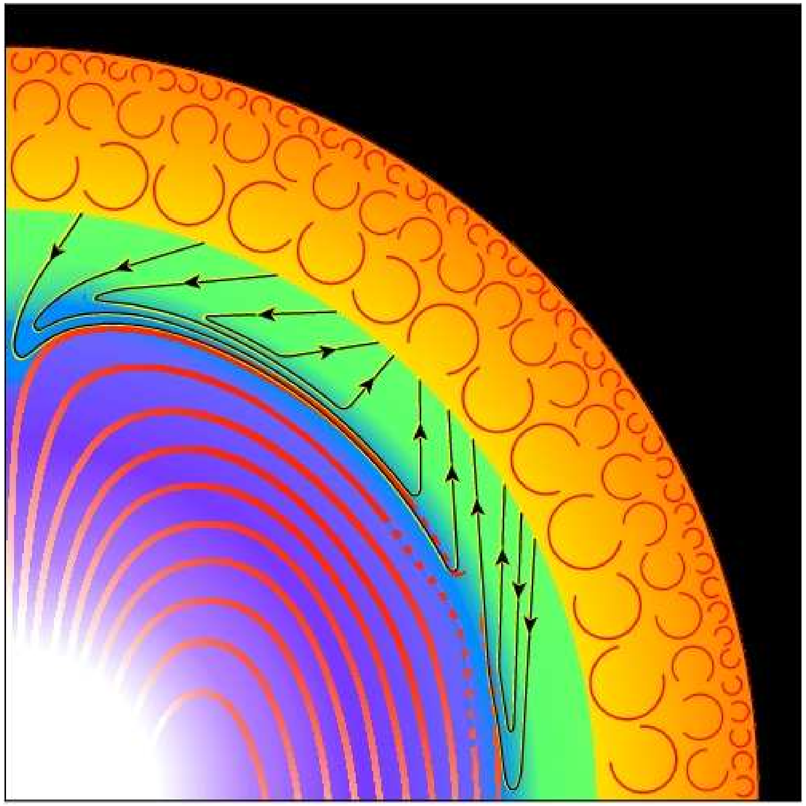


Hence if hydrodynamic and isotropic differential rotation propagates in along cylinders.

If magnetic then differential rotation propagates in along field lines

- A relic field in the interior can keep the interior rotating as a solid body (Mestel & Weiss 1987 $10^{-3} - 10^{-2}$ G)
- But if magnetic coupling spins down the radiative interior, why doesn't angular velocity propagate in along field lines
- MacGregor & Charbonneau (1999) suggested that all the field lines must be contained in the radiative interior (no magnetic coupling)

Gough & McIntyre (Part II)



Gough & McIntyre (Nature 1998)

- Meridional circulation due to gyroscopic pumping → 2 cells with upwelling at mid-latitudes
- Field held down where shear is large, brought up where no shear.
- Coupling still there, but no differential rotation brought in
- Delicate balance (Garaud 2002) crucial parameter Elsasser number $\Lambda = \dfrac{B_0^2}{\rho\Omega^2 r^2}$

Current Thinking…

NOMINAL BASE

OVERSHOOTING (AND MAYBE PENETRATIVE) CONVECTION
TURBULENCE IS 3D
MAGNETIC
STRONG MEAN DYNAMO FIELD…
BUOYANCY INSTABILITIES

END OF OVER-SHOOTING

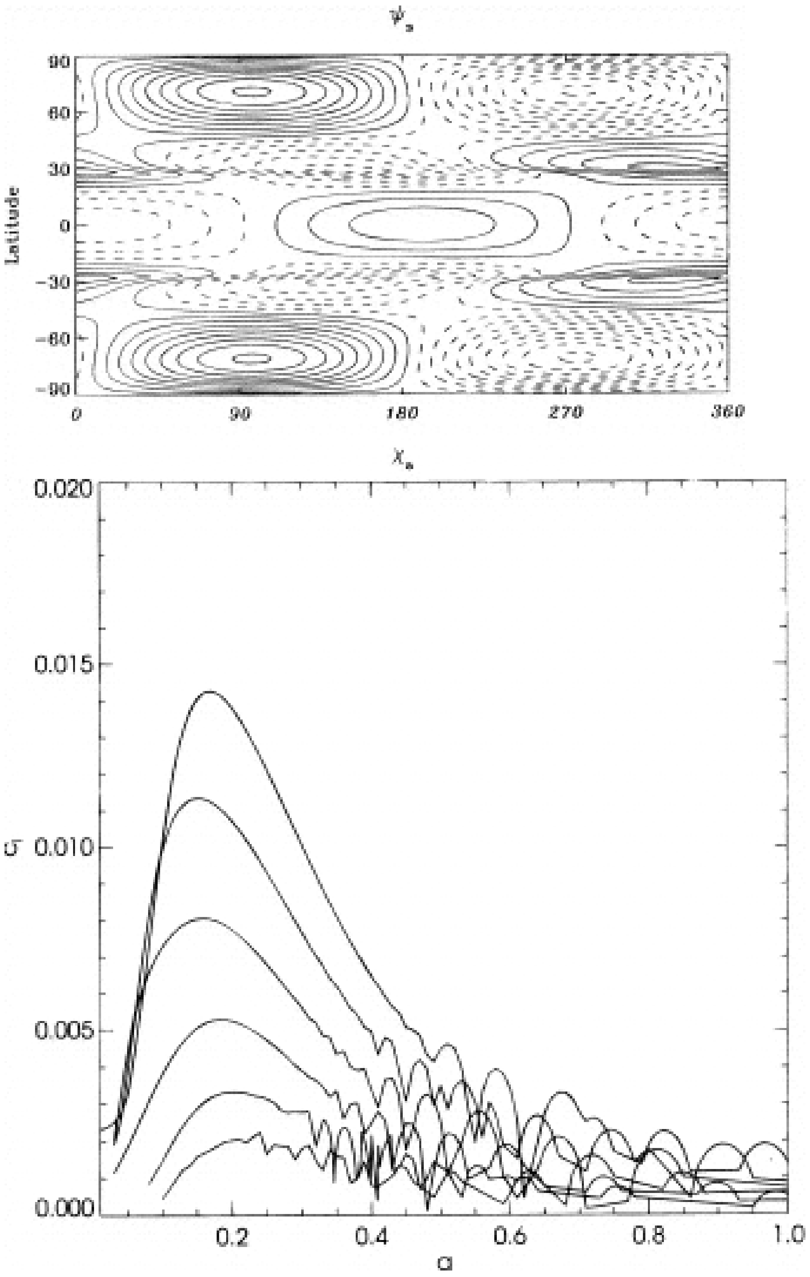
MHD TURBULENCE DRIVEN FROM ABOVE
VERY STABLY STRATIFIED – 2D
WEAK MERIDIONAL FLOW
LATITUDINAL ANGULAR MOMENTUM TRANSPORT
WEAK MEAN FIELD, BUT MHD IMPORTANT…

BASE OF TACHOCLINE

Stability of the Tachocline

- "Chicken & Egg"
- Hydrodynamic instabilities
  - Radial shear instability (K-H type)
    - $$Ri = \frac{N^2}{U'^2} \sim 10^2 - 10^4 > \tfrac{1}{4}$$
    - (radial motions suppressed, diffusion can change picture… hydro statement)
  - Latitudinal differential rotation instability (2D, (θ,ϕ))
    - Fjortoft (1950) stable if $$\frac{d^2}{d\mu^2}[(1-\mu^2)\omega(\mu)] \neq 0$$
    - Watson (1981)
      - Hydrodynamically stable if equator to pole difference $<$ 29 %
      - (in reality ~ 12%)
    - Lots of others (shallow water hydro etc) similar conclusions – at best marginally stable.

MHD & joint instabilities

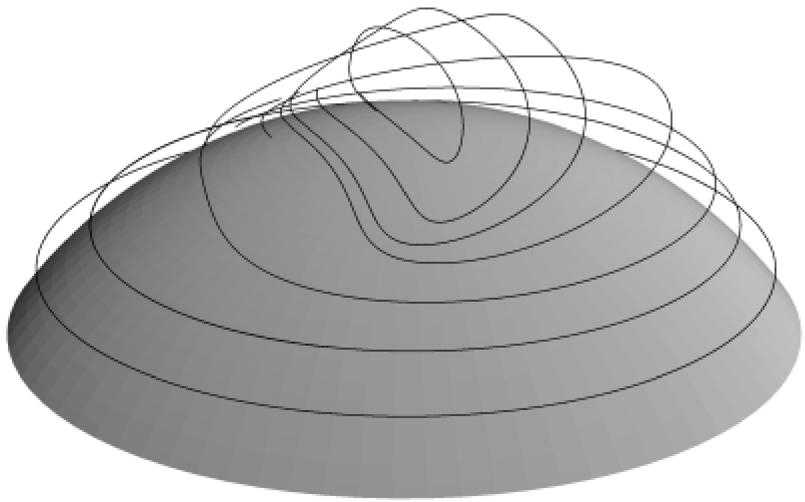


- Of course the tachocline has a magnetic field.
- Field has 2 major effects
  - Magnetic buoyancy
  - Imparts tension to plasma
- Gilman & Fox (1997)
  - Joint instability of toroidal flow and toroidal field

$$b = a \sin\theta \cos\theta$$

  - cf MRI (Balbus & Hawley 1992, Ogilvie & Pringle 1996)
  - m=1 mode.
- Maxwell stresses in the nonlinear regime would act so as to transport angular momentum towards pole (opposite to Reynolds stresses)

Follow up Papers…



Cally (2003) -- three dimensional disturbances whole nature of instability can change…

Cally (2000), Hughes & Tobias (2001)
-- simplified problem aligned shear flow and field.
-- modified semi-circle Theorem

- Gilman & Fox (1999)
  - Node between pole and equator and pole
- Dikpati & Gilman (1999)
  - Narrow band of toroidal field
- Gilman & Dikpati (2000)
  - Narrow band of toroidal field at different latitudes…
- Gilman & Dikpati (2002)
  - Broad profile in shallow water MHD (I)
  - Derive an α-effect (ho-hum)

Other Magnetic Instabilities: Magnetic Buoyancy

A stratified horizontal magnetic field that increase with depth supports more gas than would be possible in its absence. Atmosphere is, to some extent, top-heavy. Release of gravitational potential energy can result in instability (Newcomb 1961, Parker 1966).

Instability to three-dimensional modes (under ideal MHD) if:

$$\frac{d\rho}{dz} < \frac{\rho^2 g}{\gamma p}$$

$$\left(\frac{B^2}{\mu_0 p}\right)\frac{d \ln B}{dz} > -\frac{d \ln(p\rho^{-\gamma})}{dz}$$

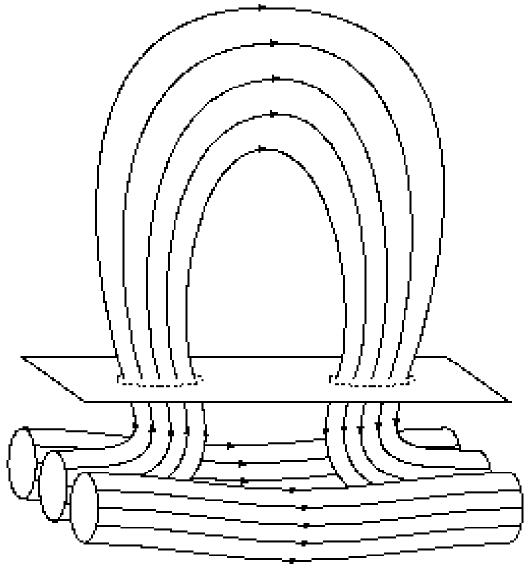
STATIC ATMOSPHERE

Instability to interchange modes (no bending of the field lines) if:

$$\frac{d\rho}{dz} < \frac{\rho^2 g}{\gamma p + B^2/\mu_0}$$

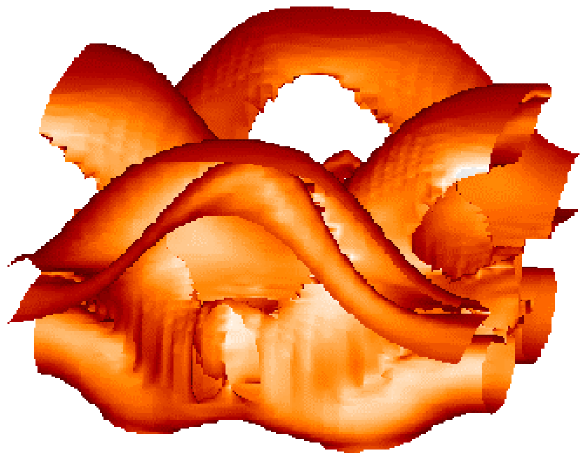
$$\left(\frac{B^2}{\mu_0 p}\right)\frac{d}{dz}\ln\left(\frac{B}{\rho}\right) > -\frac{d \ln(p\rho^{-\gamma})}{dz}$$

Sketch of emergence of magnetic field as bipolar regions (after Parker 1979).



Simulation of 3d nonlinear evolution of magnetic buoyancy instability of a layer of magnetic gas.

(Matthews, Hughes & Proctor 1995)



Magnetic Buoyancy and shear

- It is possible to address
  - The effect of buoyancy on shear flow instabilities
    - Via energy principle – can derive stability criteria (next slide)
    - Magnetic buoyancy can encourage radial motions
  - The effect of shear flows on magnetic buoyancy instabilities
    - Shear flows can stabilise and axisymmetrise magnetic buoyancy instabilities

    (Tobias & Hughes 2004)

    (see also Brummell et al 2002, Cline et al 2003, Howes et al 2002)

Magnetic Buoyancy with Shear

Provided that $U^2 < c_T^2$ everywhere, stability of the equilibrium state is assured if the following inequality is everywhere satisfied:

$$(a^2 + c^2)(c_T^2 - U^2)\left(\rho k^2 (a^2 - U^2) + g\frac{d\rho}{dz}\right) \geq \rho g^2 (a^2 - U^2).$$

In particular, stability to *all* modes (for any finite wavenumber) is guaranteed if, everywhere,

$$U^2 \leq \frac{a^2(c^2\rho' - \rho g)}{(a^2 + c^2)\rho' - \rho g}.$$

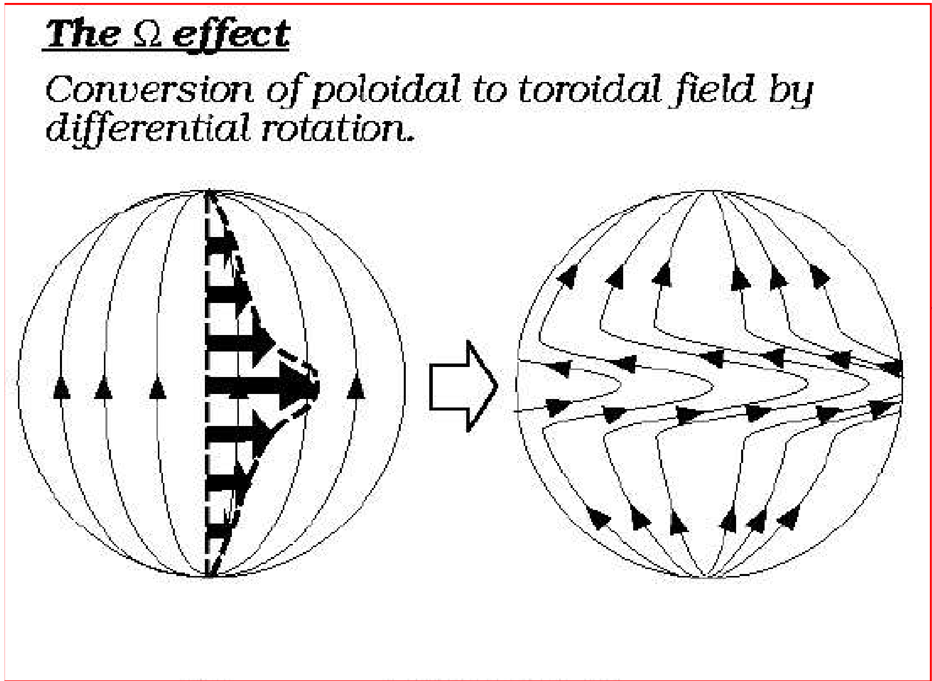
(+ Galilean transformations)

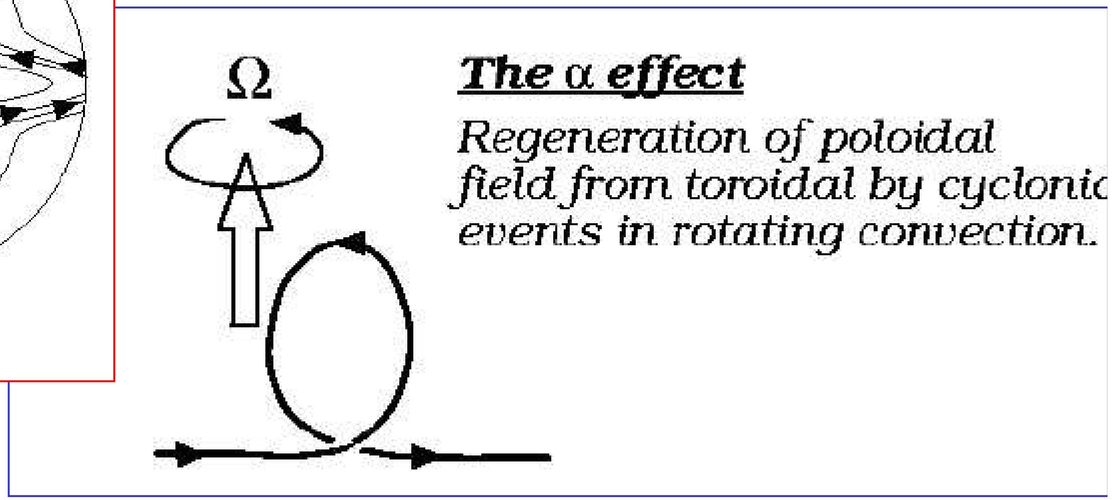
Extension of Adam's (1978) result.

$$-\frac{d}{dz}\left(\frac{\rho g k^2 U^2 (a^2 - U^2)}{(k^2 + l^2)(a^2 + c^2)(c_T^2 - U^2) + k^2 U^4}\right) < g\frac{d\rho}{dz} + \rho k^2 (a^2 - U^2) - \frac{\rho g^2 (k^2 + l^2)(a^2 - U^2)}{(k^2 + l^2)(a^2 + c^2)(c_T^2 - U^2) + k^2 U^4}.$$

The tachocline and the solar dynamo

Most solar and stellar dynamo modelling invokes *mean field theory*.
Simple to think of a Physical Picture





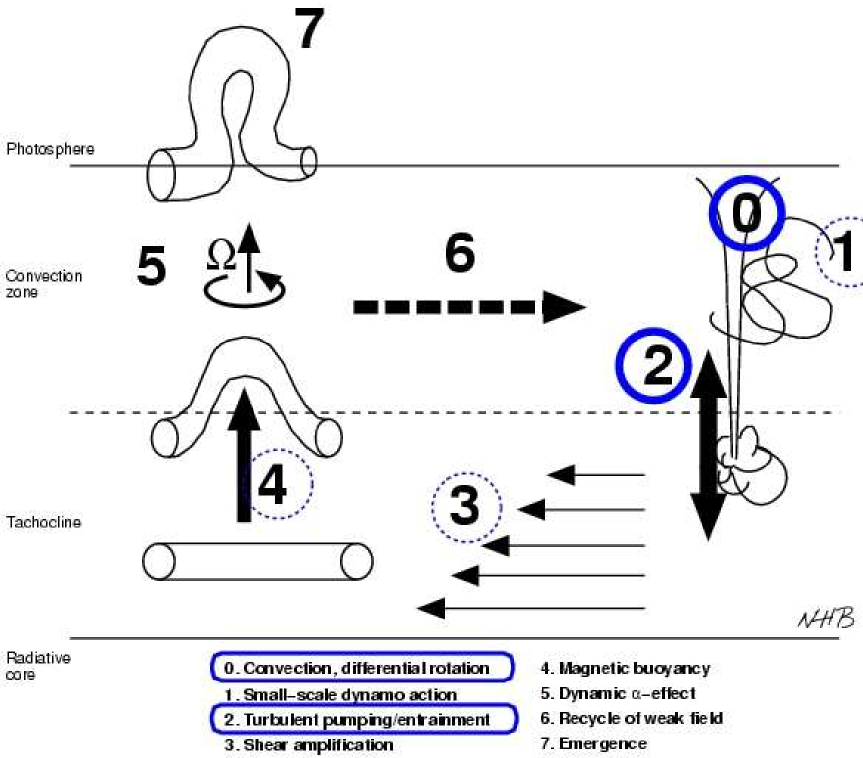
Interface Dynamo scenario

- **Considerations of MHD turbulence (Vainshtein & Cattaneo 1991, Cattaneo & Hughes 1996) suggest that the α-effect may be switched off dramatically in the presence of a mean magnetic field at high Rm.**

$$\alpha = \frac{\alpha_0}{1 + Rm^{\gamma} \dfrac{\langle B \rangle^2}{U_{eq}^2}}$$

- **Yet the Sun seems to be a jolly good dynamo…**
- **This has led many authors to consider an "interface dynamo scenario" (Parker 1993) where the strong mean toroidal (sunspot field) is created and stored in the tachocline.**
- **And the α-effect relies on turbulence in the convection zone.**
  - **Although other "dynamic α-effects" are possible**

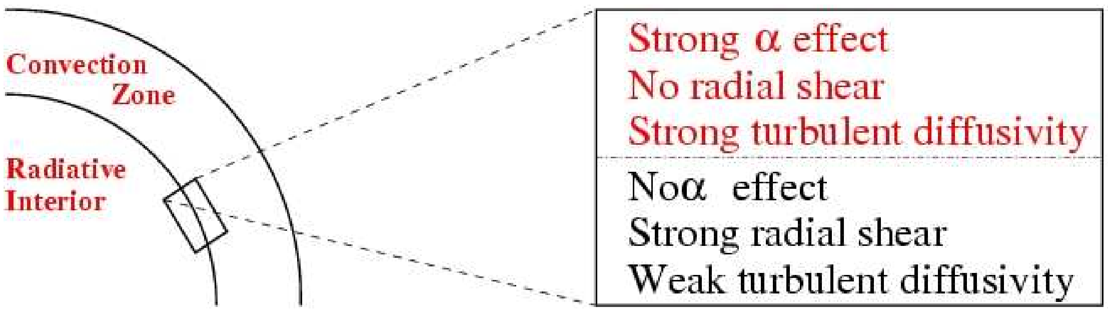
Interface Dynamo scenario



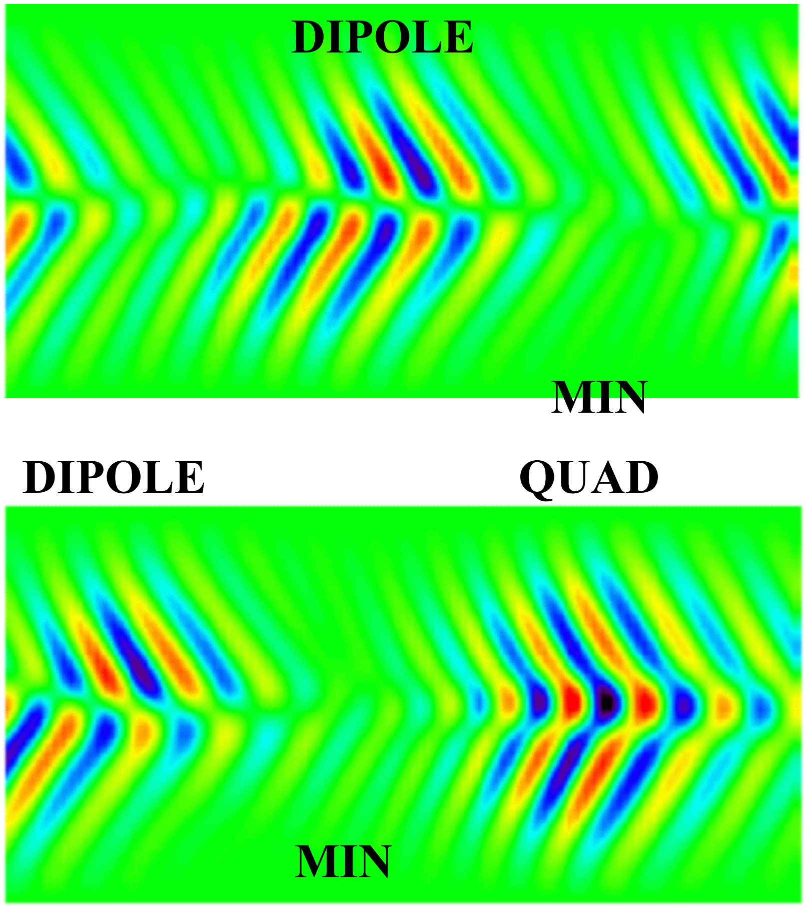
STRONG MAGNETIC FIELD IS GENERATED JUST BELOW THE INTERFACE.

MEAN FIELD IN THE CONVECTION ZONE IS WEAK.

DELICATE BALANCE…



Results from Interface Mean Field Models



TOBIAS (1996,1997) BEER ET AL (1998)

- Interface models have proved very successful at reproducing large-scale structure of Solar magnetic field.
- Even when α is catastrophically quenched (Charbonneau & MacGregor 1997)
- Computer-generated Butterfly diagrams
- Rich behaviour
- Modulation increases as rotation and activity increases – dynamo acts as a relaxation oscillator

But…

- **These mean-field models parameterise the effects of the turbulence.**
  - **"Fitting an elephant with 4 parameters"**
  - **Useful for discovering dynamic role of field once it has been generated.**
- **What is real role of MHD turbulence?**
- **Not so easy to do…**
- **Have to understand underlying physics**
- **E.g. transport via magnetic pumping.**
  - **How does magnetic flux get transported from the convection zone into the stably stratified tachocline below – see Nic Brummell's talk**

Friction or Antifriction?

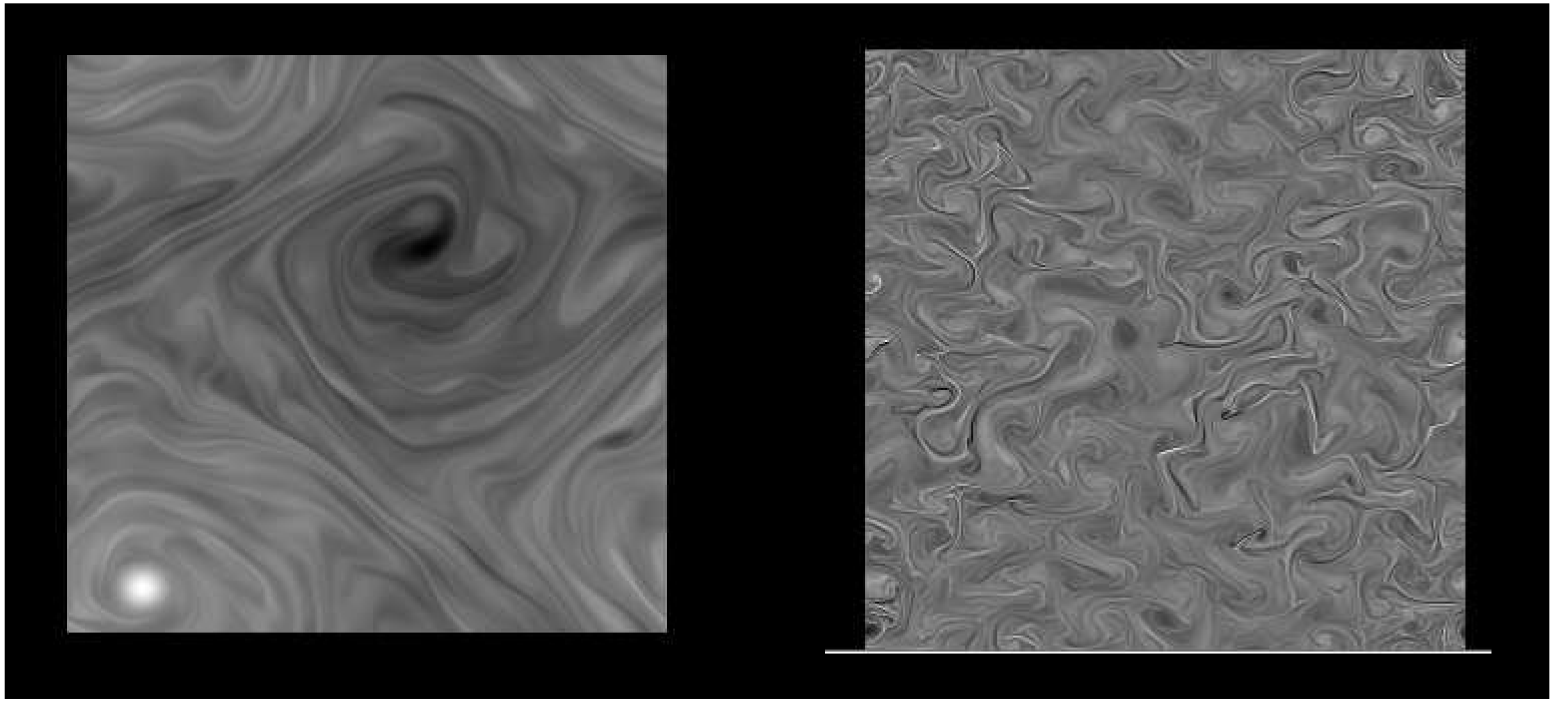
- How does the turbulence in the tachocline act so as to transport angular momentum?
  - Spiegel & Zahn – acts as an anisotropic viscosity
  - Gough & McIntyre – acts to redistribute PV
    - Antifriction (Hydrodynamic)
    - Argue that dynamo generated mean field does not penetrate far into tachocline. "Field Free Hypothesis"
  - For what values of mean field does turbulence act as 2D hydro?

$$\partial_t \omega + J(\psi, \omega) = B_0 \partial_x \nabla^2 A + J(A, \nabla^2 A) + \upsilon \nabla^2 \omega + F_\omega$$

$$\omega = -\nabla^2 \psi$$

$$\partial_t A + J(\psi, A) = B_0 \partial_x \psi + \eta \nabla^2 A$$

Friction or Antifriction



2D-HYDRO

Inverse-Cascade

Dominated by large-scale coherent structures.

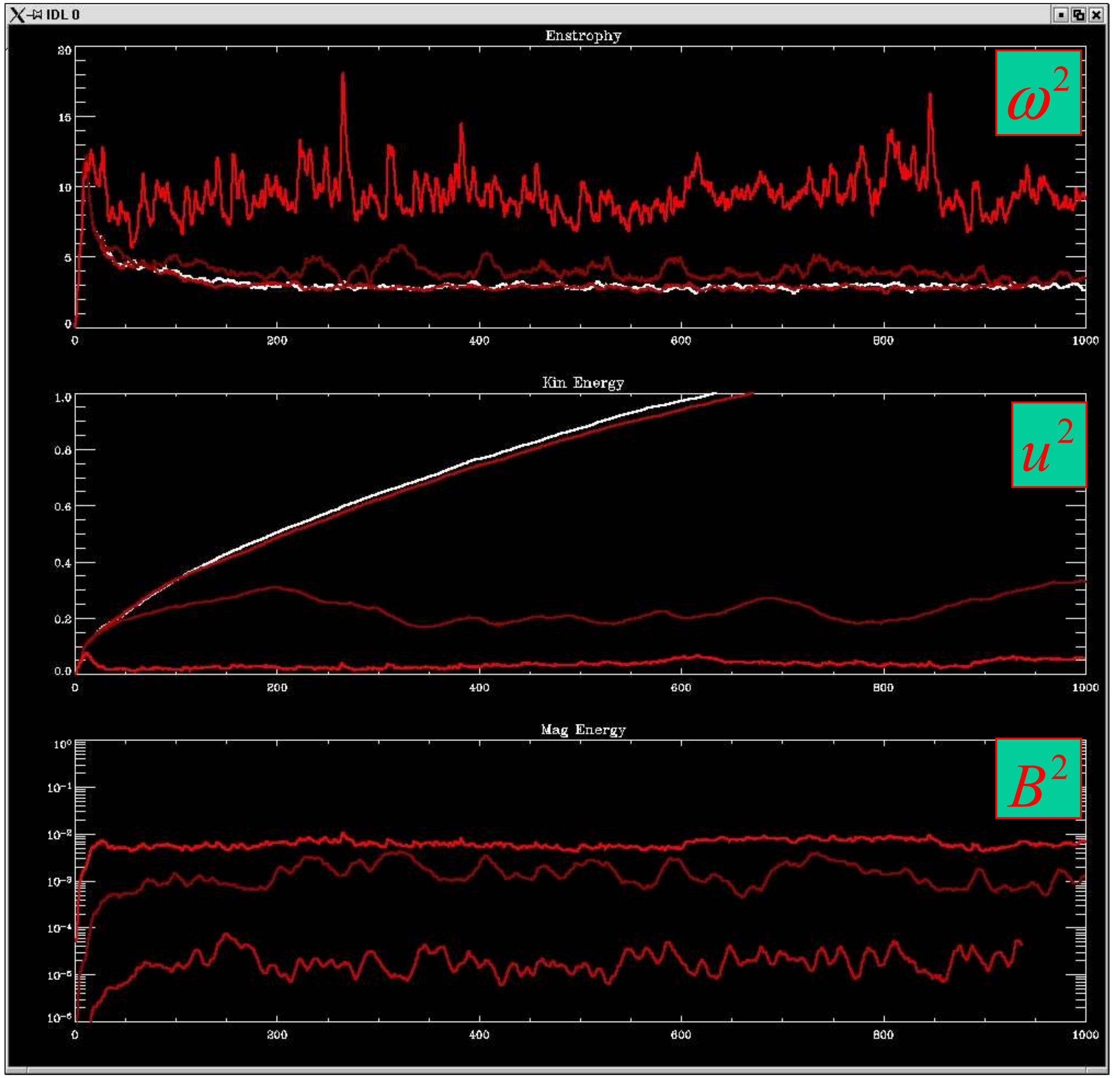
2D-MHD

Cascade

Dominated by small-scale Alfvenic turbulence.

DIFFERENT MOMENTUM TRANSPORT PROPERTIES

Friction or Antifriction



- As magnetic field strength is increased 2 distinct transitions occur

$$B_0^{\ 2} \approx Re^{-1} Rm^{-1} \langle U^2 \rangle$$

- Coupling to ohmic dissipation, energy in large-scale flows drops significantly

$$B_0^{\ 2} \approx Rm^{-1} \langle U^2 \rangle$$

- Inverse cascade halted → forward cascade. Enstrophy jumps.(see Pat tomorrow)
- Hydro studies likely to have extremely limited range of applicability

2D β-plane MHD

Simplest to address interaction of rotation and magnetic fields in the tachocline by studying β-plane MHD.

- In particular study
  - Formation of mean flows (a la Rhines)
  - Momentum transport due to β-plane MHD
  - (Diamond et al 2005)

β-plane MHD

- B-plane MHD (x-toroidal, y latitudinal, z radial – 2d in x and y)
  - Assume simplest configuration B_0 in x-direction
  - Rossby waves travel in x-direction.

$$\partial_t \omega + J(\psi,\omega) + \beta\partial_x \psi = B_0 \partial_x \nabla^2 A + J(A,\nabla^2 A) + \upsilon \nabla^2 \omega + F_\omega$$

$$\partial_t A + J(\psi, A) = B_0 \partial_x \psi + \eta \nabla^2 A + F_A$$

  - dispersion reln → identification of $k_{LR} = \left(\frac{\beta}{B_0} \right)^{1/2}$
- $k>k_{LR}$ Alvenically dominated
- $k<k_{LR}$ Rossby dominated

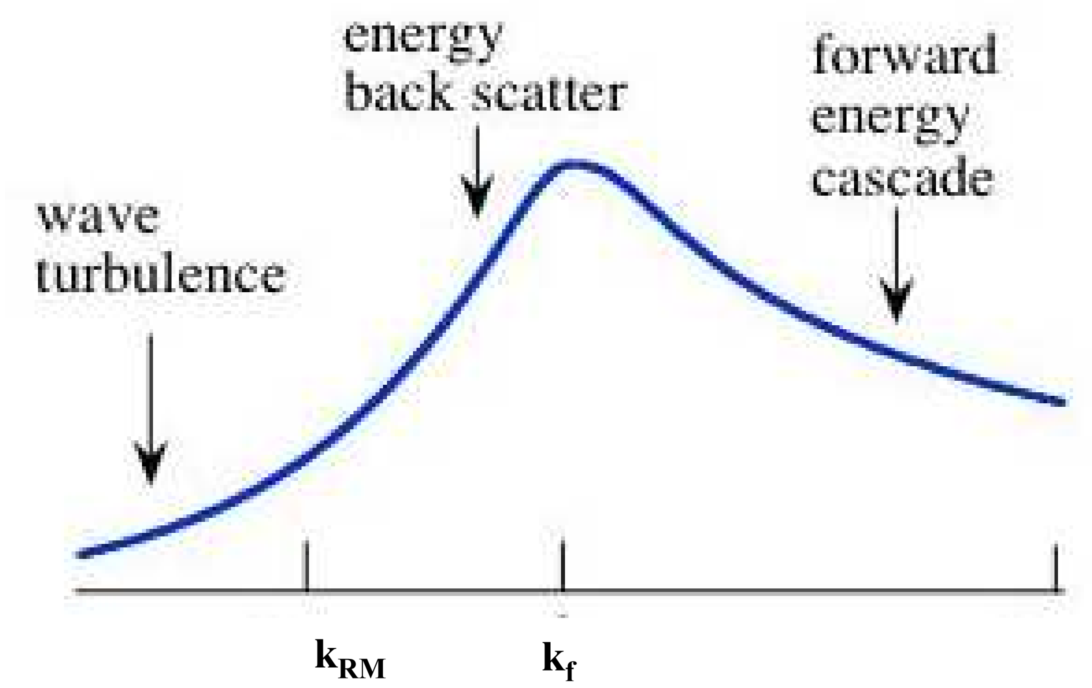
β-plane MHD

- Identification of "Magnetic Rhines Scale"
  - Balance between decorrelation rate and frequency determines boundary between turbulence being controlled by either turbulent interaction and resonant wave interaction (in hydro this can only occur with a zonal flow)
  - If decorrelation scales as kV defines Rhines scale
- Similar for MHD – but now decorrelation set by $k\tilde{V}_A$ where $\tilde{V}_A$ is Alfven speed due to fluctuating field
- Magnetic Rhines Scale given by

$$k_{RM} = \left(\frac{\beta}{\tilde{V}_A} \right)^{1/2} = Rm^{-1/4} k_{LR}$$

- But NO zonal flow formation.

2D MHD on a β-plane



Momentum transport depends on whereabouts in spectrum you are
Small-scales are 'Alfvenised' – very weak contribution to transport
Large-scales 'Rossby wave gas' may act as a viscosity, but sensitive To the form of the waves…

Some interesting questions

- Formation of tachocline.
  - Can delicate Gough/McIntyre balance really be achieved?
  - What is role of dynamo generated field?
- Stability of tachocline.
  - How 2D are the motions?
  - Where does the turbulence spread to? Convective Overshoot…
  - How is angular momentum transported – role of waves/turbulence?
    - Some way to an answer…
  - How does field escape to form active regions
    - Helicity injection from shear?
- Dynamo properties
  - Transport properties
    - How is turbulent transport affected by magnetic field/shear?
    - Is transport anisotropic of magnetic field/angular momentum?
    - Given that mean field dynamos can do so well, is there any "truth" in the formulation? If so, what of the objections?
  - What happens in fully convective stars/galaxies/discs?