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Geometry Changes Transient Transport in Plasmas

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Role of ballooning effect in toroidal plasmas on the transient transport problems is investigated. Due to the mode localization along the magnetic field line, a meso scale appears in a radial correlation length of fluctuating fields. This scale length introduces the interference of the gradient and flux in different radial locations. For the fluctuation which gives the gyro-Bohm-like diffusion in a stationary state, this long radial correlation of the fluctuating field causes a fast propagation of response against a rapid transient perturbation. Upper bound of transient thermal diffusivity is derived.

keywords: toroidal plasma, transient transport, ballooning mode, Kubo number, turbulence, meso scale

Problems of anomalous transport in toroidal plasmas have been one of the central subjects of modern plasma physics. Observation on transport phenomena has been performed via various methods, and understanding has been provided by the progress of the theory of strong turbulence in inhomogeneous plasmas [1]. One of the remaining enigma is known as 'transient transport problem' [2, 3]: It has been well known in experiments that the estimate of thermal diffusivity based on the stationary energy balance, χ_{pb} , and the one based on the transient response like heat pulse propagation, χ_{Hp} , have shown unambiguous discrepancy. χ_{Hp} is usually larger than χ_{pb} . This discrepancy is partly understood by noting the explicit dependence of χ_{pb} on the temperature gradient. If it has a dependence like $\chi \propto \left| \nabla T \right|^{\alpha}$, the effective thermal diffusivity measured by heat pulse propagation satisfies $\chi_{Hp} \simeq \left(1 + \alpha\right) \chi_{pb}$. However, this does not suffice. For instance, the change of thermal diffusivity in the core plasma after the onset of L/H transition at edge requires $\alpha \sim 50$ in order to explain the observation on JET [4]. The value $\alpha \sim 50$ contradicts to the transport property in stationary state, which requires $\alpha = 1 \sim 3/2$ [1]. In addition, a hysteresis relation in the local gradient and local heat flux has been reported from W7-AS experiments [3].

In order to understand these transient transport problems, there have been several theoretical achievements, which include a theory of heat clump propagation [5] and a non-local model of heat flux [6]. The latter model assumes the existence of the fluctuations with long radial correlation length, and has successfully explained various features of the transient transport problems. However, the physics basis for the fluctuations with long correlation length has not been given.

In this article, we address the problem of the transient transport in toroidal plasmas. It is shown that the turbulent fluctuations, which have short poloidal wave lengths ("micro scale", being of the order of ion gyroradius or collisionless skin depth) can have a long radial correlation length owing to the ballooning effect ℓ_E . The length ℓ_E is a meso scale, i.e., is intermediate length between the micro scale and the global scale which is characterized by the system size. As a result, the Kubo number of the turbulence, K , which is defined by the ratio of the correlation time to the eddy-turn-over time [7], becomes much smaller than unity $K << 1$. It is shown that the heat flux at a radius r is an integral of nonlocal heat over the radius $r - \ell_E < r < r + \ell_E$. Solving a integro differential equation for the transient response of temperature, fast transient response of transport is explained. The upper bound of χ_{Hp}/χ_{pb} is estimated.

Microinstabilities in tokamak plasmas have been analyzed, and it is well known that the mode amplitude is localized in a region of bad magnetic curvature [1]. This phenomenon is knows as 'ballooning' effect. Analyses have been given in, e.g., [8-11]. We study the simplest case of electrostatic ballooning mode in the absence of the radial electric field inhomogeneity. The eigenmode by the toroidal mode number n and central poloidal mode number M, $\phi_{n,M}(r, \theta, \zeta)$ (r, θ, ζ are minor radius, poloidal angle and toroidal angle, respectively). The eigenvalue and radial extent are given by the plasma parameters (such as density gradient, diamagnetic drift frequency ω_* and the gradient of ω_*) at the radius $r = r_{M,n}$ where $M = n\,q(r_{M,n})$ holds ($q = RB_\theta/rB_\zeta$ is the safety factor). In the limit of short wave number, the eigen mode is approximately given as

$$\phi_{n,M}(r, \theta, \zeta) \simeq \sum_m \phi_{n,M}(r - m\Delta)\exp\left(im(\theta + \theta_0) + iM\theta - in\zeta\right) f(m) \tag{1}$$

where Δ is the distance of neighbouring mode rational surfaces for fixed n, $\Delta = 1/nq'$, and θ_0 is the poloidal angle where the mode is localized [8-10]. The factor $f(m)$ ($f(0) = 1$) decays at large $|m|$ and limits summation over side-band Fourier components m as $|m| < m_* = \ell_E a^{-1} M$ (a : the minor radius of the plasma and ℓ_E : the radial extent of the eigenmode). When the radial derivative of real part of the eigenfrequency does not vanish, $d\,\Re e\omega_{M,n} / dr_{M,n} \neq 0$, one has an evaluation $m_* = 2\,\gamma \sin\theta_0 \left(s\,\Delta\, d\,\Re e\omega_{M,n}/dr_{M,n}\right)^{-1} M$ [11] where s is the magnetic shear. This bound m_* corresponds to the radial coherence length of the mode ℓ_E

$$\ell_E \simeq \sqrt{s^{-1} a \rho_i}\ . \tag{2}$$

This scale length is the hybrid between the microscopic length and global scale length. Within this radial extent, the eigenmode Eq.(1) evolves coherently. A one-time coherence length (Euler view of correlation length) is evaluated by Eq.(2).

The plasma flux by the ballooning mode has been obtained. The particle flux at radius r by the kinetic ballooning mode has been derived as [12]

$$\Gamma_r(r) = \frac{nq}{B_0 r}\frac{e}{T_e}\sum_{n,M} F_{M,n}(r) \tag{3}$$

where

$$\begin{aligned} F_{M,n}(r) = &\sum_{|m|<m_*} \Im m\left(\frac{\omega - \omega_*}{\omega}\,\xi_e Z\right)\phi_m\phi_m^* \\ &+ \sum_{|m|<m_*} + \frac{r}{2R}\frac{L_n}{R}\frac{\omega_*}{\omega}\frac{\omega-\omega_*}{\omega}\, m\, \Re e\left(\xi_e Z + 2\xi_e^2(1 + \xi_e Z)\right) \\ &\times \Im m\left(e^{-i\theta_0}\phi_{m-1}\phi_m^* + e^{i\theta_0}\phi_{m+1}\phi_m^*\right), \end{aligned} \tag{4}$$

Z is the plasma dispersion function, the argument of which is given as $\xi_e = \omega/|k_\parallel| v_{\mathrm{th\,e}}$, $k_\parallel = (M + m - nq)/qR$, $v_{\mathrm{th\,e}}$ is the electron thermal velocity, L_n is the density gradient scale length, and an abbreviation $\phi_m = \phi_{n,M}(r - m\Delta)$ is used. This result shows that the transport by the (M, n) -mode, $F_{M,n}(r)$, is extended in the region

$$r_{n,M} - \ell_E < r < r_{n,M} + \ell_E \ . \tag{5}$$

Transport at the radius r is given by the summation of $F_{M,n}(r)$ over (M, n) . Because each $F_{M,n}(r)$ has finite value in the region Eq.(5), those (M, n) -modes that satisfy $nq(r) - m_* \le M \le nq(r) + m_*$ contribute to $\Gamma_r(r)$. The (M, n) -mode is also labeled by the location of the central mode rational surface $r_{M,n}$, and the summation over M in Eq.(3) is approximately replaced by the integral over $r_{M,n}$ as

$$\Gamma_r(r) = \frac{nq}{B_0 r} \frac{e}{T_e} \sum_n \int_{r-\ell_E}^{r+\ell_E} \mathrm{d}r_{M,n}\, \Delta^{-1} \hat{F}_{M,n}\left(r - r_{M,n}\right) \tag{6}$$

where we rewrite as $F_{M,n}(r) = \hat{F}_{M,n}\left(r - r_{M,n}\right)$. Each flux $\hat{F}_{M,n}\left(r - r_{M,n}\right)$ is contributed from the plasma parameters at $r = r_{M,n}$. A similar integral form is also derived for the energy flux. When one employs the quasilinear estimate of transport coefficient $\gamma / k_\perp^2$ for the integrand, the growth rate and the mode number of the most unstable mode are determined by the dispersion relation on the radius $r = r_{M,n}$. The heat flux at the radius r is influenced not only by the parameters at r but also by those in the region Eq.(5). A nonlocal form of the transport flux is explicitly derived when the fluctuation mode has the ballooning forms.

The turbulence level of $e\tilde{\phi}/T_e \simeq 1/k_\perp L_n$ with $k_\perp \rho_i \simeq 1$ and the decorrelation rate of $\gamma_{dec} \simeq c_s / L_n$ have been obtained for the fluctuations in the range of drift wave frequencies [1]. For this level of fluctuation amplitude, the thermal diffusivity has been given to be of the order of $\rho_i T / L_n eB$, i.e., the so-called gyro-Bohm diffusion. The Kubo number for this case is much smaller than unity. The $E \times B$ velocity is estimated as $\tilde{V}_{E \times B} \simeq \rho_i c_s L_n^{-1}$, and the eddy-turn-over time is given as $\tau_{et} \simeq \ell_E L_n / \rho_i c_s$. The Kubo number $K = \gamma_{dec}^{-1} \tau_{et}^{-1}$ is evaluated as

$$K \simeq \rho_i / \ell_E \simeq \sqrt{s \rho_i / L_n} \tag{7}$$

and is much smaller than unity. This result means that the one-time autocorrelation length of fluctuation fields (Euler's view) is much longer than the autocorrelation length of fluctuating motion of plasma elements.

The transient response of transport is analyzed based on the nonlocal expression of fluxes. An essential element of the result in Eq.(6) is that the flux at r is influenced by the gradient at $r = r_{M,n}$, where $\left| r - r_{M,n} \right| < \ell_E$ is satisfied. The simplest model which includes this nonlocal interference is given for the gradient-flux relation as

$$q = - \int dr' \, \chi \, K(r - r') T'(r') \tag{8}$$

where χ is a thermal transport coefficient and is

$$K(r - r') = \pi^{-1/2} \ell_E^{-1} \exp\left(- (r - r')^2 \ell_E^{-2} \right) \tag{9}$$

In general, χ can be a nonlinear function of the gradient of temperature and this nonlinear dependence introduces a difference between χ_{Hp} and χ_{pb} . A simple model of constant χ is used here, because we are analyzing a mechanism to cause the large deviation of χ_{Hp} from χ_{pb} , which is not explained by the nonlinear dependence of χ on dT/dr .

A transient response of the form $T(x, t) \propto \exp(-i\Omega t)$ is studied for the transport equation

$$\frac{\partial}{\partial t} T = - \frac{\partial}{\partial x} q \; . \tag{10}$$

Study in a slab geometry suffices, because $\ell_E << a$ is satisfied. A temporal evolution of the form $T(x, t) \propto \exp(-i\Omega t)$ is given at a boundary, and the asymptotic response in the downstream of the form

$$T(x, t) \propto \exp(-i\Omega t + ikx) \tag{11}$$

is searched. Substituting Eq. (8) into Eq.(10), one has the equation for $k(\Omega)$ as

$$i\Omega = \chi \, k^2 \exp\left(- k^2 \ell_E^2 / 4 \right) . \tag{12}$$

If one interprets the transport as a diffusive process, the phase of perturbation is given by use of the effective diffusivity χ_{eff} as $T(x) \propto \exp\left(i \sqrt{\Omega / 2\chi_{eff}} \, x \right)$. By use of $\Re\mathrm{e}\, k(\Omega)$, χ_{eff} is formally given as

$$\chi_{eff} = \frac{\Omega}{2 \left(\Re\mathrm{e}\, k \right)^2} \; . \tag{13}$$

A solution of the equation Eq.(12) with χ_{eff} is illustrated in Fig.1. In the limit of $\Omega \to 0$, we have $\Re\mathrm{e}\, k(\Omega) \to \sqrt{\Omega/2\chi}$ and $\chi_{eff} \to \chi$. In the case of larger Ω , $\Omega > \chi \ell_E^{-2}$, numerical solution of Eq.(12) provides a relation

$$\chi_{eff}/\chi \simeq 3\Omega\, \ell_E^2/4\chi \ . \tag{14}$$

The faster the temporal change, the larger the effective diffusion coefficient.

The upper bound of the effective transport coefficient is expressed in terms of the Kubo number. The form of the transport flux, e.g., Eq.(6), is derived by averaging over the period which is longer than the correlation time, γ_{dec}^{-1} . That is, the oscillation frequency Ω must be lower than γ_{dec} , $\Omega \le \gamma_{dec}$. Combining this bound with Eq.(14), one has $\chi_{eff}/\chi < 3\gamma_{dec}\, \ell_E^2/4\chi$. Noting that the thermal conductivity is given by an estimate of $\chi \simeq \gamma_{dec} k_\perp^{-2} \simeq \gamma_{dec} \rho_i^2$, this upper bound of the effective thermal conductivity is rewritten as

$$\chi_{eff}/\chi \le K^{-2} \tag{15}$$

in the case of $K << 1$. For the case of the ballooning mode, Eq.(7), one has

$$\chi_{eff}/\chi \le a/s\rho_i \ . \tag{16}$$

This analysis suggests an important role of plasma geometrical configuration on the transient transport. A case of Heliotron configurations is discussed for illustration. In this system, plasmas can be either in the magnetic hill or the well configurations. In a hill case, the interchange mode turbulence can dominate, which has short radial correlation length. In the magnetic well geometry, the ballooning effect becomes strong and a meso scale correlation length appears: in addition, due to the toroidal asymmetry, the mode is also localized in toroidal direction. This toroidal localization further increases the radial extent of the eigenmode [13]. It is predicted that the fast transient response against high frequency modulation is much stronger for Heliotron plasmas in magnetic well configuration and is much weaker for those in magnetic hill configurations, in comparison with equivalent tokamak cases.

The form of the transient response Eq.(14) is consistent with observations in large tokamaks. For instance, the Euler view of the one-time correlation length of fluctuations has been observed in JT-60U plasma. In the region out of the transport barrier, one time correlation length is measured as $\ell_c \simeq 0.1m$, which is of the order of Eq.(7) [14]. When one uses a typical value of the L-mode of $\chi \simeq 10\ m^2/s$, Eq.(14) gives a relation $\chi_{eff}/\chi \sim 10^{-3}\, \Omega$. For the sudden change which is characterized by the time scale of the order of $100\mu s$, such as the L/H transition, the effective transport coefficient deduced from the transient response is given as $\chi_{eff}/\chi \sim 60$. This enhancement factor is in the range of experimental observations in large tokamaks at L/H transitions. Nonlocal effect could become ineffective in the transient response, if the modulation frequency is much lower than χ/ℓ_E^2 . In such a case, deviation of the transient transport coefficient from the one in a stationary power balance is contributed from, e.g., a nonlinear dependence of heat flux on the temperature gradient.

In summary, we have studied the role of ballooning effect in toroidal plasmas on the transient transport problems. A radial correlation length of fluctuating field has a meso scale. This introduces the interference of the gradient and flux in different radial locations. The fluctuating motion of plasma element is decorrelated before it circumnavigates the fluctuating vortex, so that the long radial correlation length does not necessarily appear explicitly in the transport coefficient of the stationary state. In a transient response, however, the long radial correlation of the fluctuating field causes a rapid propagation of perturbations. An enhancement factor χ_{eff}/χ in the transient response is obtained. The analysis explains an enigma of in transient transport problems. A hysteresis, in which the flux changes in advance to the gradient, is derived by use of the nonlocal transport model as has been shown in [6]. This study is to be extended to the case where stable, long-wave-length fluctuations are excited (passively) through nonlinear noise of microscopic fluctuations. Such a case will be reported elsewhere [15].

Action at distance has been discussed in conjunction with the Bohm scaling [16]. Such a nonlinear simulation could be extended for the study of transient transport problem. An important mechanism in the nonlocal transport comes from the strongly inhomogeneous radial electric field. The inhomogeneous radial electric field reduces the radial coherence length of fluctuations [1]. Suppression of transient transport across the transport barrier is known. This will be explained by the reduced radial correlation length of fluctuations.

The result in this article also illustrates an importance of Kubo number in characterizing plasma transport [17]. In a case of large Kubo number, $K >> 1$, a memory effect becomes important in the transient transport problems [18]. A subdiffusive transport is predicted and is different from the case of the nonlocal transport. Study on the one-time correlation length of fluctuating field (Euler view) and the correlation length of fluctuating motion (Lagrange view) must be further pursued in the problems of plasma turbulence. Observations on transient transport phenomena, together with those on fluctuations, will improve our understanding of plasma turbulence.

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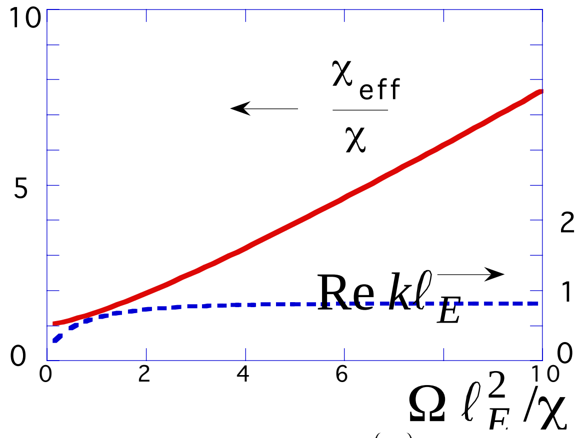

Fig.1: Wave number of temperature perturbation $\mathrm{Re}\, k(\omega)$ as a function of the modulation frequency Ω (dashed line). Effective thermal diffusivity χ_{eff} is illustrated by the solid line.

Figure Caption:

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