An area-preserving second order integration scheme for use in particle-in-cell codes

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An essential ingredient of particle-in-cell (PIC) codes with many particles and evolving on long time scales is a sufficiently simple, accurate and stable integration scheme for the electron and ion equations of motion. The usual, well-known [1], leapfrog (LF) integration scheme is perfectly time-centered, 2nd order accurate with respect to the time step Δt, and conditionally stable in the interval $\omega_0 \Delta t<2$ for linear oscillations ω_0. Unfortunately, the LF method cannot be used in many situations of interest where the force depends on velocity. Significant examples are dissipative systems, or the Langevin process which can represent particle velocity-space flow due to collisions or radio-frequency heating. In the present work an implementation of the 2nd order Runge–Kutta (RK) integration scheme – a semi-implicit midpoint RK scheme - is presented, which preserves phase space measure and possesses the same numerical stability as the LF scheme. We test the new integrator in three examples of interest. First, in the non-linear interaction of particles with a plane wave, emphasizing the importance of intrinsic stochasticity in the destruction of orbits of an otherwise exactly integrable equation [2]. Second, in the case of of particle motion in static magnetic field, and finally, in a non-linear dissipative system leading to a limit cycle. In particular, for the plane wave we show that the semi-implicit midpoint RK scheme is equivalent to the standard map. We find the stochasticity threshold $\omega_B \Delta t<1$, where ω_B is the particle bounce frequency and Δt is the time step.

[1] C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulation* (Hilger, Bristol, 1991).

[2] A. Friedman and S. P. Auerbach, J. Comp. Phys. 93, 171 (1991).