Nonlinear dynamics of magnetic islands

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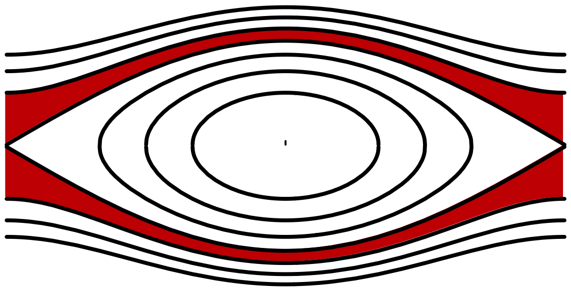
IFS

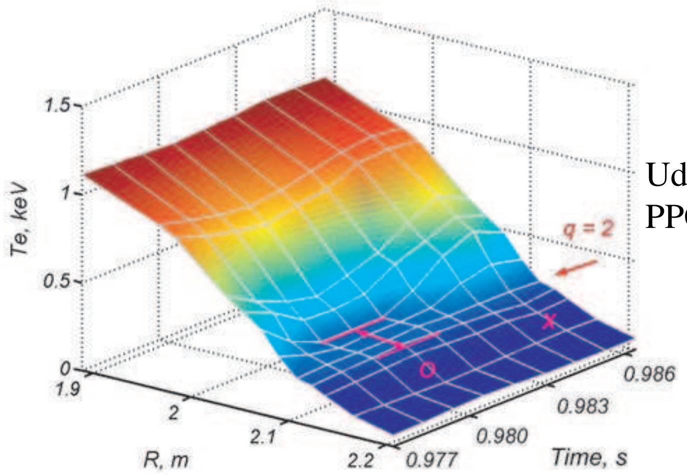
Outline

- Introduction
- Part I: MHD model
- Part II: Drift model

Introduction

- Magnetic islands are regions of *good flux surfaces* surrounding a secondary magnetic axis.
- Transport is much faster along the magnetic field lines than across the magnetic field lines. As a result of this, magnetic islands provide a short-circuit for the transport fluxes.



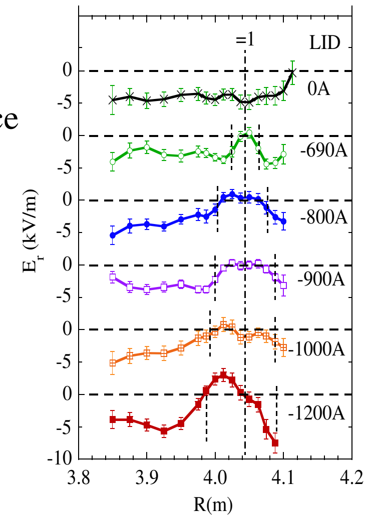


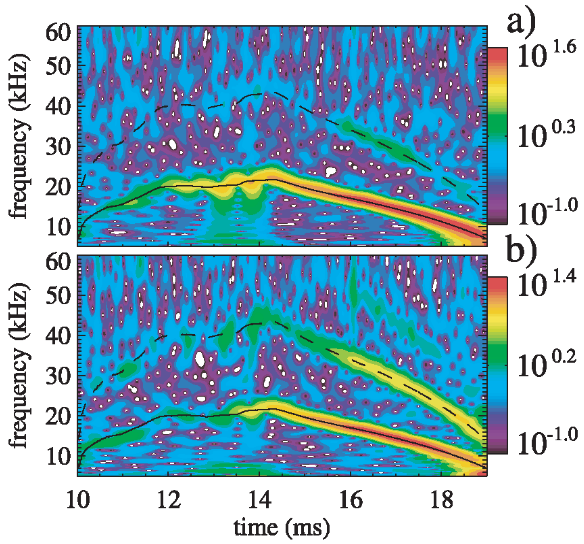
Udintsev et al.
PPCF 2003

Magnetic islands affect confinement

- Magnetic islands play an important role in all toroidal magnetic confinement devices including stellarators, reversed-field pinches and tokamaks. They affect confinement directly as well as through their effect on plasma rotation.

Large Helical Device
Ida et al., NF2004

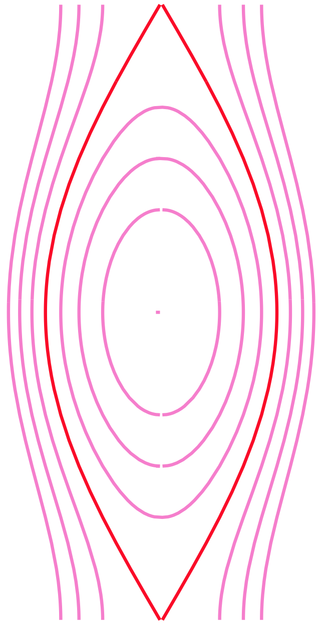


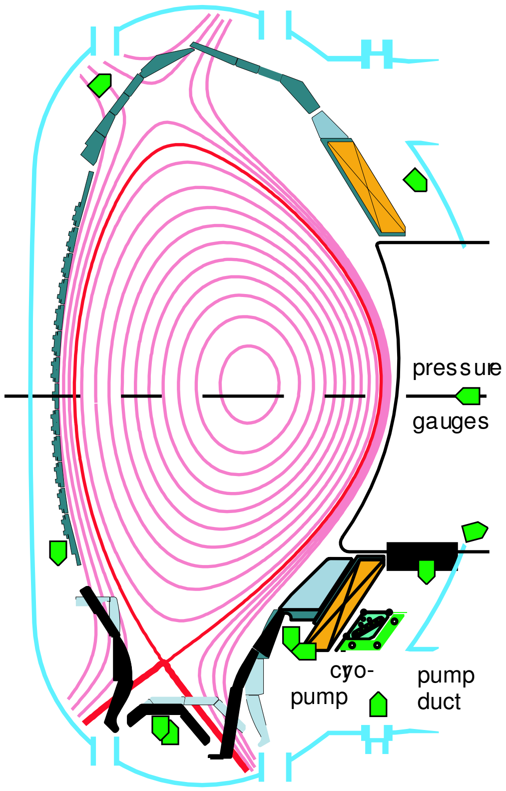


Madison Symmetric Torus
Franz et al. PRL 2004

Islands are similar to full plasma

- Predicting the evolution of magnetic islands is conceptually similar to predicting the evolution of a discharge.





Equilibrium and transport (II)

- Predicting the evolution of a discharge involves two interdependent steps:
  1. Calculate the equilibrium given the pressure and current profiles. In a tokamak,
  $$R^2\nabla(R^{-2}\nabla\psi) = -4\pi R^2 \frac{dP}{d\psi} - I\frac{dI}{d/\psi}.$$
  2. Calculate the evolution of the profiles given the geometry. This involves solving transport equations of the type
  $$\frac{\partial T}{\partial t} = \frac{d}{d\psi}\left(\langle\kappa\rangle\frac{\partial T}{\partial\psi}\right).$$
- The role of the external currents (coils) are often specified through the shape of the last closed flux surface.

For an island, the external currents are specified by Δ'

- Solution of the linear MHD equations outside the island determine

$$\Delta' = \frac{\tilde{\psi}'(0^+) - \tilde{\psi}'(0^-)}{\tilde{\psi}(0)} = \frac{1}{\tilde{\psi}(0)} \int dx\, \tilde{J} \sim \frac{\tilde{B}_y}{\tilde{E}} \sim Y.$$

- Δ' can be thought of as the admittance of the plasma for the shear-Alfvén wave.

- Δ' can also be shown to be proportional to the free energy available for reconnection (Furth et al. 1973).

- Near marginal stability $\Delta' \sim 1/\delta W$, where δW is the ideal MHD energy.

Part I: MHD model

- Current diffusion
- Rutherford regime
- Kink-tearing regime
- Transition

There are 2 types of islands.

- Magnetic islands can be divided into slowly growing and rapidly growing islands depending on how their growth rate compares to the inverse skin time for the island.
  - The skin time is $\tau = W^2/\eta$ where W is the island half-width.
  - The growth rate is $\gamma = \eta \tilde{J}/\tilde{\psi} = \eta \Delta'/W$.
- For $\gamma\tau = \tilde{J}/J_0 = W\Delta' \ll 1$ The island grows slowly and the profiles are at all time equilibrated. This is the constant-$\tilde{\psi}$ regime first studied by Rutherford (1973).
- For $\gamma\tau = \tilde{J}/J_0 = W\Delta' \gg 1$ the island grows faster than the current can diffuse, resulting in current singularities and fast reconnection.

$W\Delta' \ll 1$: the Rutherford regime

- For $W\Delta' \ll 1$, $\tilde{J}_z = \nabla^2\tilde{\psi} \ll J_{z0}$ so

$$\psi = \psi_0(x) + \tilde{\psi}(t)\cos y.$$

- The island geometry is entirely determined in terms of its half-width, $W = 2\sqrt{\tilde{\psi}/B'_y}$.
- The growth is determined by the Ohm's law

$$\langle \frac{\partial \psi}{\partial t} \rangle = \eta I(\psi) \quad \Longrightarrow \quad \dot{W} = 1.22\eta\Delta'/\mu_0.$$

$W\Delta' \gg 1$: the kink-tearing regime

- The equilibrium equation is $\mathbf{B} \cdot \nabla J_{\|} = 0$. The solution is

$$J_{\|} \simeq \frac{\partial^2 \psi}{\partial x^2} = J(\psi) = \frac{dI}{d\psi}.$$

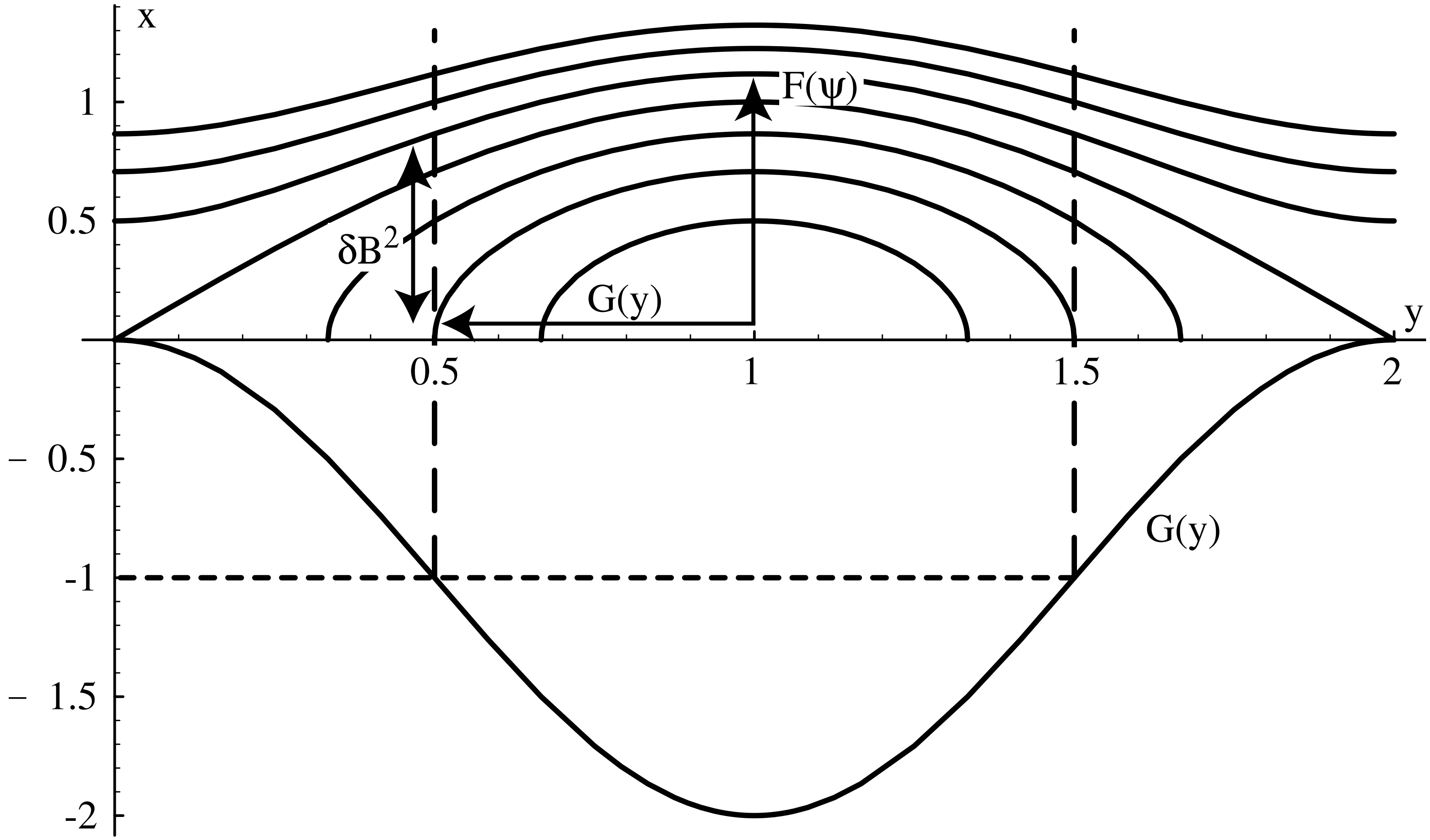
- Compare this to Newton's equation for a conservative force, $m\ddot{z} = eE(z) = -ed\varphi/dz$. Integration yields

$$B_y = \sqrt{2[I(\psi) - G(y)]}.$$

- Integrating the inverse of $B_y = \partial\psi/\partial x$ yields

$$x(\psi, y) = x_t(y) \pm \int_{\psi_t}^{\psi} \frac{d\psi}{B_y}.$$

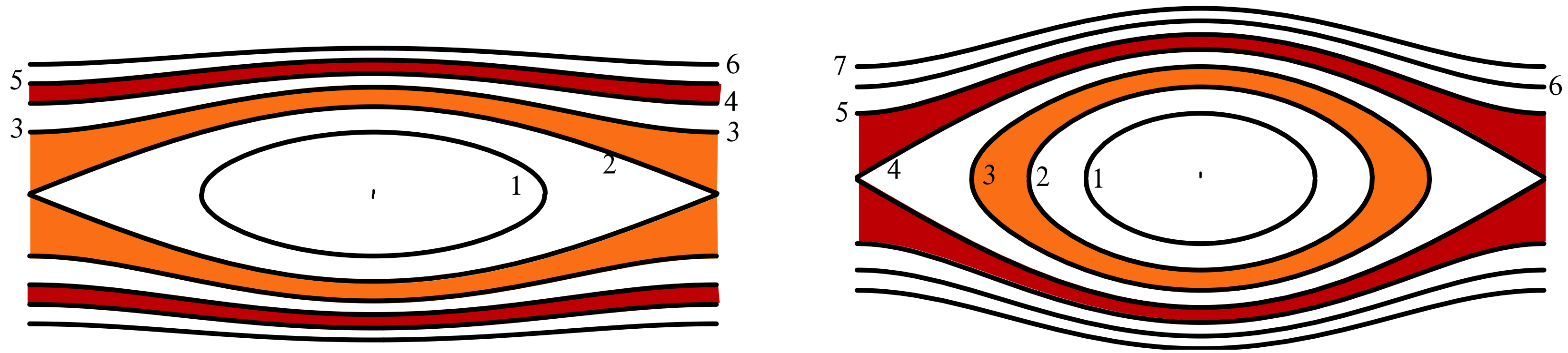
$W\Delta' \gg 1$: geometric interpretation



$W\Delta' \gg 1$: detailed helicity conservation

- If the island grows much faster than the skin time W^2/η, the flux is frozen in and helicity is conserved *for all pairs of flux tubes*:

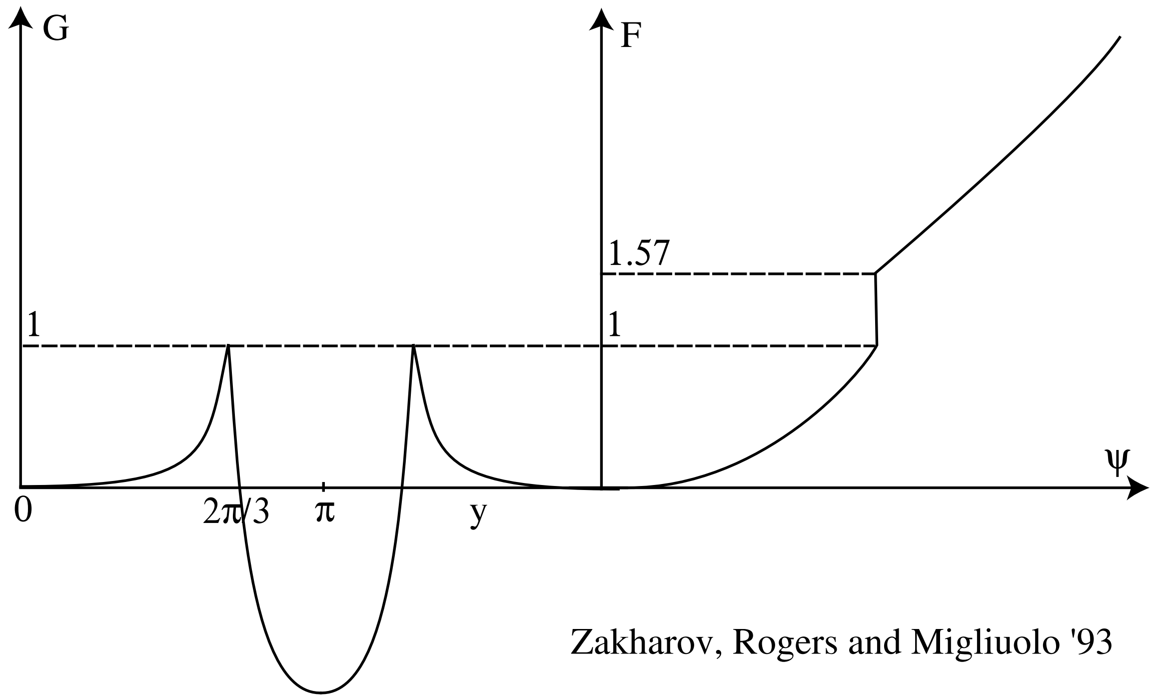
$$\oint_{3-4} \frac{dy}{B_y} = \oint_{3'-4'} \frac{dy}{B_y}.$$



Kadomtsev '75, Waelbroeck '89.

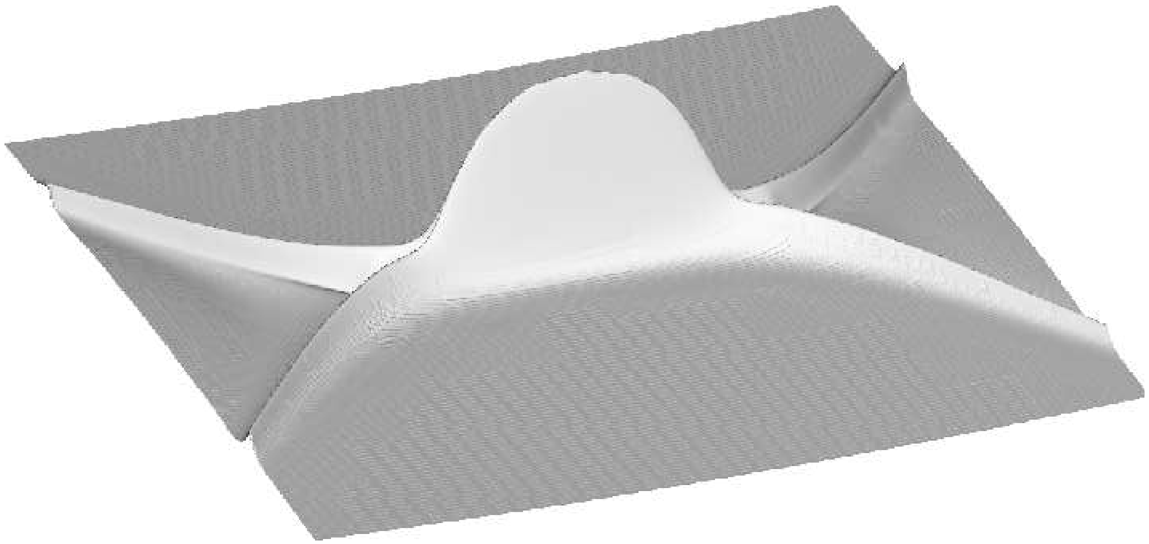
$W\Delta' \gg 1$: X-point → 2 Y-points separated by a current ribbon

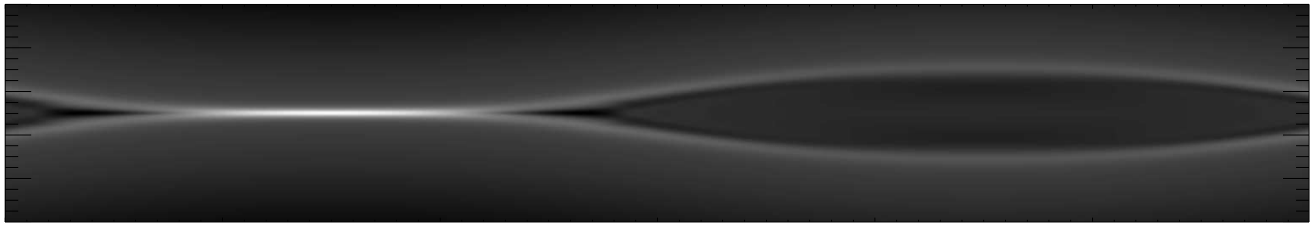
$$\frac{1}{x} = \frac{1}{\sqrt{\psi}} = \oint \frac{dy}{\sqrt{F(\psi) - G(y)}}$$



Zakharov, Rogers and Migliuolo '93

The ribbon is clearly observed in simulations



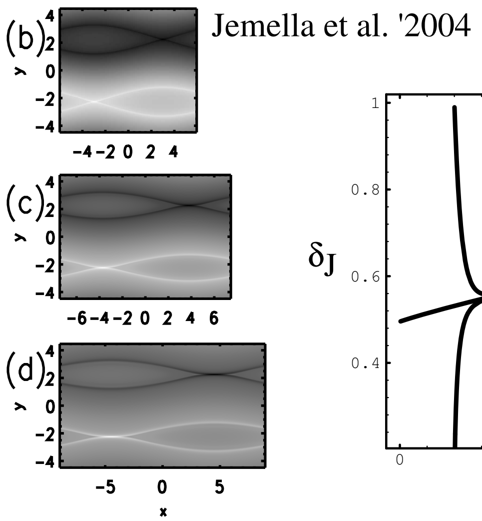


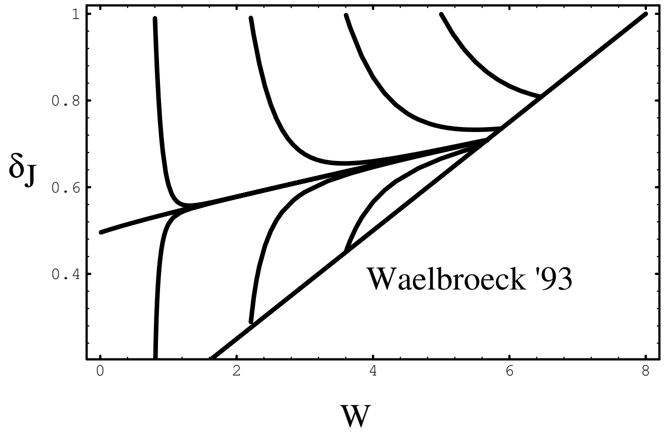
$W\Delta \simeq 1$: transition and ribbon formation

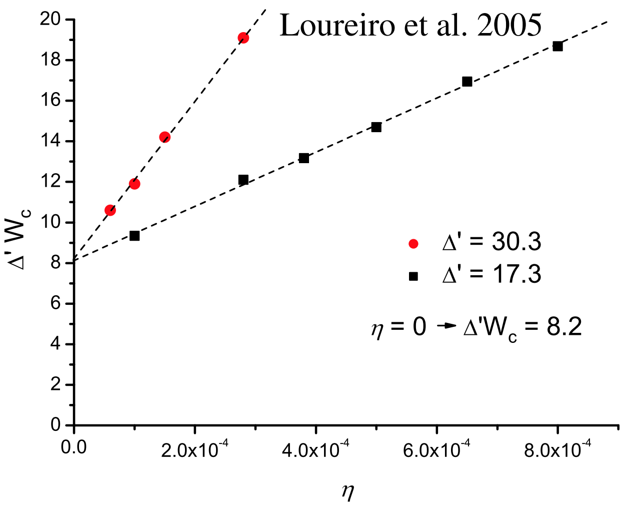
- In general ($\Delta' W \sim 1$ the current is determined by averaging Ohm's law:

$$\langle \frac{\partial \psi}{\partial t} \rangle = \eta J(\psi).$$

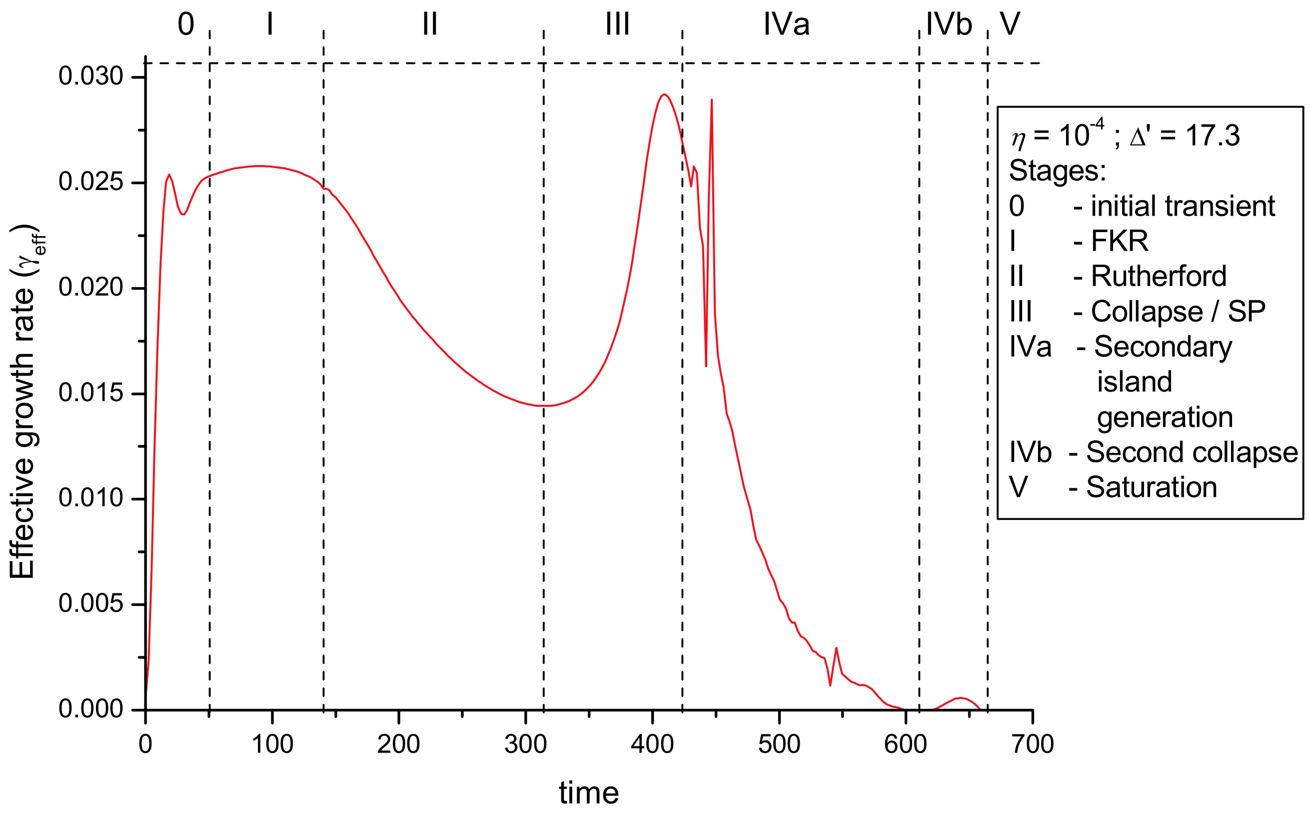
- Using parametrized model current profiles leads to the conclusion that the X-point collapses and splits into Y-points.







Summary of island magnetohydrodynamics



Loureiro et al., 2005.

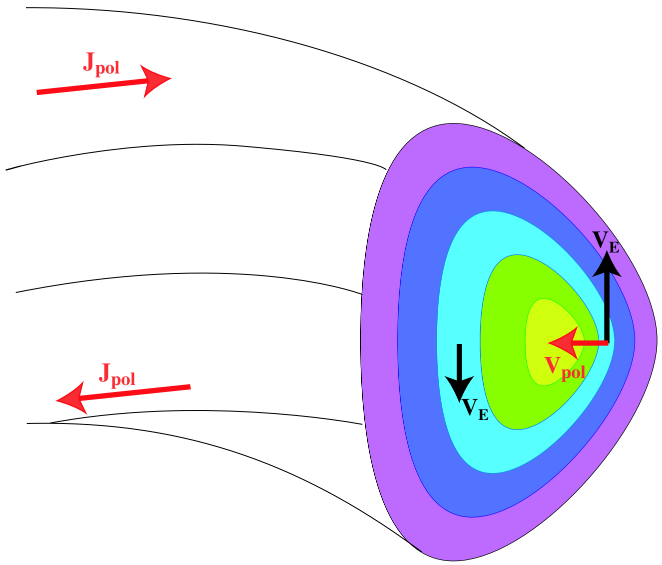
Part II: Drift (2-fluid) model

- Role of island propagation
- Numerical solutions
- Microscopic islands
- Intermediate regime
- Subsonic islands

The Polarization current plays an important role in the growth of thin islands

$$\mathbf{v}_{\rm pol} = \frac{\mathbf{b}}{\Omega_i} \times \frac{d\mathbf{V}_E}{dt} = \frac{1}{\Omega_i B}\frac{d\mathbf{E}_\perp}{dt}.$$

- $\boldsymbol{\nabla} \cdot \mathbf{v}_{\rm pol} \neq 0$, so $J_\|$ needed for neutrality.
- Compare $v_{\rm pol} \sim \phi'\phi'' \sim \phi/L^3$ with $v_D \sim p' \sim p/L$.



The properties of magnetic islands depend on their propagation velocity

- The propagation velocity is the difference between the velocity of the island and the velocity of the surrounding plasma.
- The effect of the propagation velocity on the width of the island is described by the generalized Rutherford equation,

$$\tau_R \frac{dW}{dt} \simeq \Delta'(W) + \frac{a_{bs} W}{W^2 + W_d^2} + a_{\text{pol}} \frac{(V - V_{EB})\,(V - V_i)}{W^3}.$$

- The calculation of the island propagation velocity is the subject of active research (Mikhailovskii *et al.* PoP'00, Shaing PoP '03, '04, Ottaviani, Porcelli and Grasso PRL '04).
- Here we consider the two-fluid theory in slab geometry.

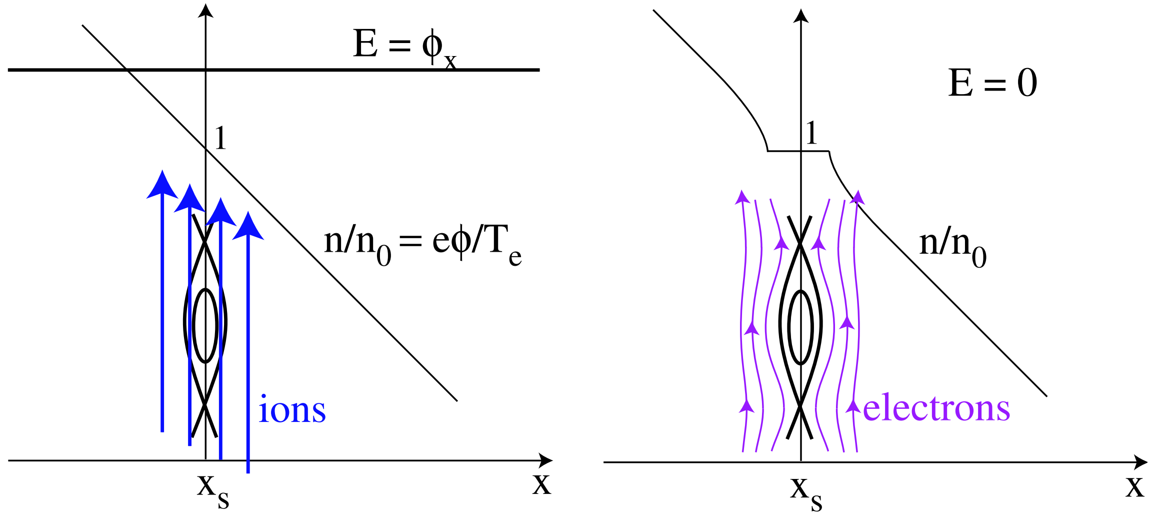
Profile flattening determines the natural velocity

- Electrons are frozen-in:

$$v_{\rm ph} = v_{\perp e} = v_{*e} + v_E.$$

This implies that if the background ($k_y = 0$) profiles are unaffected by the island, so that in the frame where $E = 0$,

$$v_{\rm ph} = v_{*e}.$$



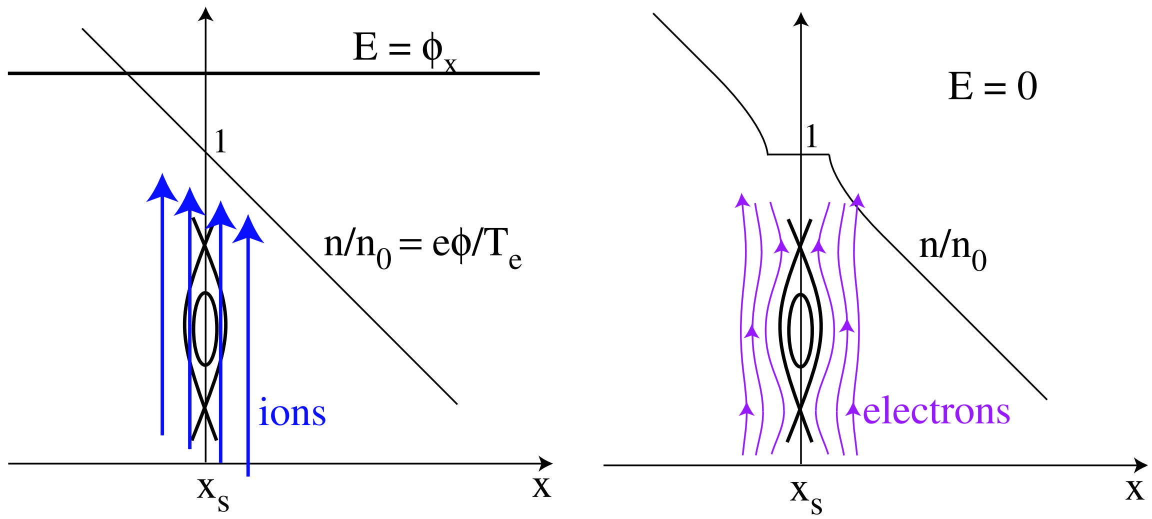
Profile flattening determines the natural velocity

- Momentum transport determines the electric field:

$$\mu_i \nabla^2 v_{\perp i} + \mu_e \nabla^2 v_{\perp e} = 0.$$

Assuming that $\mu_e = 0$, momentum transport implies that $v_{\perp i}$ is continuous. Thus,

$$v_{\rm ph} = v_{\perp e}^{\rm in} = v_E^{\rm in} = v_{\perp i}^{\rm out} = v_{*i}.$$

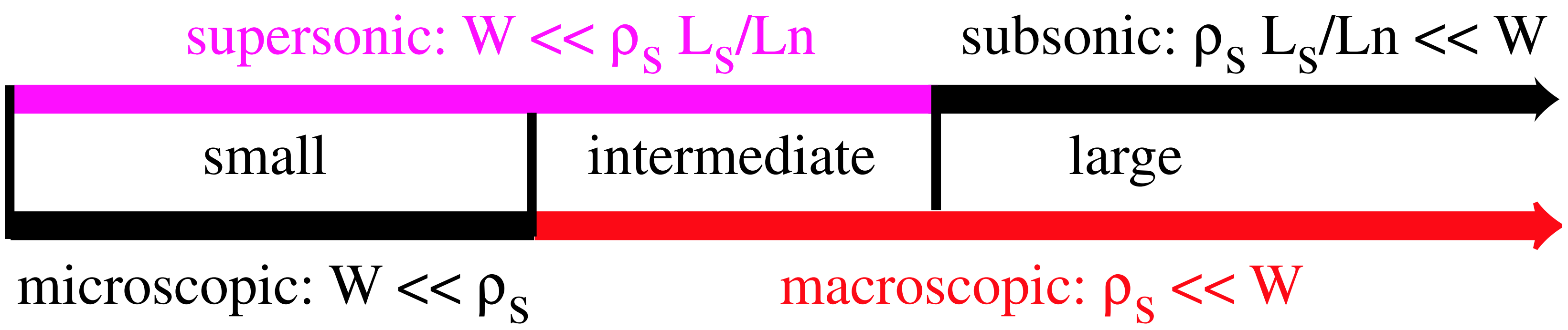


The ion-acoustic wave flattens the density

- Scott, Hassam and Drake (PF '85) have shown that the sound wave flattens the density in the *subsonic regime*:

$$\omega_* < k_{||} c_s, \qquad \text{or for} \qquad W > \rho_s L_s / L_n .$$

- This gives rise to three regimes for weak shear:



Model

We use a 2D drift model in slab geometry. The model advances

- the density n using the continuity equation;
- the electrostatic potential φ using the vorticity equation, including FLR terms and electron viscosity;
- The parallel ion velocity v using the parallel momentum-conservation equation;
- The helical flux ψ using Ohm's law;
- The electron temperature using the electron heat equation.

Steady-state solutions

The transport ordering reduces the steady-state system to the following three equations:

- Equilibrium equation

$$\nabla^2 \varphi = K(\varphi) + H(\psi) \tag{1}$$

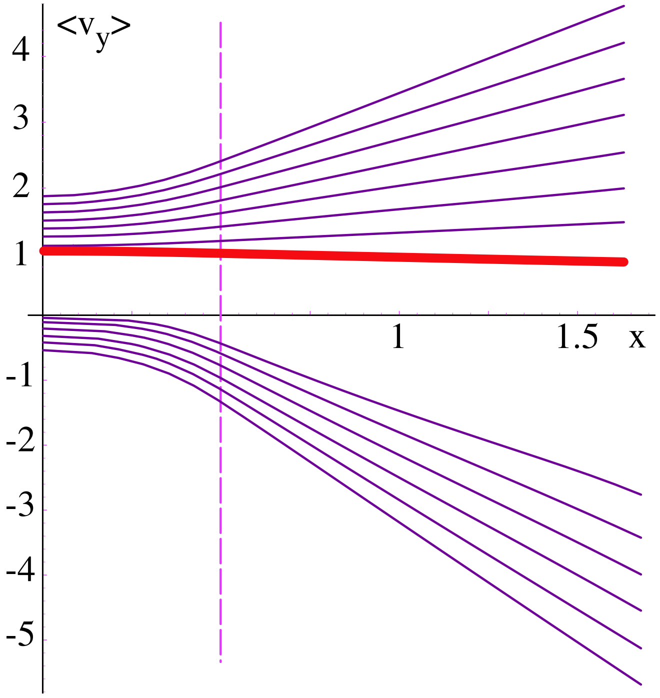
- Transport equations:

$$\frac{dH}{d\psi} = F(\varphi, \psi) \tag{2}$$

$$\frac{dK}{d\varphi} = G(\varphi) \tag{3}$$

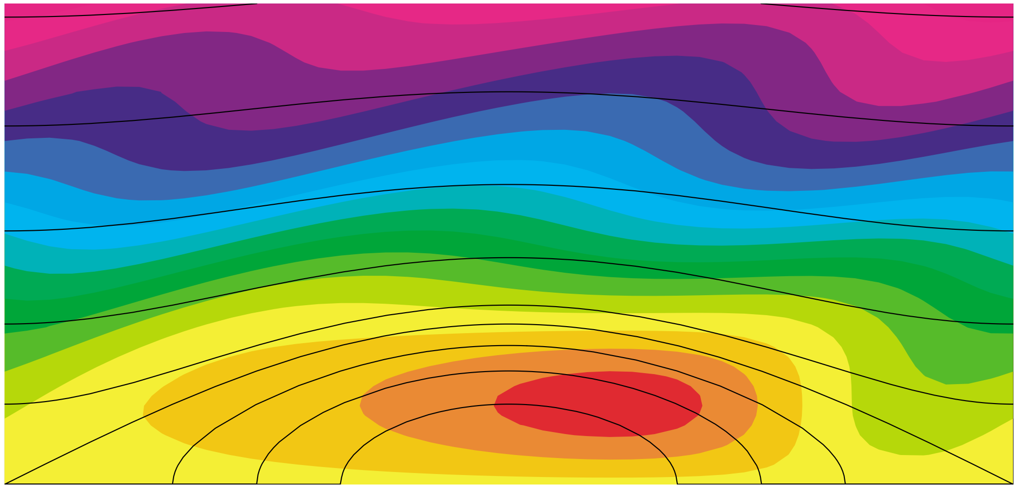
We find the natural velocity iteratively

We specify $v_y = \partial_x \varphi$ at each y, and determine the natural value of v_y by requiring that the torque vanish, $\lim_{x \to \infty} \mu \langle v_y' \rangle = 0$.



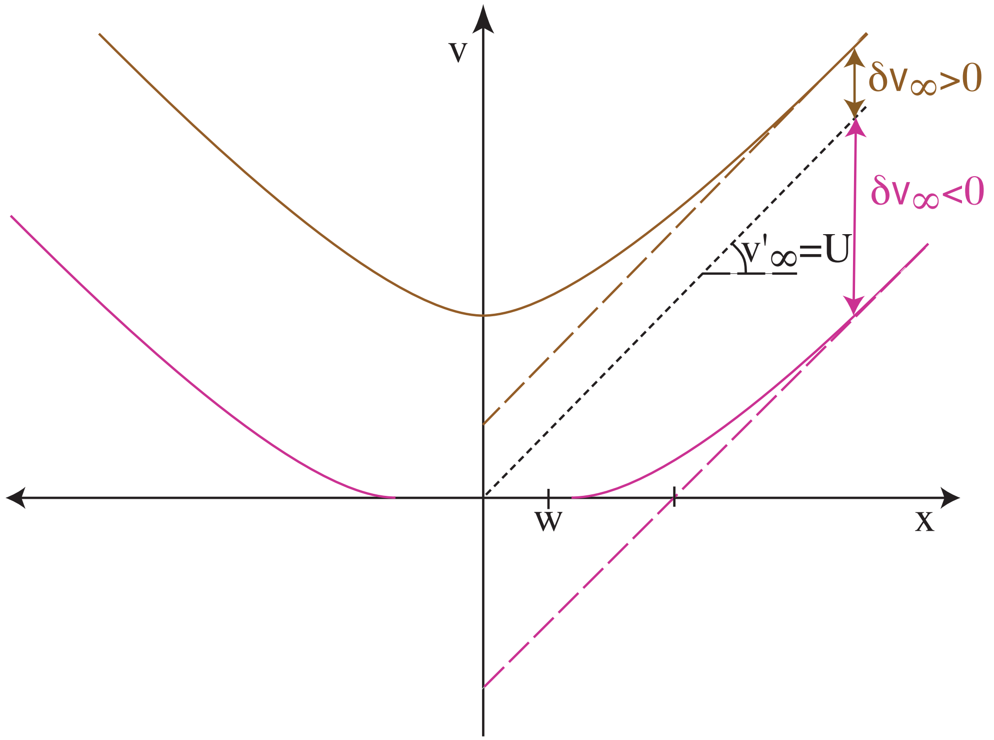
For $0 < \omega < \omega_{*e}$, the island excites drift waves

- The island emits a bow wave as it rotates.
- The island acts as a cavity resonator for the drift waves.
- Convection cells inside the island may act as ball bearings,

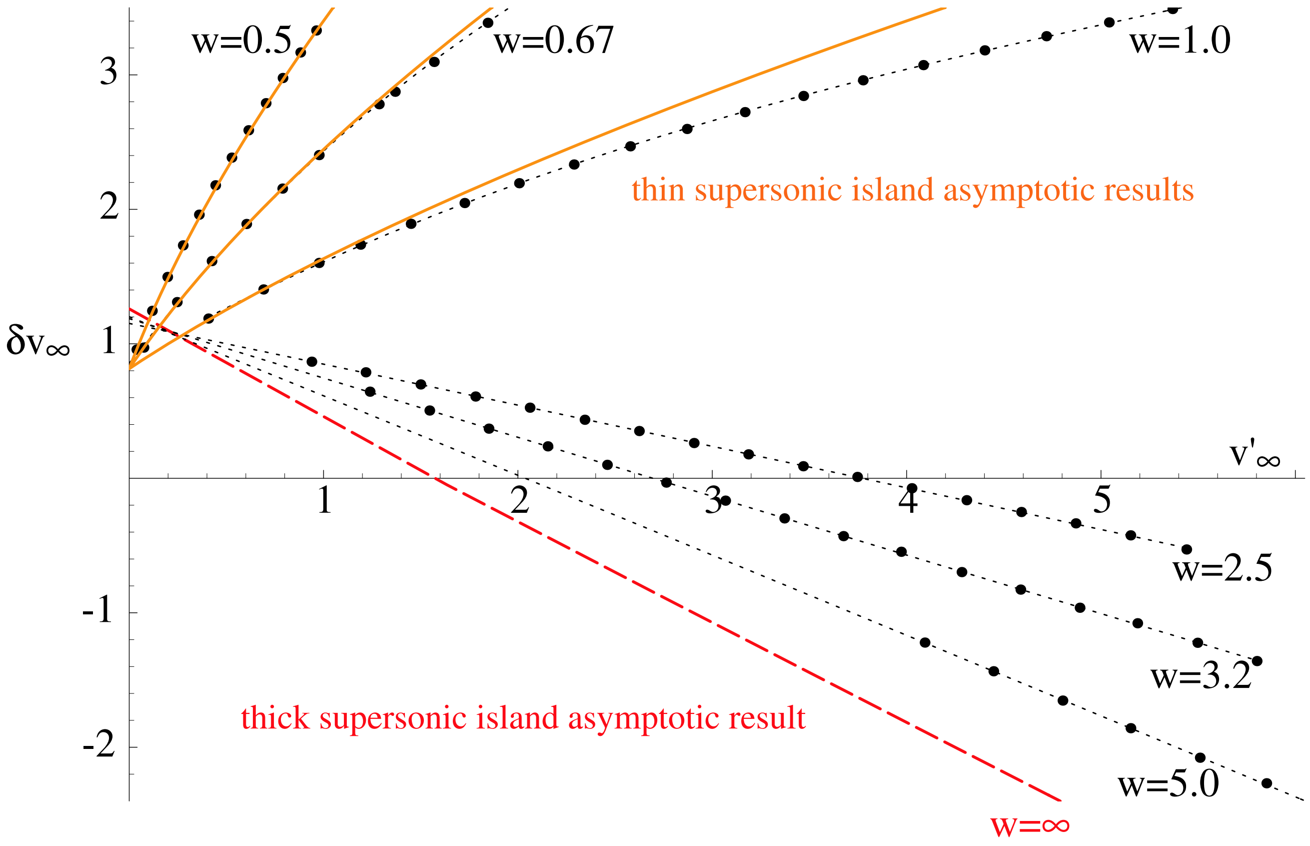


The slip velocity measures island permeability

- $\delta V_\infty > 0$ indicates the plasma is streaming through the island.
- $\delta V_\infty < 0$ indicates the island is flattening the velocity profile.



Islands become impermeable for $W > \rho_s$



Slip velocity as a function of the torque.

For large islands ($W \gg \rho_s$), the propagation is driven by the Reynolds stress

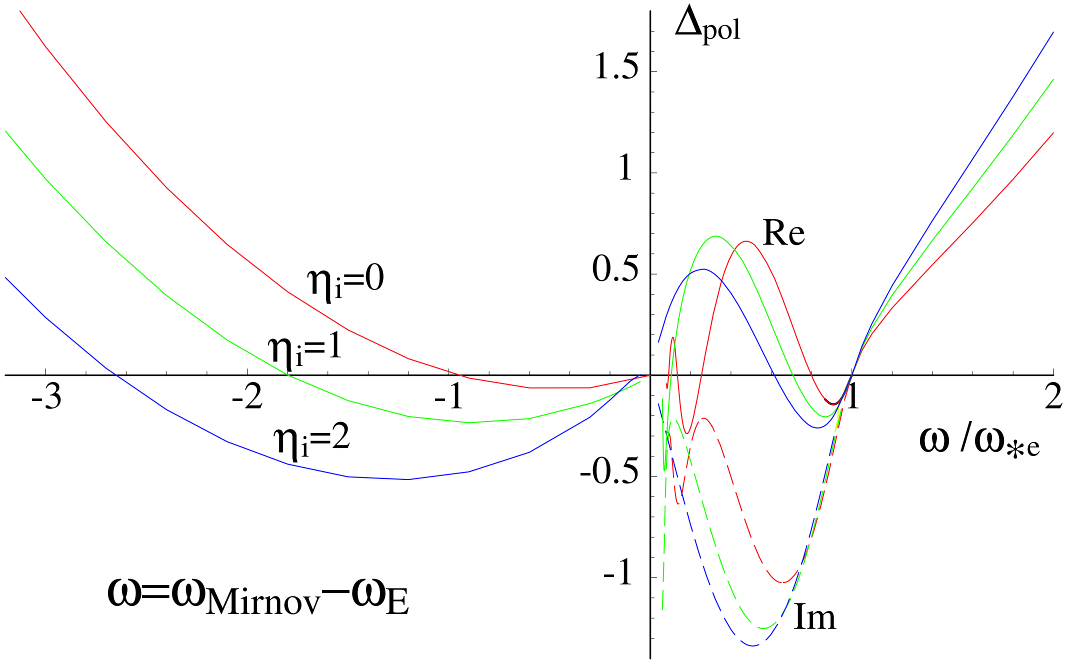
- For large islands, $\varphi = \Phi(\psi)$ and $n = N(\psi)$ outside the separatrix.
- The "Mach" number $M = d\Phi/d\psi$ is determined by

$$\text{“}\nabla^2\text{”}\left(M + \frac{\mu_{\perp i}\tau - \mu_{\perp e}}{\mu_{\perp i} + \mu_{\perp e}}\,M_*\right) = \frac{R(M, M_*)}{M(M - M_*) + s^2},$$

  where $M_*(\psi) = dn/d\psi$ and $R(M, M_*)$ is proportional to the Reynolds stress, $\langle \mathbf{v} \cdot \nabla \mathbf{v} \rangle$.
- For supersonic islands ($W < \rho_s L_s/L_n$), the density gradient persists and the appropriate boundary condition is $M = M_*$ at the separatrix.

Finite ion temperature opens a stable rotation window

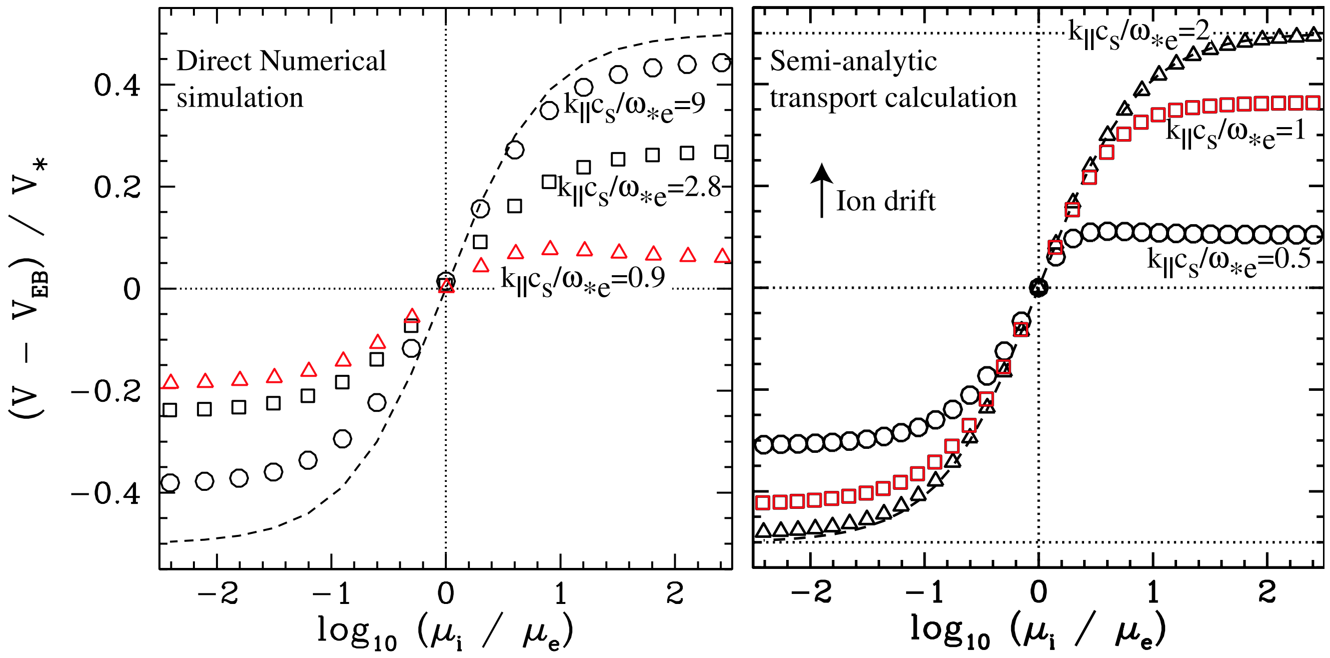
The linear solution of ion gyrokinetic equation shows that the frequency band $\omega_i < \omega < 0$ is stable.



Waelbroeck, Connor and Wilson '01; Fitzpatrick and Waelbroeck '05

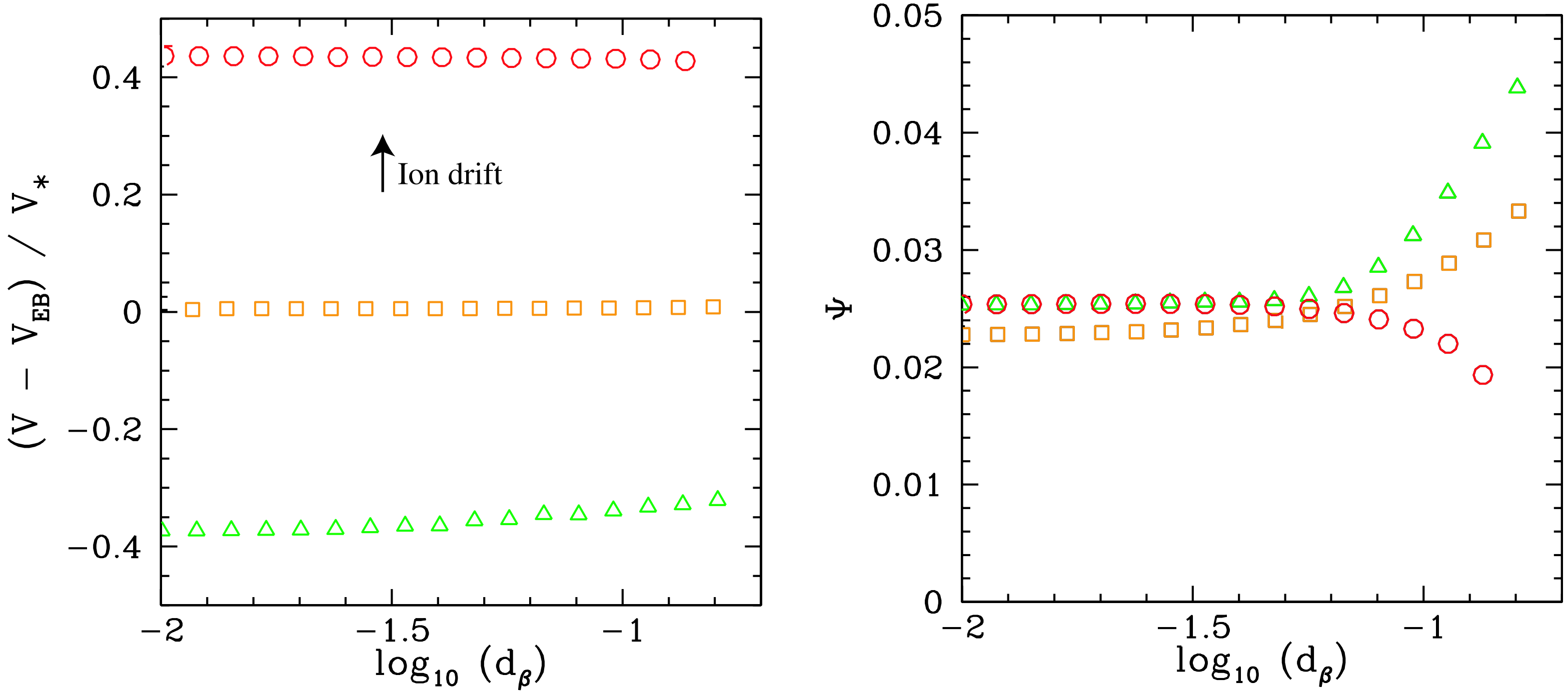
The semi-analytic results agree with direct numerical simulations for subsonic islands

- For subsonic islands, the density is flattened. The appropriate boundary condition is $M = M_* = 0$ at the separatrix.



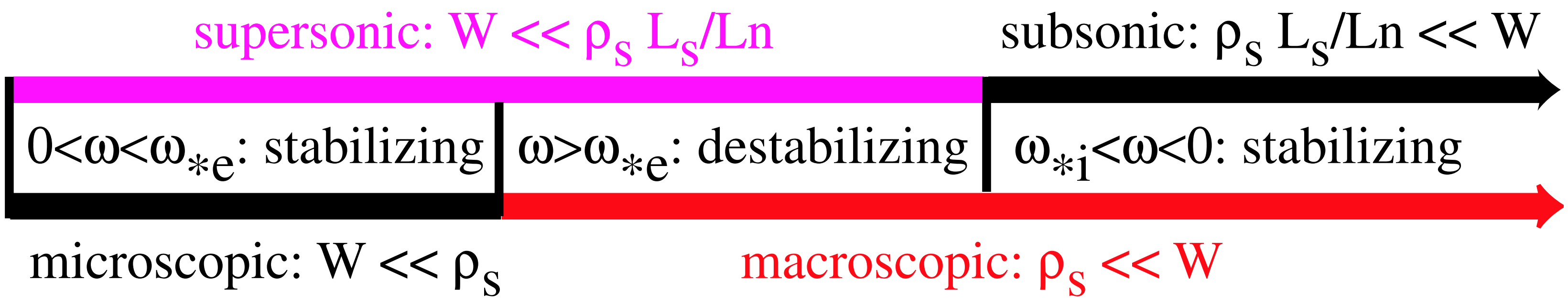
Fitzpatrick and Waelbroeck '04

The effect of the polarization current is stabilizing for subsonic islands with $\mu_i > \mu_e$



velocity and amplitude as a function of $d_\beta = \rho_s/\sqrt{1+\tau}$ for various ratios of μ_i/μ_e.

Summary


supersonic: W << ρs Ls/Ln
subsonic: ρs Ls/Ln << W
0<ω<ω*e: stabilizing
ω>ω*e: destabilizing
ω*i<ω<0: stabilizing
microscopic: W << ρs
macroscopic: ρs << W